$p\overline{p}$ final state interaction in $J/\psi \rightarrow \gamma p\overline{p}$*

ZENG Zhuo-Quan(曾卓全)[1;1)] GUO Xin-Heng(郭新恒)[1;2)]
SHEN Peng-Nian(沈彭年)[4,2,5;3)] DING Yi-Bing(丁亦兵)[3;4)]

1 College of Nuclear Science and Technology, Beijing Normal University, Beijing 100875, China
2 Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China
3 College of Physical Sciences, Graduate University of Chinese Academy of Sciences, Beijing 100049, China
4 College of Physics and Technology, Guangxi Normal University, Guilin 541004, China
5 Center of Theoretical Nuclear Physics, National Laboratory of Heavy Ion Accelerator, Lanzhou 730000, China

Abstract: The phenomenon of the near $p\bar{p}$-threshold enhancement observed in the $J/\psi \rightarrow \gamma p\overline{p}$ decay is studied by using the enhancement factor method with a simpler one-pion-exchange potential between p and $\bar{p}$. The Jost function caused by the mentioned potential is perturbatively calculated in the zero-th order approximation, and the corresponding enhancement factor is obtained. It is found that such a final state interaction offers an important contribution to the decay width near the $p\bar{p}$-threshold, although it is not large enough. To explain the decay data, a phenomenological factor $G(p)$ with the form of $285500/(m_\pi^2+p^2)$ should be introduced. A further calculation including the p-dependent bare T-matrix, a more realistic $N\bar{N}$ potential and the contribution from the higher-order wave functions would provide a better understanding of the decay data and even the existence of the baryonium $p\bar{p}$. The near $p\bar{p}$-threshold behavior of the decay width in the $J/\psi \rightarrow \pi^0 p\overline{p}$ process is also discussed.

Key words: near-threshold enhancement, final state interaction, one pion exchange potential, Jost function

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1 Introduction

In recent years, the BES collaboration has observed a near-threshold enhancement in the proton-antiproton ($p\overline{p}$) invariant mass spectrum of the radiative decay $J/\psi \rightarrow \gamma p\overline{p}$ [1, 2]. As suggested by BES Collaboration, this enhancement can be fitted with an S-wave Breit-Wigner resonance function with a resulting peak mass, $M=1861^{+6}_{-13}(\text{stat})^{+7}_{-26}(\text{syst})$ MeV, and a narrow width, which is smaller than 38 MeV at the 90% confidence level [2]. In fact, the Belle Collaboration also reported similar phenomena, namely the enhancement in the $p\overline{p}$ invariant mass distribution near $2m_p$ in $B^+ \rightarrow K^+ p\overline{p}$ [3] and $\overline{B}^0 \rightarrow D^0 p\overline{p}$ [4] decays. The narrow peak mass is slightly lower than the $p\overline{p}$ threshold, 1876.54 MeV. These observations may shed new light on the investigation of the $N\overline{N}$ interaction and the binding behavior of $N\overline{N}$ at low energy.

Some physicists regard this near $p\overline{p}$ threshold enhancement as a resonance state. Datta and Donnell used a toy model [5] to describe the enhancement, assuming the formation of a baryonium ${}^1S_0$ state. Loiseau and Wycech [6] used a coupled-channel scattering length approximation with Paris $N\overline{N}$ potential to study this phenomenon. They thought that the observed $p\overline{p}$ structure is due to the strong attraction between p and $\bar{p}$ in the ${}^1S_0$ state, so that a near-threshold quasi-bound state could possibly be formed.

Other physicists attribute this enhancement to the effect of the $p\overline{p}$ final state interaction. Kerbikov et al. [7] used scattering length approximation to show that this enhancement may come from the fi-

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1) E-mail: zengzhuoquan@foxmail.com
2) E-mail: xhguo@bnu.edu.cn
3) E-mail: shenpn@ihep.ac.cn
4) E-mail: ybding@gucas.ac.cn

nal state interaction. Using the same approximation, Bugg [8] claimed that this enhancement is due to the cusp effects. In terms of the K-matrix method, Zou et al. [9] found that the one pion exchange potential, as a part of the final state interaction, can offer an important contribution to this enhancement. Further employing a more realistic Jülich $N\overline{N}$ potential in the distorted wave Born approximation, Sibirtsev et al. [10] and Haidenbauer et al. [11] announced that this enhancement can be understood by the final state interaction between p and $\bar{p}$. Until now, there has been no definite explanation for such an enhancement.

In this work, we use the enhancement factor method [12] to further study the effect of the final state interaction and to understand the physics behind the decay data of the $J/\psi\to\gamma p\overline{p}$ process more clearly. We will employ a simpler one-pion-exchange potential for the $N\overline{N}$ interaction. Then, based on the perturbation theory, we analytically derive the enhancement factor and compare it with the experimental data.

The paper is organized as follows. In Section 2, the enhancement factor method for studying the near-threshold behavior observed in the $J/\psi\to\gamma p\overline{p}$ process is briefly introduced. A conclusion is given in the last section.

2 Final state interaction in $J/\psi\to\gamma p\overline{p}$

The first two papers dealing with final state interaction in a nuclear reaction were completed by Watson [13] and Migdal [14]. According to their analysis, one can use the following formula to deal with the S-wave final state interaction between p and $\overline{p}$ in the $J/\psi\to\gamma p\overline{p}$ decay as

$$T^{(\mathrm{FSI})}\propto T\cdot f_0(p), \tag{1}$$

where p is the module of the proton momentum in the $p\overline{p}$ center-of-mass frame, $T^{(\mathrm{FSI})}$ and T are the T-matrix with and without final state interaction, respectively, and f_0 is the S-wave scattering amplitude due to the final state interaction.

Taylor derived the following T-matrix relation [12]

$$T^{(\mathrm{FSI})}=\frac{T}{J(p)}, \tag{2}$$

where $J(p)$ is the Jost function. For the S partial wave and small p, the above two formulae agree with each other since if we retain only the terms in the order of p, $f_0=\frac{1}{2\mathrm{i}p}\left(\frac{J_0(-p)}{J_0(p)}-1\right)$ which is in proportion to $\frac{1}{J_0(p)}$, and $J_0(p)$ is the S wave Jost function. However, in general, the two formulae are different, and Eq. (2) is more preferable, because for large p, $J_0(p)\to 1$, and hence, $T^{(\mathrm{FSI})}\to T$ as one would expect.

In terms of Eq. (2), we are able to study the effect of the final state interaction between N and $\bar{N}$ in the $J/\psi\to\gamma p\overline{p}$ decay.

The enhancement factor is defined $\mathcal{F}(p)$ as,

$$\mathcal{F}(p)=\frac{1}{|J(p)|^2}. \tag{3}$$

As pointed by Tayor [12], for a repulsive potential

$$\left(\frac{\partial V}{\partial r}\leqslant 0,\ \forall r\in\mathbb{R}\right),\ \mathcal{F}(p)\leqslant 1,$$

and for an attractive potential

$$\left(\frac{\partial V}{\partial r}\geqslant 0,\ \forall r\in\mathbb{R}\right),\quad \mathcal{F}(p)\geqslant 1.$$

Now, we investigate the effect of the one-pion-exchange potential between N and $\bar{N}$ in the enhancement factor. The Lagrangian for the π-N interaction reads

$$\mathcal{L}=\mathrm{i}g_{NN\pi}\overline{\psi}_N\left(\vec{\tau}\cdot\vec{\phi}_\pi\right)\gamma^5\psi_N, \tag{4}$$

where ψ_N and $\vec{\phi}_\pi$ are the nucleon and the pion fields, respectively, $\vec{\tau}$ describes the Pauli matrix, and $g_{NN\pi}$ denotes the nucleon-pion coupling constant with $\frac{g^2_{NN\pi}}{4\pi}=13.67$. In the tree diagram approximation, one can obtain the following one-pion-exchange potential between N and $\overline{N}$

$$V_{N\overline{N}}(\vec{r})=-\frac{g^2_{NN\pi}}{4\pi}\cdot\frac{m^3_\pi}{12m^2_N}\frac{\mathrm{e}^{-m_\pi r}}{m_\pi r}\times(\vec{\sigma}_1\cdot\vec{\sigma}_2)(\vec{\tau}_1\cdot\vec{\tau}_2), \tag{5}$$

where the nucleon mass m_N and the pion mass m_π take the values of 939.0 MeV and 138.0 MeV, respectively. Denoting the potential between N and $\overline{N}$ with isospin I and spin S by $V^{(I,S)}_{N\overline{N}}(\vec{r})$, we have

$$V^{(0,0)}_{N\overline{N}}(\vec{r})=-30.64\ \frac{\mathrm{e}^{-m_\pi r}}{m_\pi r}, \tag{6}$$

and

$$V^{(1,0)}_{N\overline{N}}(\vec{r})=V^{(0,1)}_{N\overline{N}}(\vec{r})=10.22\ \frac{\mathrm{e}^{-m_\pi r}}{m_\pi r}, \tag{7}$$

for various combinations of I and I.

The Jost function is calculated in the lowest order of approximation. As well known, the integral equation for the radial regular solution of a two-body system can be written as

$$\phi_{l,p}(r)=\hat{j}_l(pr)+\frac{2m}{p}\int_{\xi=0}^{r}\mathrm{d}\xi\ g_{l,p}(r,\xi)V(\xi)\phi_{l,p}(\xi), \tag{8}$$

where the Green function $g_{l,p}(r,\xi)$ is written as

$$g_{l,p}(r,\xi) = \left(\hat{j}_l(pr)\hat{n}_l(p\xi) - \hat{n}_l(pr)\hat{j}_l(p\xi)\right) \times \Theta(r-\xi), \quad (9)$$

with $\hat{j}_l(z)$ and $\hat{n}_l(z)$ being the Riccati-Bessel function [12] and $\Theta(r-\xi)$ the Heaviside step function. The Jost function for the l partial wave can be expressed by

$$J_l(p) = 1 + \frac{2m}{p}\int_{r=0}^{+\infty} \mathrm{d}r \hat{h}_l^{+}(pr)\ V(r)\ \phi_{l,p}(r), \quad (10)$$

where $\hat{h}_l^{\pm}(z)$ is the Riccati-Hankel function [12].

In the zero-th order approximation, the regular solution is simplified into $\phi_{l,p}^{(0)}(r) \approx \hat{j}_l(pr)$. Accordingly, the Jost function for the l partial wave can be described by

$$J_l^{(0)}(p) \approx 1 + \frac{2m}{p}\int_{r=0}^{+\infty} \mathrm{d}r \hat{h}_l^{+}(pr)\ V(r)\ \hat{j}_l(pr). \quad (11)$$

Consequently, the Jost function for the S-wave can be given as

$$J_0^{(0)}(p) = 1 + \frac{2m}{p}\int_{p=0}^{+\infty} \mathrm{d}r \mathrm{e}^{\mathrm{i}pr}\ V(r)\ \sin(pr). \quad (12)$$

If the potential in Eqs. (6) and (7) take the form of $V(r) = V_0\ \frac{\mathrm{e}^{-m_\pi r}}{m_\pi r}$ with V_0 being the potential strength, then

$$\begin{aligned} J_0^{(0)}(p) &= 1 + \frac{2mV_0}{p}\ \int_{r=0}^{+\infty} \mathrm{d}r \mathrm{e}^{\mathrm{i}pr}\ \frac{\mathrm{e}^{-m_\pi r}}{m_\pi r}\ \sin(pr) \\ &= 1 + \frac{mV_0}{pm_\pi}\ \tan^{-1}\left(\frac{2p}{m_\pi}\right) \\ &\quad + \mathrm{i}\frac{mV_0}{2pm_\pi}\ \lg\left(1+\frac{4p^2}{m_\pi^2}\right). \end{aligned} \quad (13)$$

The enhancement factor due to the one-pion-exchange interaction between N and $\overline{\mathrm{N}}$ is plotted in Fig. 1. It can be seen that for the $(I,\ S)$=(1, 0) and (0, 1) states, the scattering cross-section is reduced due to a repulsive potential between N and $\overline{\mathrm{N}}$, while for the $(I,\ S)$=(1, 1) and (0,0) states, the cross-section is enhanced because of an attractive potential. Moreover, since $V_{\mathrm{N}\overline{\mathrm{N}}}^{(0,0)}(\vec{r}) = 9\ V_{\mathrm{N}\overline{\mathrm{N}}}^{(1,1)}(\vec{r})$ due to different spin-isospin quantum number of the $\mathrm{N}\overline{\mathrm{N}}$ system, the final state interaction in the $^{11}S_0$ state would be much stronger than that in the $^{33}S_1$ $\mathrm{N}\overline{\mathrm{N}}$ state. Thus, if the $p\overline{p}$ system in the $J/\psi\to\gamma p\overline{p}$ decay is in the $^{11}S_0$ state, potential $V_{\mathrm{N}\overline{\mathrm{N}}}^{(0,0)}(\vec{r})$ would enhance the scattering cross-section near the $p\overline{p}$ threshold. This deduction is compatible with the experimental data.

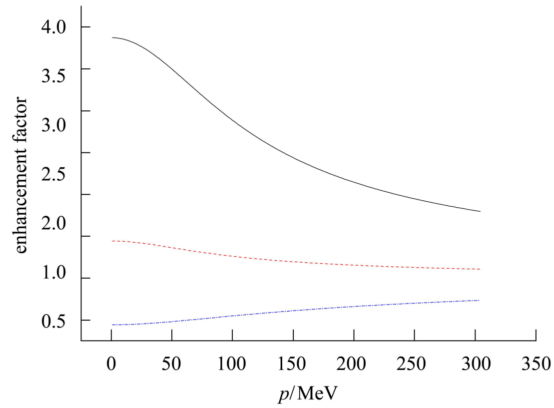

Fig. 1. (color online) The enhancement factor in an S-wave. The black solid, red dashed and blue dotted-dashed curves stand for the cases with $V_{\mathrm{N}\overline{\mathrm{N}}}^{(0,0)}(\vec{r})$, $V_{\mathrm{N}\overline{\mathrm{N}}}^{(0,1)}(\vec{r})$ and $V_{\mathrm{N}\overline{\mathrm{N}}}^{(1,1)}(\vec{r})$, respectively.

The squared amplitude $|A(J/\psi\to\gamma p\overline{p})|^2$ can be extracted from the experimental data [1, 2] by using the partial decay width of the decay process [10, 11]

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}m_{p\overline{p}}} = \frac{\left(M_0^2 - m_{p\overline{p}}^2\right)\sqrt{m_{p\overline{p}}^2 - 4m_p^2}}{(2\pi)^3 \cdot 16\ M_0^3}\left|A(J/\psi\to\gamma p\overline{p})\right|^2, \quad (14)$$

and plotted in Fig. 2. In the figure, the physical quantity in the abscissa is

$$m_{p\overline{p}}^2 - 2m_p = 2\left(\sqrt{m_p^2+p^2} - m_p\right), \quad (15)$$

with $m_{p\overline{p}}$ being the invariant mass of $p\overline{p}$, and the black squares representing the $|A(J/\psi\to\gamma p\overline{p})|^2$ values extracted from the data.

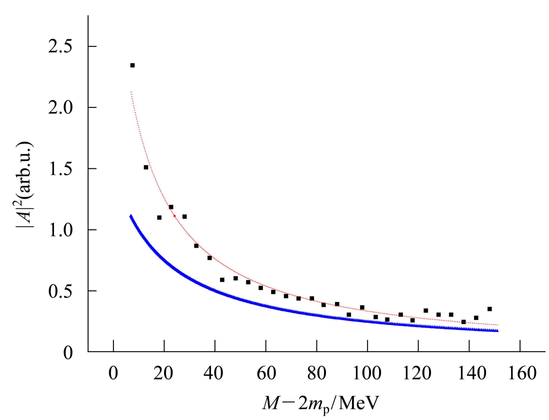

Fig. 2. (color online) The squared amplitude of $J/\psi\to\gamma p\overline{p}$. The thin red and thick blue curves denote the fitted result and the enhancement factor, respectively.

According to Eq. (2), the phenomenological expression of $|A^{\mathrm{ph}}(J/\psi\to\gamma p\overline{p})|^2$ can formally be expressed by

$$\left|A^{\mathrm{ph}}(J/\psi\to\gamma p\overline{p})\right|^2 \propto \frac{G(p)}{|J(p)|^2}, \quad (16)$$

where $G(p)$ is a phenomenological factor respected to p. By fitting the experimental data shown in Fig. 2, the deduced factor $G(p)$ can be written as

$$G(p)=\frac{285500}{m_\pi^2+p^2}. \quad (17)$$

The fitted curve is shown in Fig. 2 by the thin red curve. For comparison, the enhancement factor $\mathcal{F}(p)$ in Eq. (3) is also plotted in Fig. 2 by the thick blue curve. Apparently, the phenomenological factor $G(p)$ should include the effects from the momentum-dependent T matrix in which the effect of the final state interaction is not considered, the higher-order $p\overline{p}$ wave functions and the incomplete $N\overline{N}$ potential used in the work.

The same treatment can also be applied to study the effect of the final state interaction between $p\overline{p}$ in the $J/\psi \to \pi^0 p\overline{p}$ decay. In the strong interaction, the isospin of the system is conserved. Since $I_{\pi^0}=1$ and $I_{J/\psi}=0$, we have $I_{p\overline{p}}=1$. According to the generalized Pauli exclusion principle, the $p\overline{p}$ system satisfies $(-1)^{I+1+S+1+L+1}=-1$. This means that $L+S+I$ =even, the $p\overline{p}$ system is in the $^{33}S_1$ state. According to the one pion exchange potential $V_{N\overline{N}}^{(1,1)}(\vec{r})$ shown in Fig. 1, the near-threshold enhancement of the $^{33}S_1$ state is much less than that of the $^{11}S_0$ state. This explains why the near-threshold enhancement in this decay process is not observed in experiment.

3 Conclusion

In this work, the enhancement factor method with a simpler one pion exchange potential between p and $\overline{p}$ is applied to study the near-threshold enhancement phenomenon observed in the $J/\psi\to\gamma p\overline{p}$ decay. The Jost function is perturbatively calculated in the zero-th order approximation. It is found that the final state interaction from the one-pion-exchange potential between p and $\bar{p}$ offers an important contribution to the decay width near the $p\bar{p}$ threshold, although it is not large enough. In order to fit the experiment data, a phenomenological factor $G(p)$ with the form of $285500/(m_\pi^2+p^2)$ is introduced. We attribute this factor to: (1) The T matrix without final state interaction for such a decay process is p dependent. However, in this work, and many previous papers as well, a constant has been used. (2) The effects of higher-order wave functions in the $p\overline{p}$ system may provide contributions that exceed those from the zero-th order. (3) A simpler one-pion-exchange potential between N and $\overline{N}$ is used. Clearly, it is an incomplete form for the $N\overline{N}$ interaction, the σ-meson exchange potential should be included at least. Therefore, a further investigation with a p dependent T without final state interaction, the effect from the higher-order wave functions and a more realistic $N\bar{N}$ potential, such as the Bonn $N\overline{N}$ potential and the Paris $N\overline{N}$ potential, is preferable.

Finally, we would emphasize that if the future calculation mentioned above can fit the obtained phenomenological factor $G(p)$, it means that the near-threshold enhancement observed in the $J/\psi \to \gamma p\overline{p}$ decay is caused by the final state interaction between p and $\bar{p}$. In terms of the employed more realistic $N\bar{N}$ potential, if a reliable calculation for the binding behavior of a N-$\bar{N}$ system can produce a bound state or resonance in the corresponding I-S case considered in the $J/\psi\to\gamma p\overline{p}$ decay process, it would imply that the observed enhancement is due to the effect of the $N\bar{N}$ bound state or resonance. Namely, one can extract the binding properties of baryonium $p\bar{p}$ from the $J/\psi\to\gamma p\overline{p}$ decay data.

Moreover, we have semi-quantitatively explained why the enhancement near the $p\overline{p}$ threshold in the $J/\psi \to \gamma p\overline{p}$ decay process was observed, but not in the $J/\psi\to\pi^0 p\overline{p}$ decay process.

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