Nucleon effective mass in symmetric nuclear matter from the extended Brueckner-Hartree-Fock approach^{*}

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Abstract: We have calculated the nucleon effective mass in symmetric nuclear matter within the framework of the Brueckner-Bethe-Goldstone (BBG) theory, which has been extended to include both the contributions from the ground-state correlation effect and the three-body force (TBF) rearrangement effect. The effective mass is predicted by including the ground-state correlation effect and the TBF rearrangement effect, and we discuss the momentum dependence and the density dependence of the effective mass. It is shown that the effect of ground state correlations plays an important role at low densities, while the TBF-induced rearrangement effect becomes predominant at high densities.

Key words: nucleon effective mass, extended BHF approach, ground state correlations, TBF rearrangement effect

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1 Introduction

A nucleon in nuclear medium behaves as a quasiparticle characterized by its effective interaction and effective mass. The nucleon interacts with the surrounding nucleons, which in turn act back on itself, contributing to self-energy that drives its effective mass to depart from its bare mass. Changes in the surrounding nuclear environment, such as temperature and density, have an impact on the effective interaction known as the G-matrix in the Brueckner theory, and consequently modify the momentum dependence of the single-particle (s.p.) potential. As a result, the nucleon effective mass, which reflects the non-locality of the s.p. potential, changes according to the perturbations of the nuclear environment. The nucleon effective mass provides information about how a nucleon responds to perturbations in the nuclear medium.

The nucleon effective mass is closely related to many interesting physical quantities and nuclear phenomena [1], such as the physics of stellar collapse [2], the damping of nuclear excitations, the giant resonances [3], the properties of nucleon superfluidity in nuclear matter [4], nucleon-nucleon (NN) cross sections in dense nuclear matter [5], the nuclear level density around the Fermi surface, the dynamical evolution of heavy-ion collisions (HIC) at intermediate energies [6] and so on. For example, in HIC at intermediate energies, the nucleon density in the reaction region can reach a value of more than two times the saturation density of nuclear matter, and the momentum dependence of the s.p. potential plays a significant role for predicting the transverse flow data.

Up to now, the nucleon effective mass has been studied extensively by various theoretical approaches.

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In Ref. [7], the temperature dependence of the nucleon effective mass is investigated within the Brueckner-Hartree-Fock (BHF) approach without considering the three-body force (TBF) rearrangement contribution and the effect of ground state correlations. Ref. [8] calculated the nucleon effective mass in symmetric nuclear matter by adopting the BHF approach extended to include the effect of ground state correlations. In Ref. [14], the isospindependence of the nucleon effective mass was studied at the lowest order BHF approximation. In Ref. [9], we extended the investigation of the ground state correlation effect to asymmetric nuclear matter and discussed the influence of isospin-dependence on the effect of ground state correlations. In these previous investigations, the TBF was not considered. In Refs. [10, 11], we have studied the TBF rearrangement effect within the Brueckner approach and have shown that the TBF may induce a repulsive and strongly momentum-dependent contribution to the nucleon s.p. potential. However, the effect of ground state correlations was completely ignored in our previous investigations [10, 11].

In the present work, we will extend our previous investigation of nucleon effective mass in symmetric nuclear matter by including both the effect of ground state correlations and the TBF-induced rearrangement contribution within the extended BHF framework.

2 Theoretical approaches

2.1 BHF approximation

The present calculation is based on the Brueckner-Bethe-Goldstone (BBG) theory [9, 12, 13]. The extension of the BBG scheme to include three-body forces can be found in Ref. [15]. Here we give a brief review for clarity. The starting point of the BBG scheme is the Brueckner reaction G matrix, which satisfies the following Bethe-Goldstone (BG) equation,

$$G(\rho,\omega) = v_{\rm NN} + v_{\rm NN} \sum_{k_1k_2} \frac{|k_1k_2\rangle Q(k_1,k_2) \langle k_1k_2|}{\omega - \epsilon(k_1) - \epsilon(k_2) + i\eta} G(\rho,\omega),$$
(1)

where $k_i \equiv (\vec{k}_i, \sigma_i, \tau_i)$ denotes the momentum, the z-component of spin and isospin of a nucleon, respectively. ω is the starting energy and $Q(k_1, k_2) =$ $[1-n(k_1)][1-n(k_2)]$ is the Pauli operator which prevents the two intermediate nucleons from being scattered into the states below the Fermi sea. n(k) is the Fermi distribution at zero temperature. ρ denotes the nucleon number density. The BHF single particle (s.p.) energy is given by $\epsilon(k) = \hbar^2 k^2/(2m) + U^{\rm bhf}(k)$. In solving the BG equation for the *G*-matrix, we adopt the continuous choice for the s.p. potential $U^{\rm bhf}(k)$ since it has been proved to provide much faster convergence of the hole-line expansion for the energy per nucleon of nuclear matter up to high densities than the gap choice [16]. In addition, under the continuous choice, the s.p. potential describes physically at the lowest BHF level the nuclear mean field felt by a nucleon in nuclear medium [17] and is calculated from the real part of the on-shell *G*-matrix, i.e.

$$U^{\rm bhf}(k) = \sum_{k'} n(k') \operatorname{Re} \langle kk' | G(\epsilon(k) + \epsilon(k')) | kk' \rangle_A .$$
(2)

The realistic nucleon-nucleon (NN) interaction $v_{\rm NN}$ in our calculation contains two parts, i.e., the Argonne V_{18} (AV_{18}) two-body interaction [18] plus the contribution of a microscopic TBF [15].

2.2 TBF and its rearrangement effect

The TBF adopted in the present calculation was originally proposed by Grangė et al. [19] based on the meson-exchange current approach. The parameters of the TBF, i.e., the coupling constants and the form factors, have been determined in Ref. [15] from the one-boson-exchange potential (OBEP) model to meet the self-consistent requirement with the adopted AV_{18} two-body force.

In our BHF calculation, the TBF contribution has been included by reducing the TBF to an equivalently effective two-body interaction V_3^{eff} according to the standard scheme as described in Ref. [19]. In *r*-space, the equivalent two-body force V_3^{eff} reads:

$$\langle \vec{r}_{1}' \vec{r}_{2}' | V_{3}^{\text{eff}} | \vec{r}_{1} \vec{r}_{2} \rangle$$

$$= \frac{1}{4} \text{Tr} \sum_{n} \int d\vec{r}_{3} d\vec{r}_{3}' \phi_{n}^{*} (\vec{r}_{3}')$$

$$\times (1 - \eta(r'_{13})) (1 - \eta(r'_{23})) W_{3} (\vec{r}_{1}' \vec{r}_{2}' \vec{r}_{3}' | \vec{r}_{1} \vec{r}_{2} \vec{r}_{3})$$

$$\times \phi_{n} (\vec{r}_{3}) (1 - \eta(r_{13})) (1 - \eta(r_{23})). \qquad (3)$$

It is worth stressing that the effective force V_3^{eff} depends strongly on density. It is the density dependence of the V_3^{eff} that induces a TBF rearrangement contribution to the s.p. properties in nuclear medium within the BHF framework. According to Ref. [11], the TBF rearrangement contribution to the s.p. potential is given by:

$$U^{\rm tbf} \approx \frac{1}{2} \sum_{k_1 k_2} n_{k_1} n_{k_2} \left\langle k_1 k_2 \left| \frac{\delta V^{\rm eff}}{\delta n_k} \right| k_1 k_2 \right\rangle_A.$$

2.3 Ground state correlation effect

In the spirit of the BBG theory, the mass operator $M(k, \omega)$ can be written as an expansion according to the number of hole lines [8]

$$M(k,\omega) = M_1(k,\omega) + M_2(k,\omega) + \dots, \qquad (4)$$

where k and ω denote the momentum and energy of the nucleon respectively. If k and ω fulfil the energy-momentum relation $\omega = \epsilon(k) = \hbar^2 k^2 / 2m +$ $\operatorname{Re}M(k,\omega(k)), M(k,\omega)$ is then called the on-shell mass operator. The mass operator is a complex quantity, and its real part is identified as the s.p. potential felt by a nucleon in nuclear medium, i.e., $U(k,\omega) = \operatorname{Re}M(k,\omega)$. The first term $M_1(k,\omega)$ in the expansion is the mass operator at the lowest BHF level and its real part corresponds to $U^{\rm bhf}$. The second term $M_2(k,\omega)$ is the so-called Pauli rearrangement contribution [8] which stems from the density dependence of the Brueckner G-matrix and describes the effect of ground state two-hole-line correlations on the s.p. potential. The Pauli rearrangement contribution $M_2(k,\omega)$ in symmetric nuclear matter can be calculated as follows:

$$M_{2}(k,\omega) = \frac{1}{2} \sum_{k'k_{1}k_{2}} [1 - n(k')]n(k_{1})n(k_{2})$$
$$\times \frac{|\langle kk'|G|k_{1}k_{2}\rangle_{A}|^{2}}{\omega + \epsilon(k') - \epsilon(k_{1}) - \epsilon(k_{2})}, \qquad (5)$$

where $n(k) = \theta(k - k_{\rm F})$ is the Fermi distribution at zero temperature.

By including both the TBF-induced rearrangement contribution and the ground state correlation effect, we get finally the full s.p. potential in nuclear matter:

$$U(k) = U^{\rm bhf}(k) + U_2(k) + U^{\rm tbf}(k).$$
 (6)

The nucleon effective mass $m^*(k)$ which describes the non-locality of the s.p. potential U(k) felt by a nucleon propagating in nuclear medium, is defined as

$$\frac{m^*(k)}{m} = \frac{k}{m} \left[\frac{\mathrm{d}\,k}{\mathrm{d}\,\epsilon(k)} \right],\tag{7}$$

or equivalently

$$\frac{m^*(k)}{m} = \left[1 + \frac{m}{k} \frac{\mathrm{d}U(k)}{\mathrm{d}k}\right]^{-1},\tag{8}$$

where $\epsilon(k) = \hbar^2 k^2 / 2m + U(k)$ is the s.p. energy. *m* is the bare mass of nucleon.

3 Results and discussion

In order to see clearly the ground state correlation effect and the TBF rearrangement effect, we display in Fig. 1 the U_2 (a) and $U^{\text{tbf}}(k)$ (b) vs. the momentum k at four different densities $\rho = 0.085, 0.17,$ 0.34 and 0.5 fm⁻³. It is seen that both the ground state correlations and the TBF rearrangement effect lead to a repulsive contribution to the s.p. potential. The ground state correlation plays an important role mainly in the low momentum region around and below the Fermi surface and its effect is crucial for satisfactorily reproducing the depth of the empirical nuclear optical potential [8] and for restoring the Hugenholtz-Van Hove theorem [9, 20], whereas, the TBF rearrangement effect turns out to be quite different. At low densities, the TBF rearrangement effect is fairly small and increases quickly as the density increases. The TBF rearrangement contribution is shown to be an monotonically increasing function of momentum, and remarkably enhances the momentum dependence of the s.p. potential at high momentum and high densities. This is supported by our previous results in Ref. [10], where we found that the TBF rearrangement contribution is crucial for reducing the disagreement between the BHF s.p. potential (at large-densities and high-momentum) in symmetric matter, and the parameterized potential by Danielewicz for describing the elliptic flow data [21]. In Fig. 2 we make a comparison between the lowest order BHF s.p. potential $U^{\text{bhf}}(k)$ (a) and the full s.p. potential U(k) (b) at four typical densities $\rho = 0.085, 0.17, 0.34$ and 0.5 fm⁻³. At low momentum, $U^{\rm bhf}(k)$ becomes more attractive as the density increases up to 0.34 fm^{-3} . At high enough densities (for example, $\rho = 0.5 \text{ fm}^{-3}$), $U^{\text{bhf}}(k)$ may become less attractive, mainly due to the TBF effect via its modification of the G-matrix at the lowest BHF level [11]. Compared with the s.p. potential $U^{\rm bhf}(k)$ at the lowest order BHF approximation, the full potential U(k) is much more repulsive in the whole momentum range, especially at high densities due to the extra repulsion from the ground state correlation effect and the TBF rearrangement effect. It is worth mentioning that the repulsive contributions from U_2 and $U^{\text{tbf}}(k)$ become stronger quickly at higher densities, so that the full potential U(k) for 0.5 fm⁻³ stavs above that for 0.085 fm^{-3} in the whole range of momentum, which is not the case for $U^{\text{bhf}}(k)$.

The nucleon effective mass $m^*(k)$ is determined by the momentum dependence of the s.p. potential.





In order to see clearly the TBF rearrangement effect and the ground state correlation effect, we show in Fig. 3 the effective mass $m_{\rm BHF}^*(k)/m$, $m_{12}^*(k)/m$ and $m^*(k)/m$, respectively calculated at three different approximations. By approximating the s.p. potential at the lowest BHF approximation, i.e. by assuming $U(k) = U^{\text{bhf}}(k)$, we get the nucleon effective mass $m_{\text{BHF}}^*(k)$ at the BHF level (left panel of Fig. 3). By including the ground state correlation effect but without the TBF rearrangement contribution, i.e., by assuming $U(k) = U^{\text{bhf}}(k) + U_2$, we obtain $m_{12}^*(k)$ (middle panel). Finally, by using the full s.p. potential of Eq. (6), we get the full effective mass $m^*(k)$ (right panel). The momentum dependence of the effective mass is characterized by a peak around the Fermi surface due to the high probability amplitude for particle-hole excitations near the Fermi surface [8]. By comparing the results in the left and middle panels, it is seen that the ground state correlations lead to a strong enhancement of the effective mass around the Fermi surface, in agreement with the previous results [8, 9]. The TBF rearrangement effect is quite small at low densities, whereas it reduces the effective mass at high densities remarkably in the whole momentum range considered here, since it provides a strongly momentum dependent contribution to the s.p. potential. The reduction due to the TBF rearrangement effect is shown to be more significant at higher densities.



Fig. 2. Momentum dependence of $U_{-}(k)$ and U(k) in symmetric nuclear matter at four typical values of density.

In order to see more clearly the density dependence of the nucleon effective mass, we show in Fig. 4 the nucleon effective mass at the Fermi momentum $k_{\rm F}$ vs. density for the three cases described above. One may notice that there is a competition between the ground state correlation effect which enhances the nucleon effective mass and the TBF rearrangement effect which decreases the nucleon effective mass. Without the TBF rearrangement effect (black and red curves in the figure), the effective mass decreases at low densities below and around the saturation density 0.17 fm^{-3} and its density dependence becomes quite weak at high densities. By comparing the black and red curves, it is seen that the ground state correlations lead to an overall enhancement of the effective mass in the whole density range. At low densities, the effective mass is governed mainly by the effect of ground state correlations and inclusion of the TBF rearrangement contribution decreases only slightly the



Fig. 3. Momentum dependence of $m_{BHF}^*(k)/m$, $m_{12}^*(k)/m$ and $m^*(k)/m$ in symmetric nuclear matter at four typical values of density.

effective mass. Inclusion of the TBF rearrangement contribution (green curve) makes the effective mass become a monotonically decreasing function of density in the whole density range considered here. At relatively low densities, the full effective mass m^* is larger than the lowest order BHF one $m^*_{\rm BHF}$, while it turns out to become smaller than the $m^*_{\rm BHF}$ at high enough densities, which implies that the TBFinduced rearrangement effect becomes predominant over the ground state correlation effect at high enough densities.



Fig. 4. Density dependence of the nucleon effective mass m_{BHF}^*/m , m_{12}^*/m and m^*/m at Fermi momentum k_{F} .

4 Summary

Within the framework of the Brueckner theory, we have calculated the s.p. potential and the nucleon effective mass in symmetric nuclear matter by taking into account the ground state correlation effect and the TBF rearrangement effect. It is shown that both

the effects of ground state correlations and the TBFinduced rearrangement lead to extra repulsive corrections to the s.p. potential, and play an important part in reliably and realistically predicting the s.p. potential and the nucleon effective mass within the Brueckner approach. The ground state correlations modify the s.p. potential, mainly at low momentum around and below the Fermi surface, and it strongly enhances the effective mass around the Fermi momentum. The repulsive contribution induced by the TBF rearrangement effect to the s.p. potential increases quickly as the momentum and density increase. The TBF rearrangement effect results in an overall reduction of the effective mass in the whole momentum region. At low densities, the TBF rearrangement contribution turns out to be reasonably small and the effect of ground state correlations governs the density dependence of the effective mass around the Fermi surface. As the density increases, the TBF rearrangement effect becomes significant quickly, and as a consequence it may become dominant in determining the effective mass and the density dependence of the effective mass at high enough densities. Due to the competition between the TBF rearrangement effect and the ground state correlation effect, the full effective mass is larger than the lowest order BHF one $m_{\rm BHF}^*$ at low densities, while it becomes smaller than the $m_{\rm BHF}^*$ at high densities. The predicted s.p. potential and nucleon effective mass provide microscopic inputs for transport models for heavy ion collisions and are expected to be important for improving the phenomenological nucleon-nucleon effective interaction (such as Skyrme interaction) in nuclear medium.

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