## Research on the detuning system of a cooling electron beam for the dielectronic recombination experiment at CSRm

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Abstract: A storage ring equipped with an electron cooler is an ideal platform for dielectronic recombination (DR) experiments. In order to fulfill the requirement of DR measurements at the main Cooler Storage Ring, a detuning system for the precision control of the relative energy between the ion beam and the electron beam has been installed on the electron cooler device. The test run using 7.0 MeV/u  $C^{6+}$  beam was performed with recording the Schottky spectra and the ion beam currents. The influence of pulse heights and widths of the detuning voltage on the ion beam was analyzed. For the small pulse height, the experimental results from the Schottky spectra were in good agreement with the theoretical results. The frequency shift in the Schottky spectra was significantly reduced for the short pulse width. For the large pulse height, an oscillation phenomenon was observed and some effective ways to reduce the oscillation were pointed out. The detailed description of the phenomenon and the theoretical model based on the plasma oscillation is discussed in this paper. The overall results show that the new detuning system works properly, and could fulfill the requirements of future DR experiments.

Key words:detuning system electron cooling Schottky spectrum plasma oscillation, dielectronic recombinationPACS:29.20.db, 29.27.BdDOI: 10.1088/1674-1137/37/1/017004

#### 1 Introduction

Dielectronic recombination (DR) is a fundamental electron-ion collision process [1], and is of great importance in various plasmas [2, 3]. It determines the chargestate distribution of atoms in ionized gases and also the spectrum of electromagnetic radiation emitted by such a gas. DR cross sections and rate coefficients are required for diagnosing the status of plasma. In the past few decades, many experiments have been conducted in an effort to acquire DR rates. Early electron-ion recombination experiments were carried out by applying plasma techniques such as theta-pinch [4]. Since the ion charge states accessible to these experiments were rather limited at the beginning, other experimental access to DR cross sections was pursued. The crossed- and mergedbeam measurements of free electrons and ions has made great progress [5–7]. However, all of these methods have similar defects, such as low counting rate, very limited energy resolution, and high background.

The use of an electron cooler [8, 9] in storage rings for recombination studies has many advantages [10] compared with the traditional methods, such as high luminosity due to the high revolution frequency of ion beams, very low background because of the ultra-high vacuum, and phase-space cooling which improves the energy resolution in the experiments, etc. Such experiments have been carried out at several ion storage ring facilities, e.g., TSR in Heidelberg [11–16], ESR in Darmstadt [17], and CRYRING in Stockholm [18, 19]. Fruitful results on the structure of a few electron ions and QED effects have been obtained by the DR spectroscopy [12–14, and 17] on these facilities. In addition, absolute recombination rate coefficients for astrophysical and other physical plasma applications were determined [15, 16, 18, and 19].

At the end of 2007, the construction of the Cooler Storage Ring (CSR) was completed and the commissioning was successful [20]. Later, a test experiment for the

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detection of the recombined ions was performed at the CSRm [21]. It proves that the measurement of electronion recombination at the CSRm is feasible. In order to well control the relative energy in DR measurements at the CSRm, a detuning system developed in cooperation with the Budker Institute of Nuclear Physics was installed on the electron cooler device [22] of the CSRm. The details of the detuning system and the experimental test results are reported in this paper.

# 2 The related issue of the electron beam energy

DR is a two-step resonant process [1]. The first step is radiation-less capture of a free electron by a resonant process, also called dielectronic capture. The second step, which completes the DR process, is a radiative stabilization process: a photon is emitted leaving the recombined ion in a bound but usually still excited state. The whole process can be expressed as

$$e^{-} + X^{q+} \to (X^{(q-1)+})^{**} \to (X^{(q-1)+})^{*} + hv,$$
 (1)

the DR process which only occurs at specific electron-ion collision energies meets the condition:

$$E_{\rm trans} = E_{\rm n} + E_{\rm rel}, \qquad (2)$$

where  $E_{\text{trans}}$  is the transition energy of two atomic states.  $E_{\text{n}}$  is the binding energy of the lying state of the captured electron.  $E_{\text{rel}}$  is the electron-ion relative (collision) energy.

In the storage ring, the velocity of the electrons is equal to the average velocity of the ions under electron cooling. In order to obtain the well-defined electron-ion collision energies in DR measurements, the electron energy needs to be detuned. However, when the electron energy is changed, the ions are continuously dragged by Coulomb interactions with the electrons until the velocities are matched. To solve the problem, a technique of fast modulation of the electron beam energy is necessary. With this technique, the electron energy can be detuned for a very short time, therefore the drag effect can be neglected.

#### 3 The detuning system and experiment

The working principle of the fast modulation of the electron beam energy is based on the idea of two power supplies connected in series. The 35 kV power supply (PS) produces the constant voltage for the acceleration of the electron beam to the fixed energy. The value of pulse height relative to the cooling point can be changed with the help of an independent power supply. The  $\pm 3$  kV switch PS produces fast switching between two values of electron energy and realizes the detuning energy of the electron beam. The detuning system consists of two subsystems (see Fig. 1). One is located in the high-voltage terminal (HVT) and modulates the energy of the electron beam. The other one is located at the ground potential (ST) and modulates the potential of the electrostatic plates. Electrostatic plates realize the electrostatic bending of the electron beam; therefore the impulses on the HVT and the plates should be synchronized with high accuracy to keep the position of the electron beam at the interaction zone unchanged. A waveform generator produces the synchronizing signal for both systems, and before and after each positive or negative pulse, the waveform generator will output separate standard NIM signals for data analysis, see the description of Fig. 1. The waveform of the pulsed voltage is a rectangular shape with adjustable amplitude, width and polarity. The rising time for the pulse is about



Fig. 1. The detuning system for fast change of the electron beam energy. The high voltage terminal contains all power supplies for the electron beam operation. The heater PS produces cathode heating.

50 microseconds at 400 V pulse voltage (see Fig. 2). The minimum step size of the pulse height is 1 V. The pulse width and the time between two pulses can be adjusted from 1 millisecond to 2500 milliseconds.

The grid and anode PS control the shape of the electron beam and the electron beam current. The suppressor PS controls the collector efficiency and the collector PS absorbs the energy of the electron beam. The switch power supply forms the fast change of the electron beam energy. The ST module controls the voltage of the electrostatic plates for its bending in the toroid part of the magnetic field.



Fig. 2. The upper panel is a designed waveform of the detuning voltage. The lower panel is the measured signal from the output of the detuning system. The rising time is about 50  $\mu$ s at 400 V pulse voltage.

The test experiment was performed at the electron cooler section of the CSRm. The layout of the CSRm is shown in Fig. 3. A beam of 7.0 MeV/u C<sup>6+</sup> was provided by the Sector Focusing Cyclotron and injected into the cooler storage ring. The electron beam in the cooler was guided by a longitudinal magnetic field and collinearly overlapped with the ions in the cooling section, which is 4 meters long. The electron density was set at about  $1.3 \times 10^7 \text{cm}^{-3}$  in the present measurements.

To test the performance of the system, the electron energy was detuned away from the cooling point by positive pulses during the measurements. The drag effect varies depending on the parameter setting, e. g., pulse height and pulse width. In a storage ring, the change of ion revolution frequency, which is in direct proportion to the ion beam velocity, can be monitored by using a Schottky pick-up device [23–25]. The signals from the Schottky device were recorded and analyzed by a Tektronix RSA3408A spectrum analyzer. A DC current transformer (DCCT) was used to monitor the primary ion beam current. A data collecting card (National Instruments USB-6210) combined with commercial software (LabView) was used for data acquisition of the signals from the DCCT.



Fig. 3. Layout of the CSRm. In the DR measurements, the circulating ion beam is merged with the electron beam in the cooling section (EC35). After the downstream dipole magnets, the recombined ions are separated from the primary ion beam and detected by the particle detectors.

#### 4 The results and discussion

The Schottky spectra which were recorded during detuning are shown in Fig. 4, where horizontal and vertical axes of each spectrum are frequency and time, respectively. The pulse width for each voltage was set to 100 ms. In the upper panel of the figure, the data were recorded for the pulse heights of (a) 0 V, (b) 5 V and (c) 10 V. At the cooling point (0 V), the frequency of the ion beam always remained the same, which indicates the detuning system itself has no influence on the ion beam. At the pulse height of 5 V, the frequency of the ion beam was shifted (see peaks in the figure) due to the pulses. This indicates that the energy of the ion beam was changed by the detuned electron beam. At the pulse height of 10 V, the largest shift in the measurements was observed. In the lower part of the figure, the data were recorded for the pulse heights of (d) 20 V. (e) 30 V and (f) 40 V. In this range, the shift of the frequency decreases with the increase of the pulse height. The observed phenomena can be explained by the electron cooling force. The cooling force F increases linearly with the increase of relative velocity between ions and electrons until it reaches its maximum value, and then decreases as  $F \propto 1/(V_e - V_i)^2$ , where  $V_e$  and  $V_i$  are velocities of electrons and ions, respectively.



Fig. 4. The Schottky spectrum recorded with pulse heights varying from 0 to 40 V. The pulse width is 100 ms and the interval between two pulses is 1 s. The central frequencies for the upper part and the lower part of the figure are 17.988 MHz and 17.968 MHz, respectively. The span of the spectrum is 50 kHz.

According to the parameters used in the measurements, the frequency shift has been calculated. When the velocity of the electrons is detuned, the ions will have acceleration

$$a(V_{\rm i}, V_{\rm e}) = -a_0 \cdot \frac{V_{\rm i} - V_{\rm e}}{\left[(V_{\rm i} - V_{\rm e})^2 + V_{\rm eff}^2\right]^{\frac{3}{2}}},\tag{3}$$

here

$$a_0 = 4r_{\rm e}r_{\rm i}n_{\rm e}c^4\eta_{\rm c}\ln\left(\frac{\rho_{\rm max}}{\rho_{\rm L}}\right),\tag{4}$$

where  $V_{\rm eff}$  is the effective spread of the electron velocity,  $n_{\rm e}$  is the number density of the electrons, c is the speed of light,  $\eta_c = 0.025$  is the ratio between the length of interaction zone and the ring circumference.  $\rho_{\rm max}$  is the maximal impact parameter,  $\rho_{\rm L}$  is the radius of Larmor rotation in the magnetic field of the cooling section,  $r_{\rm e}$ and  $r_{\rm i}$  are the classical radii of electrons and ions, respectively.

The initial velocity variation of the ions is zero, we have

$$\Delta V_{i0} = 0, \tag{5}$$

the velocity variation of the ions at any intermediate time can be written as

$$\Delta V_{i(k+1)} = \Delta V_{ik} + a(V_{ik}, V_{ek}) \cdot \Delta t, \qquad (6)$$

the frequency shift in the Schottky spectrum is

$$\Delta f_k = f_0 \eta_{\rm p} \frac{\Delta V_{\rm ik}}{c\beta},\tag{7}$$

here

$$\eta_{\rm p} = \frac{1}{\gamma^2} - \frac{1}{\gamma_{\rm t}^2},\tag{8}$$

where  $f_0$  is the central frequency of the Schottky spectrum,  $\eta_p$  is the mean frequency dispersion function,  $\gamma$  is the relativistic Lorentz factor, and  $\gamma_t$  is the transition point of the storage ring.

The calculated results are shown in Fig. 5 corresponding to the parameters of measurements in Fig. 4. By comparing Fig. 4 and 5, it can be found that the relative shift of the frequencies is in good agreement with the experimental results, and the absolute value of the frequency shifts is about 90% of the experimental values. This indicates that the detuning system works properly in this region of detuning voltages.

To study the influence of the pulse width on the ion beam during detuning, the Schottky spectra for pulses width of 100 ms and 20 ms are compared. The pulse height is 10 V, which produces the most predominant shift in the measurements. The test results are shown in Fig. 6. The frequency shift is significantly reduced for the pulse width of 20 ms (the right panel). Therefore, the influence of detuning on the ion beam can be neglected when the pulse width is small enough.



Fig. 5. The theoretical results of frequency shift under various detuning conditions calculated according to the parameters of measurements in Fig. 4. The horizontal and vertical axes are the frequency and time, respectively.



frequency shift/kHz

Fig. 6. Schottky spectra recorded for pulse width of 100 ms (left) and 20 ms (right), respectively. The pulse height is 10 V and the interval is 1 s. The central frequency for the left panel and the right panel is 17.988 MHz. The span of the spectrum is 50 kHz.



Fig. 7. The recorded Schottky spectrum. The pulse height is 700 V and the interval is 1s for the pulse width of 100 ms (left) and 20 ms (right), respectively. The central frequency for the upper part and the lower part of the figure is 17.988 MHz. The span of the spectrum is 50 kHz.

Generally, the required small relative energy in the centre of mass frame corresponds to a large pulse height of detuning in the DR experiments, e.g., 50 eV relative energy corresponding to the pulse height of about 900 V for a 7.0 MeV/u ion beam. The large pulse height test was carried out up to 1000 V. In principle, the cooling force can be neglected when the relative energy is large. Therefore, the velocity of the ion beam should not be significantly changed for large pulses. Unexpectedly, oscillations in the Schottky spectrum were observed when the large pulse heights were applied. A typical result is shown in Fig. 7 where the pulse height is 700 V. The pulse widths from left to right are 100 ms and 20 ms, respectively. The observed oscillations are weaker for the pulse width of 20 ms. This suggests that using a short pulse width during detuning is an effective way to reduce the oscillation level when the DR experiment is performed.

To further study the phenomenon, the influence of large pulse height on the ion beam intensity was investigated. The ion beam intensities versus the stored time for pulse heights of 100 V, 200 V, and 400 V are shown in Fig. 8. The vertical axis is logarithmic. In order to facilitate the comparison, the initial intensities are normalized to the same value. For the pulse heights of 100 V and 200 V, the intensities decrease linearly and the loss rates (which are related to the slope of the line) are small. For the pulse height of 400 V, very high loss rate of the ion beam intensity was observed. There is reason to expect that the rapid beam loss is caused by the oscillation observed in the Schottky spectrum. Furthermore, we found that the loss rate of the ion beam decreased when the ion beam current was small, compared to a straight line (dash line in the figure). The result shows that the oscillation is dependent on the ion beam intensity.



Fig. 8. The ion beam intensity vs. time for the pulse height of 100 V, 200 V and 400 V. The pulse width is 20 ms and the interval between two pulses is 100 ms. The vertical axis is logarithmic. The dashed line is only for guidance.

The rapid loss of the ion beam was also observed at CELSIUS and was called "electron heating" by Reistad [26]. The lifetime of a bunched 48 MeV proton beam typically changes from 50–100 s without electron cooling to 0.5–1 s when it is exposed to 100 mA electrons with energy at the cooling point. Furthermore, when the electron beam energy was detuned far away from the cooling point, the lifetime of the proton beam became even shorter. The author considered the phenomenon as

a single-particle instability which may be due to the excitation of resonances by non-linear electrical fields from the electron beam. However, the difference is that the oscillations only appear when the electron energy is detuned away from the cooling point with a large pulse height in the CSRm.

To explain the electron heating phenomenon, Parkhomchuk proposed another hypothesis [27]. The theoretical model to describe the electron heating is based on two-beam-plasma oscillation. In the cooling section, the equation of motion for plasma oscillations can be written as:

$$\frac{\mathrm{d}^2 x_{\mathrm{e}}}{\mathrm{d}t^2} = -\frac{e}{m} E_{\mathrm{p}},\tag{9}$$

$$\frac{\mathrm{d}^2 x_{\mathrm{i}}}{\mathrm{d}t^2} = -\frac{\mathrm{Z}e}{M} E_{\mathrm{p}},\tag{10}$$

here

$$E_{\rm p} = 4\pi e \left( n_{\rm e} x_{\rm e} - n_{\rm i} x_{\rm i} \right), \tag{11}$$

where  $E_{\rm p}$  is the electric field of plasma oscillations acting on both beams, e is the electric charge, Z is the atomic number, m and M are the effective mass of the electron and the ion,  $n_{\rm i}$  is the number density of the electron and ion beams, respectively. The oscillation equation can be written in the form

$$\frac{\mathrm{d}^{2}E_{\mathrm{p}}}{\mathrm{d}t^{2}} = -(\omega_{\mathrm{e}}^{2} + \omega_{\mathrm{i}}^{2})E_{\mathrm{p}} = -\omega_{\mathrm{p}}^{2}E_{\mathrm{p}}, \qquad (12)$$

where  $\omega_{\rm e}$  and  $\omega_{\rm i}$  are plasma frequency of electron and ion beams, respectively. The ion beam runs for many turns in the storage ring and interacts with a fresh electron beam at each turn. Therefore, at the entrance of the cooling section, the ion beam has the coordinates  $x_{\rm i}(0)$  and initial velocity  $dx_{\rm i}(0)/dt$ , the electron beam has  $x_{\rm e}(0)=0$  and  $dx_{\rm e}(0)/dt=0$ .

The self-consistent solution can be written as:

$$\begin{pmatrix} x_{i} \\ dx_{i}/dt \end{pmatrix} = A(t) \times \begin{pmatrix} x_{i} \\ dx_{i}/dt \end{pmatrix}_{0}.$$
 (13)

With specific initial conditions, the determinant (det) of the matrix A(t) can be calculated

$$|A| = 1 - 2 \frac{\omega_{\rm e}^2 \omega_{\rm i}^2}{\omega_{\rm p}^4} (1 - \cos(\omega_{\rm p}\tau)) + \frac{\omega_{\rm e}^2 \omega_{\rm i}^2 \tau}{\omega_{\rm p}^3} \sin(\omega_{\rm p}\tau), \quad (14)$$

where  $\tau$  is the time of interaction in the cooling section (in the ion beam system).

In Fig. 9 we show the calculated results of det(A) for C<sup>6+</sup> ion beam with number density of 10<sup>5</sup> cm<sup>-3</sup> (solid line) and 10<sup>6</sup> cm<sup>-3</sup> (dash line), respectively. The value of det(A) determines the energy transfer between two beams. If det(A)<1, the electrons absorb the energy from ions, indicating fast cooling of the ion beam. If det(A)>1, the electrons release energy, indicating fast heating of the ion beam. Furthermore, the fast heating

effect is not obvious when the number density of ions is small. With lower ion beam current, the oscillation should be weaker. In this point the theoretical result is in agreement with the test result in Fig. 8. For the factor which provokes the instability at high velocity difference, we can formulate the following hypothesis: when the pulse starts or terminates, the average velocity of the refreshed electrons is instantly changed. If the velocity of the refreshed electrons is faster than the velocity of the cooling electrons. If the velocity of the refreshed electrons is slower than the velocity of the refreshed electrons is slower than the velocity of the cooling electrons, the situation is the opposite. Both situations will change the partial distribution of the electron density, and then lead to plasma oscillations.



Fig. 9. Variation of matrix determinant vs. electron beam current for  $C^{6+}$  ion beam with number density of  $10^5$  cm<sup>-3</sup> (solid line) and  $10^6$  cm<sup>-3</sup> (dash line), respectively.

### 5 Conclusions

The detuning system which varies the relative energy between the ion beam and the electron beam was installed at the CSRm for performing the DR experiment. The system was tested by varying pulse height and pulse width using the stored 7.0 MeV/u  $C^{6+}$  ion beam. For small pulse heights the frequency shifts in the measured Schottky spectra are very close to the results of theoretical values. To test the influence of pulse width on the ion beam quality, the results for the pulse widths of 100 ms and 20 ms with the pulse height of 10 V are compared. The frequency shift was significantly reduced for the pulse width of 20 ms. For large pulse heights, oscillations were observed in the Schottky spectra. According to the test results, the oscillation phenomenon is dependent on the pulse width and the ion beam current. The theoretical model which is based on the plasma oscillation was employed to explain the phenomenon. The results of the theoretical model show that the heating effect

has a strong dependence on the density of the electron beam and the ion beam. According to the test results and the theoretical results, there are three ways to reduce the heating effect: lower the pulse width, adjust the number density of the electrons and limit the ion beam current.

With the newly installed detuning system, the test

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run proves that the DR measurements are feasible at the CSRm. The results discussed in this paper will be helpful for the upcoming DR experiment currently under preparation.

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