An analytical model for the polarization of synchrotron radiation in a soft X-ray region*

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Abstract: Conventionally, the polarization of a synchrotron soft X-ray beam is measured through a polarimeter based on multilayer optical elements. The major drawback of the traditional approach is the difficulty in comparing different configurations due to the misalignment of each incident angle. In this paper, a new analytical model, based on the variation of reflectivity for different incident angles, is established to facilitate the extraction of important polarization-related information, i.e. angular distribution of polarization components, a tiny change of the direction of azimuth rotation axis of polarizer, etc.

Key words: soft X-ray, polarization, multilayer polarizer, stokes vector

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1 Introduction

A synchrotron has a broad spectrum tunability, high brilliance, pure polarization, etc. Among those features, the polarization of synchrotron radiation is unique and can be utilized in novel techniques (e.g., XMCD, PEEM, etc.) to study spin-related scientific issues. As is well known, synchrotron radiation (SR) from a standard wiggler source is completely linearly polarized in the orbital plane of a storage ring, while it is partial linear polarization parallel to this plane just above and below the orbital plane, approaching a completely non-polarized state far from the plane [1]. The polarization characteristic is one of the most important properties of SR, which finds wide applications in spectroscopic and microscopic studies. The experimental determination of SR polarization state is rudimentary before performing measurements on samples using polarization-based techniques. In the soft X-ray (SXR) region, the polarization is measured usually by using multilayer optical elements as polarizers [2], which consist of materials with high reflectivity such as Mo/Si, Ru/Si, Cr/C, W/C and Fe/C, etc. [3]. Wang et al. have developed all kinds of such polarization elements including broadband or narrowband reflective analyzers and transmission retarders [4–6]. However, for narrowband polarization elements the reflective peak is essentially a diffraction peak with a sharp Bragg feature, which leads to a high requirement of alignment accuracy of the polarimeter rotation axis with the optical axis. Kortright et al. even suggested that the incidence angle of the beam onto the multilayer needs to be constant within 300 μrad or better [7]. Actually, nowadays most polarimeters cannot achieve the alignment accuracy [8]. In this paper we will deal with the situation where misalignment happens.

Furthermore, for SR produced by a bending magnet and wiggler, it is essential to measure the angular distribution in the vertical direction of each polarization component, i.e. measuring the degree of linear polarization (LP) with different vertical observation angle Ψ. However, as the Ψ changes in different measurements, the incident light and azimuth angle of the polarizer cannot be restored. Thus, it is inconvenient to adjust the polarizer to compensate the changes of angle Ψ in real experiments. In this paper we propose and establish a new analytical model to facilitate the extraction of polarization-related information under non-coaxial conditions.

2 The analytical method

A theory based on the Stokes vector and the Mueller matrix formalism, which will transform the result of the state of polarization to parameters of polarization ellipse

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for easy understanding, will be employed to represent the polarization of the SR. Traditionally the intensity of light reflected by a polarizer [9, 10] can be expressed as

$$I(\alpha)=\frac{r_{\mathrm{p}}^2}{2}\left((k^2+1)S_0+(k^2-1)(S_1\cos(2\alpha)+S_2\sin(2\alpha))\right), \tag{1}$$

where k is the ratio of polarizer amplitude reflectance for the s to p component, $r_{\mathrm{s}}/s_{\mathrm{p}}$; S_0, S_1, S_2 is the Stokes vector component; α is the azimuth angle of the polarizer. Traditionally, in this formula r_{p} is treated as a constant without considering its variation with respect to the incident angle. In fact, the transformation of vertical observation angle Ψ of light can change the direction of incident light and lead to an inclined angle between the direction of light $\vec{k}$, and α axis, which can be denoted as θ_x and θ_y in $\vec{X}$ and $\vec{Y}$ directions respectively. Because we essentially utilize the one-dimensional diffraction of a multilayer to the light it is reasonable to consider the r_{p} as a Gaussian distribution around the Bragg angle, that is,

$$r_{\mathrm{p}}^2=\frac{1}{\sqrt{2\pi}\sigma}\mathrm{e}^{-\frac{1}{2}\frac{\left(\theta'(\theta_x,\theta_y,\theta,\alpha)-\theta_{\mathrm{m}}\right)^2}{\sigma^2}}, \tag{2}$$

where θ_{m} is the Bragg angle of polarizer, θ' (θ_x, θ_y, θ, α) is the incident angle of light which is the function of θ_x, θ_y and α, while θ is the intersection angle between normal vector of the surface of polarizer $\vec{n}$ and α axis. A coordinate system is introduced with original point O at the intersection of α axis and the surface of polarizer, $\vec{Z}$ axis coinciding with α axis, $\vec{X}$ and $\vec{Y}$ axis vertical and horizontal to the orbital plan of the electron in the storage ring, which is shown in Fig. 1. The normal vector of the surface of a polarizer is written as

$$\vec{n}=(\sin(\theta)\cos(\alpha),\sin(\theta)\sin(\alpha),\cos(\theta))^{\mathrm{T}}. \tag{3}$$

Generally we can regard that the propagation direction $\vec{k}$ of light is coincident with the $\vec{Z}$ axis, but here we must consider the influence of inclination angle between them. We can treat this problem as a rotation of coordination of $O\vec{X}\vec{Y}\vec{Z}$ to $O\vec{X}\vec{Y}\left(-\vec{k}\right)$ by tiny angle θ_x and θ_y in $\vec{X}$ and $\vec{Y}$ directions, respectively. The rotation matrix is given by:

$$R=\begin{bmatrix}\cos(\theta_y) & 0 & \sin(\theta_y)\\ \sin(\theta_x)\sin(\theta_y) & \cos(\theta_x) & -\sin(\theta_x)\cos(\theta_y)\\ \cos(\theta_x)\sin(\theta_y) & \sin(\theta_x) & \cos(\theta_x)\cos(\theta_y)\end{bmatrix}, \tag{4}$$

then $\vec{n}$ in the coordinate $O\vec{X}\vec{Y}\left(-\vec{k}\right)$ can be presented as

$$\vec{n}'=R\vec{n}. \tag{5}$$

The inclined angle between $-\vec{k}$ and $\vec{n}'$ denoted as θ' is

$$\cos(\theta')=(0,0,1)\cdot\vec{n}'. \tag{6}$$

The θ' can be written through combining the above expressions as

$$\theta' = \arccos(\sin(\theta_x)\sin(\theta)\sin(\alpha)+\cos\theta_x\cos(\theta_y)\cos(\theta) -\cos(\theta_x)\sin(\theta_y)\sin(\theta)\cos(\alpha)). \tag{7}$$

A reasonable simplification, based on the fact that θ_x and θ_y are very small, is necessary to give a Taylor expansion to expression (7) at the point θ_x and θ_y equal to zero, then we get

$$\theta'=\theta+\theta_y\cos(\alpha)-\theta_x\sin(\alpha). \tag{8}$$

Now we can write the modified polarization formula as

$$I(\alpha) = \frac{1}{2\sqrt{2\pi}\sigma}\mathrm{e}^{-\frac{1}{2}\frac{(\theta-\theta_{\mathrm{m}}+\theta_y\cos(\alpha)-\theta_x\sin(\alpha))^2}{\sigma^2}}((k^2+1)S_0 +(k^2-1)(S_1\cos(2\alpha)+S_2\sin(2\alpha))). \tag{9}$$

In this expression we neglect the influence of θ_x and θ_y on α in factor $((k^2+1)S_0+(k^2-1)(S_1\cos(2\alpha)+S_2\sin(2\alpha)))$ because of relatively smaller variation compared with r_{p} which presents as an exponential function.

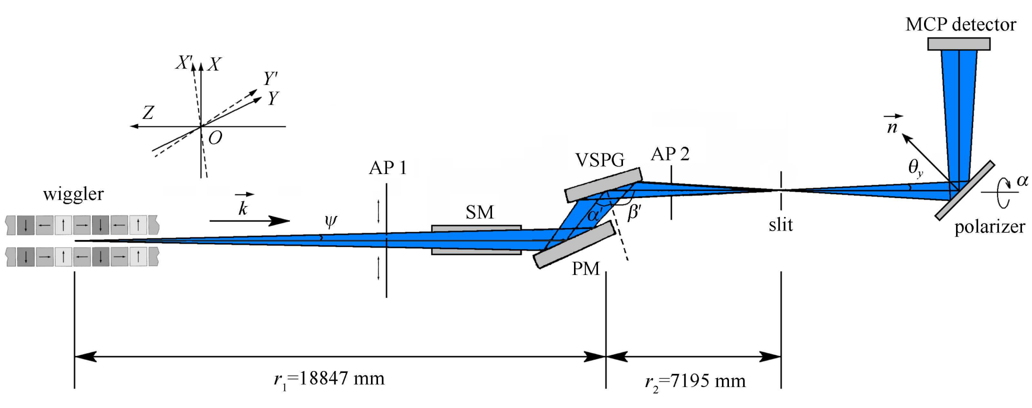

Fig. 1. Schematic diagram of the polarization measurement on the 3W1B Beamline at BSRF.

3 The experiment

The polarization measurement setup was installed at the 3W1B beamline of the Beijing Synchrotron Radiation Facility (BSRF), which is an SXR beamline with a wiggler magnet source, as shown in Fig. 1. The wiggler has 5 periods, each is 30 cm long, the deflection parameter K is 42 and the magnetic field is 1.5 T. The monochromator of this beamline is a varied-space plane grating. The W/B_4C multilayer polarizer optics is manufactured with a short period of 40 layers of W/B_4C alternately deposited on a Si wafer substrate using a high vacuum DC magnetron sputtering system. At its Brewster angle, namely 43.4 degrees, the best k value is available when the photon energy is 704 eV. The polarizations with various Ψ are measured by moving the water-cooled aperture (AP 1 in Fig. 1) vertically by a step of 0.5 mm, corresponding to a Ψ variation of 3.125×10^{-5} rad/step taking into account the distance (16 m) between AP 1 and the centre of the wiggler. At a certain position of AP1, the signal of light is so weak that we can barely detect it; then the position is defined as the origin, i.e. x=0. The width of AP 1 is 1 mm.

4 Results and discussion

First we measure the r_p with various incident angles at α=90° and the energy E=704 eV. As shown in Fig. 2, a simple Gaussian function (Eq. (2)) is adopted to fit the experimental curve. The fitting produces σ of 0.178° (0.0031 rad) and θ_m of 43.37° (0.77 rad). From Eq. (9), it is apparent that the value of S_0, S_1, S_2 and k cannot be resolved simultaneously by merely fitting $I(\alpha)$; however, it is feasible to roughly estimate the values using $I(\alpha)$ with Ψ=0. In case of Ψ=0, the light is fully linearly polarized, with P_L close to 1, while $k^2\approx\dfrac{I(0)}{I(90)}$. However, it is impossible to identify the exact position of AP 1 corresponding to Ψ=0. A compromise is to compare the value of $\dfrac{I(0)}{I(90)}$ at each position of AP 1 and assume the largest one as the ideal k^2. Finally the position is found at x=7.5 with $k^2=\dfrac{I(0)}{I(90)}$=248.6, which is quite large and implies a good quality polarizer with $\dfrac{k^2-1}{k^2+1}$=0.992. Although the actual value of k^2 is larger than the assumed value, the latter is sufficiently large for us to perform further analysis, since the value barely influences the accuracy of the results we will present later.

With the knowledge of parameters σ and k^2, it is possible to fit the experimental curve using Eq. (9). The partially fitted curve shown in Fig. 3 indicates good agreement between experiment and calculation, thus demonstrating the rationality and effectiveness of our model. The Levenberg-Marquardt algorithm is adopted to optimize the parameters in the least squares curve fitting. The correlation coefficient of most fittings is above 0.99.

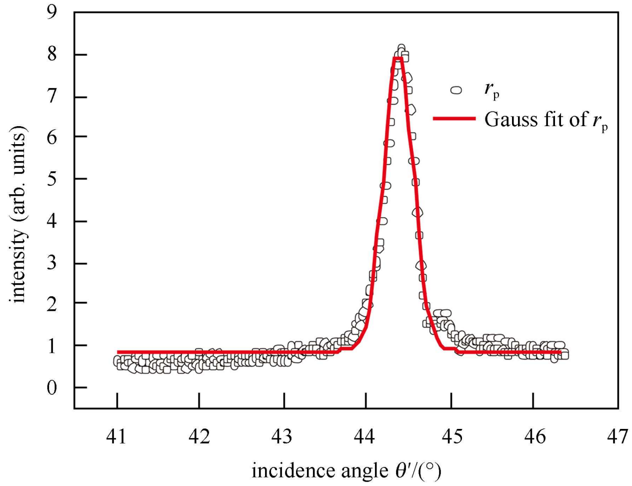

Fig. 2. Reflectivity of vertical polarized light for the polarizer at 704 eV. The open points show the measured data and the solid line is the result of Gaussian fitting.

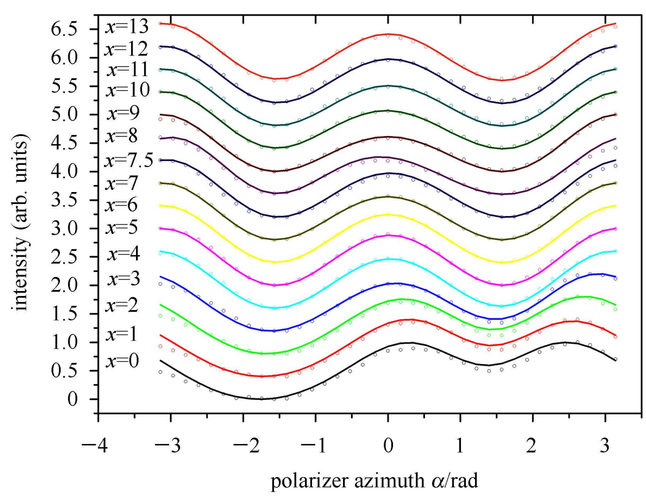

Fig. 3. The azimuth scan of polarizer at various vertical observation angle Ψ. The open points are the measured data and the solid lines are the result of fitting using our model. Here the x stands for the site of AP1.

Using the fitted value of S_0 and S_1 we can obtain the P_1 $\left(P_1=\dfrac{S_1}{S_0}\right)$ at various Ψ positions. The theoretical value of P_1 is interpolated using the method developed by J. Goulon [11] and R. P. Walker [1]:

$$P_1=\frac{K^2_{\frac{2}{3}}(\xi)-\dfrac{3900625E_e^2\psi^2}{(1+3900625E_e^2\psi^2)}K^2_{\frac{1}{3}(\xi)}}{K^2_{\frac{2}{3}}(\xi)+\dfrac{3900625E_e^2\psi^2}{(1+3900625Ee^2\psi^2)}K^2_{\frac{1}{3}(\xi)}}, \qquad (10)$$

where

$$\xi=0.7633587786\frac{E(1+3900625E_e^2\psi^2)^{3/2}}{E_eB}, \qquad (11)$$

in which E is the energy of light, E_e is the energy of electron in the storage ring and B denotes the magnetic field strength of wiggler. In our case, the parameters are E=700 eV, E_e=2.5 GeV and B=1.5 T. In Fig. 4 the curves of theory and experiment are presented, in which we assume Ψ=0 when x=7.5 mm and the value of Ψ increases by 0.03125 mrad as x increases by 0.5 mm. As shown in Fig. 4, in principle the $P_2\left(P_2=\dfrac{S_2}{S_0}\right)$ is 0 for the conventional wiggler source. We can see that the fitting of P_1 is convergent in general but to some extent the calculated curve is narrower than the measured curve, which can be attributed to the non-negligible size of the electron beam in the storage ring. The experimental value of P_2 presents a slight fluctuation around 0, which is in good agreement with the theory.

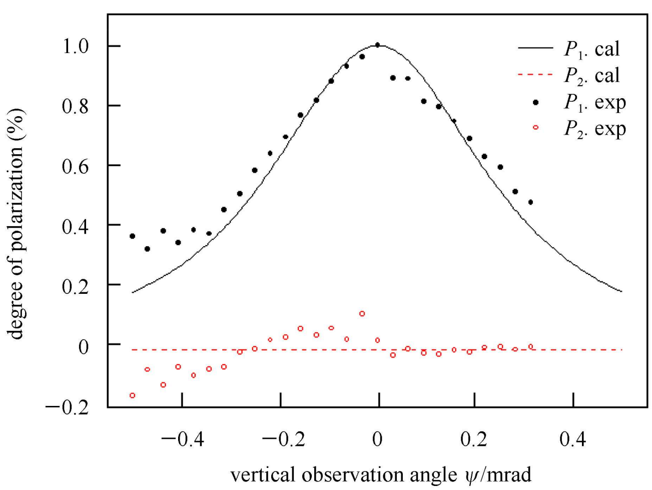

Fig. 4. The distribution of polarization as a function of vertical observation angle Ψ. The open points are the measured data and the solid lines are the result of theoretical calculation.

The θ_x and θ_y are also the function of vertical observation angle Ψ, which is shown in Fig. 5. It is not difficult to understand that the inclined angle in vertical direction θ_y mainly comes from the nonzero vertical observation angle Ψ. However, from Fig. 5 we can see that θ_x also varies with respect to Ψ, which illustrates that the shape of beam spot is inclined. This phenomenon is also observed by naked eyes for the zero-order reflection spot of the grating which probably results from the incline of the first mirror SM. Furthermore, an obviously and regularly oscillation is presented in the curves of θ_x and θ_y which corresponds to the alternation of clockwise and anticlockwise rotation of the polarizer around α axis. We can figure out that the α axis will rotate by a tiny degree when the angle is estimated to be 0.025±0.005 mrad in y direction and 0.05±0.005 mrad in x direction. Therefore we should carry out the experiment only in one direction when it is required to rotate the α axis.

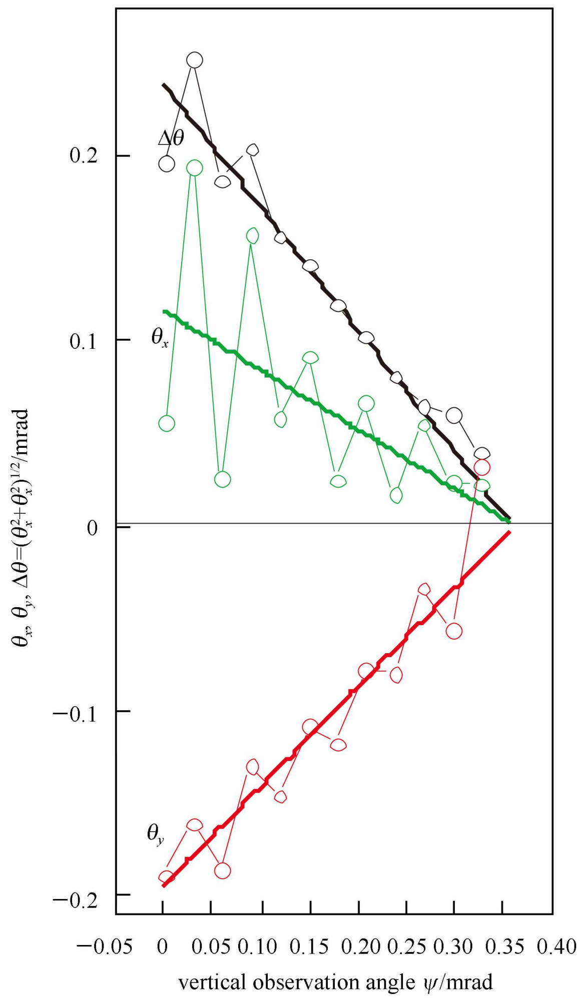

Fig. 5. The change of inclined angle between propagation direction of light and the azimuth axis of polarizer in two directions x and y in the coordination presented in Fig. 1, which is denoted as θ_x and θ_y, respectively, and the total inclined angle denoted as $\Delta\theta$ is shown as a function of vertical observation angle Ψ. The open points are the measured data and the straight lines present the tendency of change.

5 Summary

We have modified the equations for polarization measurements based on the change of reflectivity of light along with the azimuth angle of polarizer α and vertical observation angle Ψ. With the new analytical model, the experimental data show remarkable agreement with the analytical model. The polarization-related information, e.g., inclined angle, configuration of optical elements, etc., is successfully extracted by fitting the model to the experiments, which cannot be resolved by the cumbersome conventional method. The present work may also be extrapolated to polarization issues for other beam light with certain modifications of the reflectivity.

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