

One-loop on-shell renormalization of MSSM vertexes^{*}

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Abstract: The on-shell renormalization scheme for electroweak theory is well studied in the standard model (SM), but a consistent on-shell renormalization scheme for the minimal supersymmetric standard model (MSSM) is still unknown. In the MSSM, we study the on-shell scheme for three vertexes: $Z\bar{l}l$, $W^+\bar{\nu}l$ and $\tilde{L}_i^*\chi_\alpha^0l$, with virtual SUSY particles (chargino, sneutrino, neutralino and slepton) at one-loop order. Instead of the amplitude of a single triangle diagram, the sum of the amplitude of triangle diagrams belonging to one suit can be renormalized in the on-shell scheme. One suit points out that the internal virtual particles are consistent. The zero-momentum scheme is also used for the renormalization. The two schemes can make the renormalized results decoupled, and in the MSSM some of the special characters of the on-shell scheme are shown. This work is propitious in completing the on-shell renormalization scheme in the MSSM.

Key words: on-shell, MSSM, zero-momentum

PACS: 11.30.Er, 12.60.Jv, 14.80.Cp **DOI:** 10.1088/1674-1137/37/5/053101

1 Introduction

As we all know, quantum field theory is perturbative theory. That is to say, it cannot be solved exactly. To obtain finite results, renormalization is necessary, and there are some typical renormalization schemes such as MS, $\overline{\text{MS}}$, MOM, zero-momentum and on-shell schemes [1–5]. In the MS scheme, the counter term is only the pole term ($1/\epsilon$, $\epsilon \rightarrow 0$). The counter term is proportional to $1/\epsilon + \ln 4\pi - \gamma_E$ in the $\overline{\text{MS}}$ scheme. The two foregoing schemes have nothing to do with mass. For the on-shell scheme, the renormalization constants are all obtained under the on-shell condition. This is the only physical scheme. For electroweak theory, the on-shell scheme is the most appropriate.

If we can resolve the theory accurately, different renormalization schemes can give the same finite result of any physical process, though the functions of the renormalized parameters are different. However, different physical predictions are produced from different renormalization schemes and different renormalized parameters because of the curttate perturbation theory.

To obtain the counter term for the UV-divergent diagram, one can take all the external momenta of the diagram as zero, which is called a zero-momentum renor-

malization scheme. The advantage is that in an arbitrary model, each divergent diagram can easily be renormalized, and the renormalized results are decoupled [6, 7].

We focus on the on-shell renormalization scheme that is popular in electroweak theory. In the on-shell scheme, the fine structure constant α is an expansion parameter and defined in the Thomson limit. At any order of perturbation theory, the physical parameters are the same as the finite renormalized parameters. They represent clear physical meaning and can be measured directly in experiments. The renormalization procedure is summarized in the counter term approach [4].

Extending the SM, physicists have developed many new models [8, 9] to explain the experimental phenomena. The MSSM [10] is the most attractive one. Many experimentalists of high energy physics are focusing on searching for Higgs bosons in the MSSM. The colliders (the LHC, e^+e^- linear collider, etc) will provide abundant information of new physics beyond the SM. In the MSSM, the decays $h^0(H^0, A^0) \rightarrow \tilde{\chi}_m^0\tilde{\chi}_n^0$, $\tilde{\chi}_m^0 \rightarrow h^0(H^0, A^0) + \tilde{\chi}_n^0$ ($m, n = 1, 2, 3, 4$) and $\tilde{b}_a \rightarrow \chi_i^- t$ ($a, i = 1, 2$) are studied at one-loop order with the on-shell renormalization scheme [11, 12], but they do not give analytic results to show the apparent elimination of UV-divergence. Considering the one-loop contributions, the

Received 25 June 2012, Revised 16 August 2012

^{*} Supported by National Natural Science Foundation (11047002, 10975027) and Natural Science Foundation of Hebei Province in China (A2011201118)

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authors [13] completed a systematic on-shell renormalization programme for the gauge boson and Higgs parts. Radiative one-loop corrections to the process $e^+e^- \rightarrow l^+l^-$ (hadrons) are calculated with the same scheme [14].

For supersymmetric gauge theories, a consistent regularization scheme preserving supersymmetry and gauge invariance is still not known. Two equivalent ways to solve the problem are shown here. One is to use an invariant scheme to maintain manifest symmetries, where only those counterterms are necessary for renormalization, so they themselves preserve the symmetries. The other is to use a non-invariant scheme, through using appropriate non-invariant counterterms to compensate the corresponding symmetry breaking. With appropriate non-invariant counterterms, W. Hollik et al. [15] showed that supersymmetric QED can keep supersymmetry. Their study can be generalized to supersymmetric models with soft breaking and eventually to the supersymmetric extensions of the SM. Although the corresponding Slavnov Taylor identities are more involved since they have to express not only the symmetries but also the spontaneous or soft breaking, their structure is the same as in SQED. Therefore, this method can also extend to the full EW theory of the MSSM.

With the extension of the on-shell scheme of the SM, the vertexes $(Z\bar{l}l)$, $(W^+\bar{\nu}l)$ and $(\tilde{L}_i^* \tilde{\chi}_\alpha^0 l)$ are studied at one-loop order in this work. We find some special characters for the on-shell scheme in the MSSM. Compared with the zero-momentum scheme, it is easy to find that the renormalized results in the on-shell scheme are decoupled. These selected vertexes are ordinary, and can represent the general vertexes in the MSSM. The study of the on-shell scheme for these vertexes is propitious to completing the on-shell renormalization programme of the MSSM. If one studies the on-shell renormalization scheme in other models, it is also helpful.

After the introduction, in Section 2 we study both the zero-momentum scheme and the on-shell scheme of two SM vertexes in the MSSM. The corresponding results of the SUSY vertex are shown in Section 3. In Section 4, the decoupling behaviors for the counter terms in both renormalization schemes are researched. Section 5 is devoted to the discussion and conclusion.

2 Renormalization of the SM vertex $(Z\bar{l}l, W^+\bar{\nu}l)$ in the MSSM

The authors [4, 5] studied the on-shell renormalization scheme of electroweak theory in the SM successfully and completely. Extending the model from the SM to

the MSSM, the condition becomes complex and faint, so it needs more research. In the Feynman gauge, applying both the on-shell and zero-momentum schemes, we study the two SM vertexes $(Z\bar{l}l)$, and $(W^+\bar{\nu}l)$ with virtual particles $(\tilde{L}, \tilde{\chi}^0, \tilde{\nu}, \tilde{\chi}^\pm)$ in this section. The studied one loop diagrams are shown in Fig. 1. In order to obtain the counter terms, we adopt the naive dimensional regularization with the anticommuting γ_5 scheme, where there is no distinction between the first four dimensions and the remaining D-4 dimensions [16, 17].

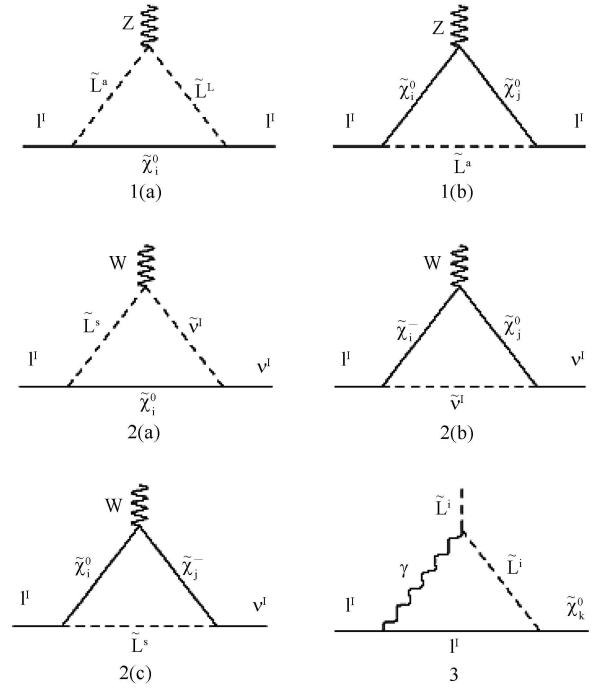


Fig. 1. The studied one-loop diagrams.

2.1 $Z\bar{l}l$ vertex with virtual SUSY particles $(\tilde{L}, \tilde{\chi}^0)$

There are two triangle diagrams for the $Z\bar{l}l$ vertex with virtual SUSY particles $(\tilde{L}, \tilde{\chi}^0)$, and they are shown as diagrams 1(a) and 1(b). The two diagrams are not complete and the results do not satisfy the gauge invariant rule, but they belong to one suit and can be renormalized with some renormalization constants. In the zero-momentum scheme, each diagram has its own counter term, and the corresponding renormalized result is decoupled. Here we show the sum of the counter terms for the two triangle diagrams in the zero-momentum scheme.

$$\delta V_{Z\bar{l}l}^{(ZM),\mu} = \frac{e^3}{64\pi^2 s_W c_W} \left\{ \frac{1-2s_W^2}{2s_W^2} \left[\left(\frac{m_{l1}}{c_\beta m_W} \right)^2 + \frac{1}{c_W^2} \right] \gamma^\mu \omega_- - \left[\frac{4s_W^2}{c_W^2} + \left(\frac{m_{l1}}{c_\beta m_W} \right)^2 \right] \gamma^\mu \omega_+ \right\} \Delta_{UV}$$

$$\begin{aligned}
 & + \left\{ \frac{e^3}{256\pi^2 s_W^3 c_W} \left(\frac{1-2s_W^2}{c_W^2} - (1+2s_W^2) \left(\frac{m_{H^\pm}}{c_\beta m_W} \right)^2 \right) + \sum_{i,\beta=1}^6 \sum_{j=1}^4 \frac{e^3}{4s_W^3 c_W} (\mathcal{G})_{i\beta} (\mathcal{D}^I)_{ij} (\mathcal{D}^I)_{\beta j}^* F_1(x_{\tilde{L}_i}, x_{\tilde{L}_\beta}, x_{\tilde{\chi}_j^0}) \right. \\
 & - \frac{e^3}{8s_W^3 c_W} \sum_{s=1}^6 \sum_{i,j=1}^4 (\mathcal{D}^I)_{si}^* (\mathcal{R}^*)_{ji} (\mathcal{D}^I)_{sj} F_1(x_{\tilde{\chi}_i^0}, x_{\tilde{L}_s}, x_{\tilde{\chi}_j^0}) + \frac{e^3}{4s_W^3 c_W} \sum_{s=1}^6 \sum_{i,j=1}^4 (\mathcal{D}^I)_{si}^* (\mathcal{R})_{ji} (\mathcal{D}^I)_{sj} \sqrt{x_{\tilde{\chi}_i^0} x_{\tilde{\chi}_j^0}} \\
 & \times F_2(x_{\tilde{\chi}_i^0}, x_{\tilde{L}_s}, x_{\tilde{\chi}_j^0}) \left. \right\} \gamma^\mu \omega_- + \left\{ \frac{e^3}{128\pi^2 s_W c_W} \left(\frac{c_W^2}{s_W^2} \left(\frac{m_{H^\pm}}{c_\beta m_W} \right)^2 - \frac{4s_W^2}{c_W^2} \right) \right. \\
 & + \frac{e^3}{2s_W c_W} \sum_{i,\beta=1}^6 \sum_{j=1}^4 (\mathcal{G})_{i\beta} (\mathcal{C}^I)_{ij} (\mathcal{C}^I)_{\beta j}^* F_1(x_{\tilde{L}_i}, x_{\tilde{L}_\beta}, x_{\tilde{\chi}_j^0}) + \frac{e^3}{4s_W c_W} \sum_{s=1}^6 \sum_{i,j=1}^4 (\mathcal{C}^I)_{si}^* (\mathcal{R})_{ji} (\mathcal{C}^I)_{sj} F_1(x_{\tilde{\chi}_i^0}, x_{\tilde{L}_s}, x_{\tilde{\chi}_j^0}) \\
 & \left. - \frac{e^3}{2s_W c_W} \sum_{s=1}^6 \sum_{i,j=1}^4 (\mathcal{C}^I)_{si}^* (\mathcal{R})_{ji}^* (\mathcal{C}^I)_{sj} \sqrt{x_{\tilde{\chi}_i^0} x_{\tilde{\chi}_j^0}} F_2(x_{\tilde{\chi}_i^0}, x_{\tilde{L}_s}, x_{\tilde{\chi}_j^0}) \right\} \gamma^\mu \omega_+. \quad (1)
 \end{aligned}$$

To get Eq. (1), we use the unitary character of the mixing matrices $Z_{\tilde{L}}, Z_N$ for sleptons and neutralinos. Additionally, we adopt the abbreviation notations $c_W = \cos\theta_W$, $s_W = \sin\theta_W$, $c_\beta = \cos\beta$, $s_\beta = \sin\beta$, where θ_W is the Weinberg angle and $\tan\beta = v_2/v_1$ represents the ratio between the vacuum expectation values of the two Higgs doublets. $x_i = m_i^2/\Lambda_{\text{NP}}^2$ with i denoting the virtual particles in these one-loop diagrams, and Λ_{NP} denotes the new physic energy scale. Here, $\Delta_{\text{UV}} = 1/\epsilon + \ln(4\pi x_\mu) - \gamma_E$, $2\epsilon = 4 - D$, $\omega_- = (1 - \gamma_5)/2$, $\omega_+ = (1 + \gamma_5)/2$ and the functions F_1, F_2 are shown as

$$F_1(x, y, z) = \frac{1}{16\pi^2} \left(1 - \frac{x^2 \ln x}{(y-x)(z-x)} - \frac{y^2 \ln y}{(x-y)(z-y)} - \frac{z^2 \ln z}{(x-z)(y-z)} \right), \quad (2)$$

$$F_2(x, y, z) = \frac{1}{16\pi^2} \left(\frac{x \ln x}{(y-x)(z-x)} + \frac{y \ln y}{(x-y)(z-y)} + \frac{z \ln z}{(x-z)(y-z)} \right). \quad (3)$$

The concrete forms of the vertex couplings used in Eq. (1) read as

$$\begin{aligned}
 (\mathcal{C}^I)_{ij} &= \frac{-\sqrt{2}}{c_W} Z_{\tilde{L}}^{(I+3)t} Z_N^{1j*} - \frac{m_{H^\pm} Z_{\tilde{L}}^{It} Z_N^{3j*}}{\sqrt{2} s_W c_\beta m_W}, \\
 (\mathcal{D}^I)_{ij} &= \frac{Z_{\tilde{L}}^{It}}{c_W} (Z_N^{1j} s_W + Z_N^{2j} c_W) - \frac{m_{H^\pm} Z_{\tilde{L}}^{(I+3)t} Z_N^{3j}}{c_\beta m_W}, \\
 (\mathcal{G})_{ts} &= \frac{1}{2} Z_{\tilde{L}}^{It} Z_{\tilde{L}}^{Is*} - s_W^2 \delta^{st}, \\
 (\mathcal{R})_{k\alpha} &= (Z_N^{4k} Z_N^{4\alpha*} - Z_N^{3k} Z_N^{3\alpha*}).
 \end{aligned} \quad (4)$$

In the on-shell scheme, the counter term for the radiative correction to the SM vertex $Z\bar{l}l$ is shown

here [4].

$$\begin{aligned}
 \delta V_{Z\bar{l}l}^{(\text{OS}),\mu} &= -\frac{e}{2} \left[\delta Z_{AZ} - \frac{1}{2s_W^3 c_W} \left(\frac{\delta m_Z^2}{m_Z^2} - \frac{\delta m_W^2}{m_W^2} \right) \right. \\
 & - \frac{(2s_W^2 - 1)}{s_W c_W} \left(\frac{\delta e}{e} + \frac{1}{2} \delta Z_{ZZ} + \delta Z_L^1 \right) \left. \right] \gamma^\mu \omega_- \\
 & - \frac{e}{2} \left[\delta Z_{AZ} - \frac{s_W}{c_W} \left(2 \frac{\delta e}{e} + \frac{1}{s_W^2} \left(\frac{\delta m_Z^2}{m_Z^2} - \frac{\delta m_W^2}{m_W^2} \right) \right) \right. \\
 & \left. + \delta Z_{ZZ} + 2\delta Z_R^1 \right] \gamma^\mu \omega_+. \quad (5)
 \end{aligned}$$

δZ_{AZ} and δe are the renormalization constants for γZ mixing and charge, respectively. Only the sum of the amplitude of the two triangle diagrams can be renormalized in the on-shell scheme. That is to say, the divergent term of each diagram can not be canceled by the counter term. Another character is that only the lepton wave function renormalization constants δZ_L^1 and δZ_R^1 are necessary to counteract the ultra-divergent terms.

The renormalization constants for the left- and right-handed lepton wave functions are deduced from the lepton self-energy with virtual SUSY particles ($\tilde{L}, \tilde{\chi}^0$).

$$\begin{aligned}
 \delta Z_L^1 &= -\frac{e^2}{64\pi^2 s_W^2} \left(\frac{1}{c_W^2} + \left(\frac{m_{H^\pm}}{c_\beta m_W} \right)^2 \right) \Delta_{\text{UV}} \\
 & - \sum_{i=1}^6 \sum_{j=1}^4 \left\{ \frac{1}{2s_W^2} |(\mathcal{D}^I)_{ij}|^2 F_4(x_{\tilde{L}_i}, x_{\tilde{\chi}_j^0}) \right. \\
 & + x_{H^\pm} \left[\frac{1}{2s_W^2} |(\mathcal{D}^I)_{ij}|^2 + |(\mathcal{C}^I)_{ij}|^2 \right. \\
 & \left. \left. + \frac{\sqrt{2}}{s_W} \text{Re} [(\mathcal{C}^I)_{ij}^\dagger (\mathcal{D}^I)_{ij}] F_3(x_{\tilde{L}_i}, x_{\tilde{\chi}_j^0}) \right] \right\},
 \end{aligned}$$

$$\begin{aligned}
 \delta Z_R^1 = & -\frac{e^2}{32\pi^2} \left(\frac{2}{c_W^2} + \left(\frac{m_{\text{H}^\pm}}{\sqrt{2}s_W c_\beta m_W} \right)^2 \right) \Delta_{\text{UV}} \\
 & - \sum_{i=1}^6 \sum_{j=1}^4 \left\{ |(\mathcal{C}^1)_{ij}|^2 F_4(x_{\tilde{L}_i}, x_{\tilde{\chi}_j^0}) \right. \\
 & + x_{\text{H}^\pm} \left[\frac{1}{2s_W^2} |(\mathcal{D}^1)_{ij}|^2 + |(\mathcal{C}^1)_{ij}|^2 \right. \\
 & \left. \left. + \frac{\sqrt{2}}{s_W} \text{Re} [(\mathcal{C}^1)_{ij}^\dagger (\mathcal{D}^1)_{ij}] \right] F_3(x_{\tilde{L}_i}, x_{\tilde{\chi}_j^0}) \right\}, \quad (6)
 \end{aligned}$$

where the functions F_3, F_4 are shown as follows

$$\begin{aligned}
 F_3(x, y) &= \frac{x^2 + 2xy(\ln y - \ln x) - y^2}{32\pi^2(x-y)^3}, \\
 F_4(x, y) &= \frac{(2y-x)(y-x+x \ln x) - y^2 \ln y}{32\pi^2(x-y)^2}.
 \end{aligned} \quad (7)$$

Considering Eqs. (5)–(7), the required counter terms for diagram 1(a) and 1(b) are obtained in the on-shell scheme.

2.2 $W^+\bar{\nu}^1\text{I}^1$ vertex with virtual SUSY particles ($\tilde{L}, \tilde{\nu}, \tilde{\chi}^0, \tilde{\chi}^\pm$)

The condition of the $W^+\bar{\nu}^1\text{I}^1$ vertex is more complex than that of the $Z\bar{\nu}^1\text{I}^1$ vertex. The three triangle diagrams (2(a), 2(b) and 2(c)) are all necessary and belong to one suit, where the virtual SUSY particles are $\tilde{L}, \tilde{\nu}, \tilde{\chi}^0, \tilde{\chi}^\pm$. We collect the counter terms of these three diagrams in the zero-momentum scheme.

$$\begin{aligned}
 & \delta V_{W^+\bar{\nu}^1\text{I}^1}^{(\text{ZM}),\mu} \\
 = & \frac{e^3}{\sqrt{2}s_W^3} \left\{ \frac{1}{64\pi^2} \left[\frac{1}{c_W^2} + \left(\frac{m_{\text{H}^\pm}}{c_\beta m_W} \right)^2 \right] \Delta_{\text{UV}} \right. \\
 & + \frac{1}{128\pi^2} \left[4 - \frac{1}{c_W^2} + \left(\frac{m_{\text{H}^\pm}}{c_\beta m_W} \right)^2 \right] \\
 & - \frac{1}{2c_W} \sum_{i=1}^2 \sum_{J=1}^3 \sum_{j=1}^4 \left((\zeta^I)_{Jj}^* (\mathcal{T})_{ji} (\mathcal{B}_i)^{IJ} F_1 \right. \\
 & + 2(\zeta^I)_{Jj}^* (\mathcal{Q})_{ji} (\mathcal{B}_i)^{IJ} \sqrt{x_{\tilde{\chi}_i^-} x_{\tilde{\chi}_j^0}} F_2 \left. \right) (x_{\tilde{\chi}_i^-}, x_{\tilde{\nu}_J}, x_{\tilde{\chi}_j^0}) \\
 & + \frac{1}{4c_W} \sum_{i=1}^6 \sum_{J=1}^3 \sum_{j=1}^4 (\eta)_{iJ}^* (\zeta^I)_{Jj}^* (\mathcal{D}^I)_{ij} F_1(x_{\tilde{L}_i}, x_{\tilde{\nu}_J}, x_{\tilde{\chi}_j^0}) \\
 & + \frac{1}{2} \sum_{i=1}^2 \sum_{s=1}^6 \sum_{j=1}^4 \left(2(\mathcal{P}^I)_{si} (\mathcal{Q})_{ji}^* (\mathcal{D}^I)_{sj} \sqrt{x_{\tilde{\chi}_i^-} x_{\tilde{\chi}_j^0}} F_2 \right. \\
 & \left. + (\mathcal{P}^I)_{si} (\mathcal{T})_{ji}^* (\mathcal{D}^I)_{sj} F_1 \right) (x_{\tilde{\chi}_i^-}, x_{\tilde{L}_s}, x_{\tilde{\chi}_j^0}) \left. \right\} \gamma^\mu \omega_-, \quad (8)
 \end{aligned}$$

with the vertex couplings

$$\begin{aligned}
 (\mathcal{A}_i)^{IJ} &= \frac{-m_{\text{H}^\pm} \mathcal{Z}_{2i}^* \mathcal{Z}_{\tilde{\nu}}^{IJ*}}{\sqrt{2}c_\beta m_W}, \quad (\mathcal{B}_i)^{IJ} = \mathcal{Z}_{1i}^+ \mathcal{Z}_{\tilde{\nu}}^{IJ*}, \\
 (\eta)_{sj} &= \mathcal{Z}_{\tilde{\nu}}^{IJ} \mathcal{Z}_{\tilde{L}}^{Is}, \quad (\mathcal{Q})_{ji} = \mathcal{Z}_{\tilde{N}}^{2j*} \mathcal{Z}_{1i}^+ - \frac{\mathcal{Z}_{\tilde{N}}^{4j*} \mathcal{Z}_{2i}^+}{\sqrt{2}}, \\
 (\mathcal{T})_{ji} &= \mathcal{Z}_{\tilde{N}}^{2j} \mathcal{Z}_{1i}^- + \frac{1}{\sqrt{2}} \mathcal{Z}_{\tilde{N}}^{3j} \mathcal{Z}_{2i}^{-*}, \\
 (\mathcal{P}^I)_{si} &= \frac{m_{\text{H}^\pm}}{\sqrt{2}c_\beta m_W} \mathcal{Z}_{\tilde{L}}^{(I+3)**} \mathcal{Z}_{2i}^{-*} - \mathcal{Z}_{\tilde{L}}^{Is*} \mathcal{Z}_{1i}^{-*}, \\
 (\zeta^I)_{Jj} &= \mathcal{Z}_{\tilde{\nu}}^{IJ*} (\mathcal{Z}_{\tilde{N}}^{1j} s_W - \mathcal{Z}_{\tilde{N}}^{2j} c_W).
 \end{aligned} \quad (9)$$

In the on-shell scheme, the counter term formula for vertex $W^+\bar{\nu}^1\text{I}^1$ can be found in Ref. [4].

$$\begin{aligned}
 \delta V_{W^+\bar{\nu}^1\text{I}^1}^{(\text{OS}),\mu} &= \frac{e}{2\sqrt{2}s_W} \left(\frac{\delta m_Z^2}{m_Z^2} - \frac{\delta m_Z^2 - \delta m_W^2}{m_Z^2 - m_W^2} \right. \\
 & \left. + 2\delta e + \delta Z_L^1 + \delta Z_L^Y + \delta Z_{\text{WW}} \right) \gamma^\mu \omega_-, \quad (10)
 \end{aligned}$$

where δe is calculated from the virtual slepton contribution. The virtual slepton and sneutrino produce the mass renormalization constants $\delta m_Z^2, \delta m_W^2$ and the W wavefunction renormalization constant δZ_{WW} . The wavefunction renormalization constants $\delta Z_{\tilde{L}}^Y$ and $\delta Z_{\tilde{L}}^1$ are deduced, respectively, from the self-energies of the neutrino and lepton with virtual SUSY particles $[(\tilde{\nu}, \tilde{\chi}^0), (\tilde{L}, \tilde{\chi}^\pm)]$ and $[(\tilde{\nu}, \tilde{\chi}^\pm), (\tilde{L}, \tilde{\chi}^0)]$.

To cancel the UV-divergent terms for these diagrams in the on-shell scheme, all the renormalization constants in Eq. (10) must be taken into account. Following the method in Refs. [4, 5], we obtain the required renormalization constants.

$$\begin{aligned}
 \frac{\delta m_Z^2}{m_Z^2} &= \frac{e^2}{32\pi^2 s_W^2 c_W^2} (1 - 2s_W^2)^2 \Delta_{\text{UV}} \\
 & - \frac{e^2}{s_W^2 c_W^2} \left\{ \frac{1}{4} \sum_{j=1}^3 F_5(x_{\tilde{\nu}_j}, x_{\tilde{\nu}_j}) \right. \\
 & \left. + \sum_{\alpha, \beta=1}^6 |(\mathcal{G})_{\alpha\beta}|^2 F_5(x_{\tilde{L}_\alpha}, x_{\tilde{L}_\beta}) \right\}, \\
 \frac{\delta m_W^2}{m_W^2} &= \frac{e^2 c_W^2}{32\pi^2 s_W^2} \Delta_{\text{UV}} \\
 & - \frac{e^2 c_W^2}{2s_W^2} \sum_{i=1}^6 \sum_{\alpha=1}^3 |(\eta)_{i\alpha}|^2 F_5(x_{\tilde{\nu}_\alpha}, x_{\tilde{L}_i}), \\
 \delta Z_{\text{WW}} &= -\frac{e^2}{32\pi^2 s_W^2} \Delta_{\text{UV}} \\
 & + \frac{e^2}{2s_W^2} \sum_{i=1}^6 \sum_{\alpha=1}^3 |(\eta)_{i\alpha}|^2 F_5(x_{\tilde{\nu}_\alpha}, x_{\tilde{L}_i}), \\
 \delta e &= \frac{e^2}{8\pi^2} \Delta_{\text{UV}} - e^2 \sum_{i=1}^6 F_5(x_{\tilde{L}_i}, x_{\tilde{L}_i}),
 \end{aligned}$$

$$\begin{aligned}
 \delta Z_L^V &= \frac{-e^2}{32\pi^2 s_W^2} \left(\frac{1}{2c_W^2} + 1 + \left(\frac{m_{H^\pm}}{\sqrt{2}c_\beta m_W} \right)^2 \right) \Delta_{UV} \\
 &\quad - \frac{e^2}{2s_W^2 c_W^2} \sum_{i=1}^4 \sum_{\alpha=1}^3 |(\zeta^I)_{\alpha i}|^2 F_4(x_{\tilde{\nu}_\alpha}, x_{\tilde{\chi}_i^0}) \\
 &\quad - \frac{e^2}{s_W^2} \sum_{i=1}^4 \sum_{\alpha=1}^6 |(\mathcal{P}^I)_{\alpha i}|^2 F_4(x_{\tilde{L}_\alpha}, x_{\tilde{\chi}_i^0}), \\
 \delta Z_L^1 &= -\frac{e^2}{32\pi^2 s_W^2} \left(\frac{1}{2c_W^2} + 1 + \left(\frac{m_{H^\pm}}{\sqrt{2}c_\beta m_W} \right)^2 \right) \Delta_{UV} \\
 &\quad - \frac{e^2}{s_W^2} \sum_{\alpha=1}^3 \sum_{i=1}^2 \left\{ |(\mathcal{B}_i)^{I\alpha}|^2 F_4 + x_{I1} \left[|(\mathcal{B}_i)^{I\alpha}|^2 \right. \right. \\
 &\quad \left. \left. + |(\mathcal{A}_i)^{I\alpha}|^2 + 2\text{Re}[(\mathcal{A}_i^\dagger)^{I\alpha} (\mathcal{B}_i)^{I\alpha}] \right] F_3 \right\} (x_{\tilde{\nu}_\alpha}, x_{\tilde{\chi}_i^-}) \\
 &\quad - e^2 \sum_{j=1}^4 \sum_{i=1}^6 \left\{ x_{I1} \left[\frac{|(\mathcal{D}^I)_{ij}|^2}{2s_W^2} + \frac{\sqrt{2}}{s_W} \text{Re}[(\mathcal{C}^I)_{ij}^\dagger (\mathcal{D}^I)_{ij}] \right. \right. \\
 &\quad \left. \left. + |(\mathcal{C}^I)_{ij}|^2 \right] F_3 + \frac{1}{2s_W^2} |(\mathcal{D}^I)_{ij}|^2 F_4 \right\} (x_{\tilde{L}_i}, x_{\tilde{\chi}_j^0}). \quad (11)
 \end{aligned}$$

The function F_5 is

$$\begin{aligned}
 F_5(x, y) &= \frac{1}{288\pi^2 (x-y)^3} [6(x-3y)x^2 \ln x + 6(3x \\
 &\quad - y)y^2 \ln y - (x-y)(5x^2 - 22xy + 5y^2)]. \quad (12)
 \end{aligned}$$

From Eqs. (10)–(12), we get the counter terms for the three diagrams (2(a), 2(b) and 2(c)). The renormalization constants in Eq. (10) are all necessary at this place, which is different from the condition of the $Z\bar{1}^1$ vertex.

3 Renormalization of the $\tilde{L}_i^* \bar{\chi}_\alpha^0 1^1$ vertex with virtual photon

In order to further research the on-shell renormalization scheme in the MSSM, we study the vertex $\tilde{L}_i^* \bar{\chi}_\alpha^0 1^1$ at one-loop order in this section. The studied triangle diagram is diagram 3 with the virtual photon, which is the simplest instance. Diagram 3 belongs to electromagnetic interaction, and can be treated separately without considering the diagrams with virtual W and Z. The counter term for this diagram in the zero-momentum scheme is

$$\begin{aligned}
 \delta V_{\tilde{L}_i^* \bar{\chi}_\alpha^0 1^1}^{(ZM)}(\gamma) &= \frac{e^3}{16\pi^2} \left\{ \frac{(\mathcal{D}_{i\alpha}^I)}{\sqrt{2}s_W} \omega_- + (\mathcal{C}_{i\alpha}^I) \omega_+ \right\} \Delta_{UV} \\
 &\quad + e^3 F_1(x_{\tilde{L}_i}, 0, x_{I1}) \left(\frac{(\mathcal{D}_{i\alpha}^I)}{\sqrt{2}s_W} \omega_- + (\mathcal{C}_{i\alpha}^I) \omega_+ \right). \quad (13)
 \end{aligned}$$

In the on-shell scheme, the counter term formula of the vertex $\tilde{L}_i^* \bar{\chi}_\alpha^0 1^1$ is complicated. Following the idea of the SM on-shell scheme, we show the formula here [17], where the counter term is determined by the on-shell condition.

$$\begin{aligned}
 \delta V_{\tilde{L}_i^* \bar{\chi}_\alpha^0 1^1}^{(OS)} &= \frac{e}{\sqrt{2}s_W c_W} \left\{ \left[\left(\frac{\delta e}{e} \delta_{IJ} + \frac{(\delta Z_L^1)_{JI}}{2} \right) \delta_{ij} \delta_{\alpha\beta} + \frac{(\delta Z_L^\dagger)_{ij}}{2} \delta_{IJ} \delta_{\alpha\beta} + \frac{(\delta Z_{\tilde{\chi}^0})_{\beta\alpha}}{2} \delta_{IJ} \delta_{ij} \right] (Z_L^\dagger)_{jJ} (Z_N^{1\beta})_{sW} \right. \\
 &\quad \left. + Z_N^{2\beta} c_W \right) - \frac{s_W}{c_W} \delta c_W (Z_L^\dagger)_{jJ} Z_N^{1\beta} \delta_{ij} \delta_{IJ} \delta_{\alpha\beta} - \frac{c_W}{s_W} \delta s_W (Z_L^\dagger)_{jJ} Z_N^{2\beta} \delta_{ij} \delta_{IJ} \delta_{\alpha\beta} - \frac{m_{H^\pm} c_W}{m_W c_\beta} \left[\left(\frac{\delta e}{e} + \frac{\delta m_{H^\pm}}{m_{H^\pm}} \right. \right. \\
 &\quad \left. \left. + \frac{\delta m_W}{m_W} - \frac{\delta s_W}{s_W} - \frac{\delta c_\beta}{c_\beta} \right) \delta_{IJ} \delta_{ij} \delta_{\alpha\beta} + \frac{1}{2} (\delta Z_L^\dagger)_{ij} \delta_{IJ} \delta_{\alpha\beta} + \frac{1}{2} (\delta Z_{\tilde{\chi}^0})_{\beta\alpha} \delta_{IJ} \delta_{ij} \right. \\
 &\quad \left. + \frac{1}{2} (\delta Z_L^1)_{JI} \delta_{\alpha\beta} \delta_{ij} \right] (Z_L^\dagger)_{j(3+J)} Z_N^{3\beta} \left\} \omega_- + \frac{\sqrt{2}e}{c_W} \left\{ - \left[\left(\frac{\delta e}{e} - \frac{\delta c_W}{c_W} \right) \delta_{IJ} \delta_{ij} \delta_{\alpha\beta} \right. \right. \\
 &\quad \left. \left. + \frac{1}{2} (\delta Z_L^\dagger)_{ij} \delta_{IJ} \delta_{\alpha\beta} + \frac{1}{2} (\delta Z_{\tilde{\chi}^0}^*)_{\beta\alpha} \delta_{IJ} \delta_{ij} + \frac{1}{2} (\delta Z_R^1)_{JI} \delta_{\alpha\beta} \delta_{ij} \right] (Z_L^\dagger)_{j(3+J)} Z_N^{1\beta*} \right. \\
 &\quad \left. + \frac{m_{H^\pm} c_W}{2m_W s_W c_\beta} \left[\frac{(\delta Z_L^\dagger)_{ij}}{2} \delta_{IJ} \delta_{\alpha\beta} + \frac{(\delta Z_{\tilde{\chi}^0}^*)_{\beta\alpha}}{2} \delta_{IJ} \delta_{ij} + \left(\frac{\delta e}{e} + \frac{\delta m_{H^\pm}}{m_{H^\pm}} + \frac{\delta m_W}{m_W} - \frac{\delta s_W}{s_W} - \frac{\delta c_\beta}{c_\beta} \right) \delta_{IJ} \delta_{ij} \delta_{\alpha\beta} \right. \right. \\
 &\quad \left. \left. + \frac{1}{2} (\delta Z_R^1)_{JI} \delta_{\alpha\beta} \delta_{ij} \right] (Z_L^\dagger)_{jJ} Z_N^{3\beta*} \right\} \omega_+. \quad (14)
 \end{aligned}$$

$\delta Z_{L,R}^1$, $\delta Z_{\tilde{\nu}}$, $\delta Z_{\tilde{L}}$, $\delta Z_{\tilde{\chi}^-}$ and $\delta Z_{\tilde{\chi}^0}$ are the renormalization constants of the wave functions for leptons and SUSY particles. The other renormalization constants come from the vertex coupling renormalization.

After tedious calculations and various compounding of the renormalization constants, we only find that the wave function renormalization constant of slepton ($\delta Z_{\tilde{L}}$) $_{ij}$ is essential. That is to say, only the renormalization constant ($\delta Z_{\tilde{L}}$) $_{ij}$ can cancel the UV-divergent term. The wave function renormalization constant ($\delta Z_{\tilde{L}}$) $_{ij}$ is collected as follows.

$$F_6(x, y) = \frac{1}{32\pi^2(y-x)^3} [(y-x)(6x^2-7xy+3y^2) + 2x(2x^2-2xy+y^2)\ln x - 2y(4x^2-5xy+2y^2)\ln y],$$

$$(\delta Z_{\tilde{L}}^\gamma)_{ij} = \frac{e^2}{8\pi^2} \Delta_{UV} \delta^{ij} + e^2 F_6(x_{\tilde{L}_i}, 0) \delta^{ij}. \quad (15)$$

($\delta Z_{\tilde{L}}^\gamma$) $_{ij}$ in Eq. (15) is obtained from the self-energy of the slepton with the virtual photon and slepton. In our calculation, Eq. (14) is predigested as

$$\delta V_{\tilde{L}_i^* \tilde{\chi}_\alpha^0 \tilde{l}^i}(\gamma) = \frac{1}{2} (\delta Z_{\tilde{L}}^\gamma)_{ij}^\dagger \left[(\mathcal{D}_{j\alpha}^I) \omega_- + (\mathcal{C}_{j\alpha}^I) \omega_+ \right]. \quad (16)$$

Combining the formulas (15) and (16), diagram 3 can be renormalized successfully in the on-shell scheme. Until now, we have obtained the counter terms for the vertexes ($Z\bar{l}l$, $W^+\bar{\nu}l$) and $\bar{L}_s^* \chi_j^0 l$ in both the zero-momentum and on-shell schemes.

4 The decoupling behavior

In this section, we discuss the decoupling behavior of the renormalized results in the two schemes. It is easy to prove that the renormalized results in the zero-momentum scheme are decoupled. Adopting the on-shell scheme, we must get decoupled renormalized results if the renormalized results can not go to infinity with the incessantly enlarging SUSY particle masses. To obtain the decoupling behavior of renormalized results in the on-shell scheme, we suppose that all SUSY particle masses are the same and much heavier than the masses of the SM particles. Compared with the decoupling character of the zero-momentum counter terms, the decoupling behavior of the counter terms in the on-shell scheme is obvious.

4.1 The SM vertex ($Z\bar{l}l$, $W^+\bar{\nu}l$)

To obtain the decoupling behavior of the counter terms for the vertex $Z\bar{l}l$ in the zero-momentum scheme, we show the decoupling approximation of the functions F_1 and F_2 . The variables x, y, z in $F_1(x, y, z)$ are all

symmetrical, and three conditions are considered here.

$$F_1(x, y, z) = \begin{cases} -\frac{\ln x}{16\pi^2} - \frac{1}{32\pi^2}, & (x=y=z) \\ -\frac{\ln x}{16\pi^2} + \dots & (x=y \gg z), \\ \frac{1-\ln x}{16\pi^2} + \dots & (x \gg y, z) \end{cases}$$

$$F_2(x, y, z) = \frac{1}{32\pi^2 x}, \quad (x=y=z). \quad (17)$$

With Eqs. (1) and (17), the decoupling behavior of Eq. (1) reads

$$\delta V_{Z\bar{l}l}^{(ZM), \mu} \sim \frac{e^3}{64\pi^2 s_W c_W} \left\{ \frac{1-2s_W^2}{2s_W^2} \left[\frac{1}{c_W^2} + \left(\frac{m_{lI}}{c_\beta M_W} \right)^2 \right] \gamma^\mu \omega_- - \left[\frac{4s_W^2}{c_W^2} + \left(\frac{m_e^I}{c_\beta M_W} \right)^2 \right] \gamma^\mu \omega_+ \right\} (\Delta_{UV} - \ln x_M) + \dots, \quad (18)$$

where the dots denote the terms that are finite, even when the SUSY particle masses turn to infinity. $x_M = M^2/\Lambda_{NP}^2$ with M representing the SUSY particle mass. In the same way, we deduce the decoupling behavior of the counter terms for the vertex $Z\bar{l}l$ in the on-shell scheme.

$$\delta V_{Z\bar{l}l}^{(OS), \mu} \sim \frac{e^3}{64\pi^2 s_W c_W} \left\{ \frac{1-2s_W^2}{2s_W^2} \left[\frac{1}{c_W^2} + \left(\frac{m_{lI}}{c_\beta M_W} \right)^2 \right] \gamma^\mu \omega_- - \left[\frac{4s_W^2}{c_W^2} + \left(\frac{m_{lI}}{c_\beta m_W} \right)^2 \right] \gamma^\mu \omega_+ \right\} (\Delta_{UV} - \ln x_M) + \dots \quad (19)$$

$$F_3(x, y) = \frac{1}{96\pi^2 x}, \quad (x=y);$$

$$F_4(x, y) = -\frac{\ln x}{32\pi^2} + \frac{1}{64\pi^2}, \quad (x=y). \quad (20)$$

It is satisfactory that the infinite terms and undecoupled large logarithm terms in Eqs. (18) and (19) are the same. Though the finite terms represented by the dots in Eqs. (18) and (19) are different, the renormalized results in both schemes are decoupled, because the zero-momentum scheme can guarantee the decoupled renormalized results.

Using the unitary character of the mixing matrixes, we obtain the expectant results for the counter terms of

the vertex $W^+\overline{\nu}^I l^I$ in both schemes.

$$\delta V_{W^+\overline{\nu}^I l^I}^{(ZM),\mu} \sim \left\{ \frac{e^3}{\sqrt{2}s_W^3} \frac{1}{64\pi^2} \left[\left(\frac{m_{l^I}}{c_\beta M_W} \right)^2 + \frac{1}{c_W^2} \right] (\Delta_{UV} - \ln x_M) \right\} \gamma^\mu \omega_- + \dots, \quad (21)$$

$$\delta V_{W^+\overline{\nu}^I l^I}^{(OS),\mu} \sim \left\{ \frac{e^3}{\sqrt{2}s_W^3} \frac{1}{64\pi^2} \left[\left(\frac{m_{l^I}}{c_\beta M_W} \right)^2 + \frac{1}{c_W^2} \right] (\Delta_{UV} - \ln x_M) \right\} \gamma^\mu \omega_- + \dots, \quad (22)$$

$$F_5(x, y) = \frac{\ln x}{48\pi^2}, \quad (x = y).$$

The two counter terms in Eqs. (21) and (22) can both completely eliminate the Δ_{UV} and $\ln x_M$ terms produced from the three triangle diagrams (2(a), 2(b) and 2(c)).

4.2 The MSSM vertex $\tilde{L}_i^* \overline{\chi}_\alpha^0 l^I$

For the MSSM vertex $\tilde{L}_i^* \overline{\chi}_\alpha^0 l^I$, the decoupling behavior of the counter term is discussed here. Assuming the SUSY particles are very heavy, the approximate results of Eq. (13) deduced from the virtual photon are shown as

$$\delta V_{\tilde{L}_i^* \overline{\chi}_\alpha^0 l^I}^{(ZM)}(\gamma) \sim \frac{e^3}{16\pi^2} \left\{ \frac{1}{\sqrt{2}s_W} (\mathcal{D}_{i\alpha}^I) \omega_- + (\mathcal{C}_{i\alpha}^I) \omega_+ \right\} (\Delta_{UV} - \ln x_M) + \dots$$

$$F_6(x, y) = -\frac{\ln x}{8\pi^2} + \frac{3}{16\pi^2} + \dots, \quad (x \gg y). \quad (23)$$

The decoupling behavior of the counter term Eq. (16) in the on-shell scheme is the same as that of Eq. (23) for Δ_{UV} and $\ln x_M$. In this way, we find that the renormalized results are decoupled, not only in the zero-momentum scheme but also in the on-shell scheme.

5 Discussion and conclusion

Until now, there have been several renormalization schemes for renormalizable theories. The on-shell renormalization scheme is approbated broadly for electroweak theory in the SM, and has been well studied by theorists. For the model including new physics beyond the SM, the on-shell renormalization scheme still has mist to clear. The MSSM is considered the most potent candidate out of the new models, and has attracted much attention from researchers for around 20 years. In the framework of the MSSM, some processes are calculated with the

on-shell renormalization scheme. However, a consummate on-shell renormalization scheme for the MSSM is still absent.

To explore the perfect on-shell renormalization scheme, at one-loop order we study two SM vertexes, ($Z\overline{l}^I l^I$, and $W^+\overline{\nu}^I l^I$), and one MSSM vertex, $\tilde{L}_i^* \overline{\chi}_\alpha^0 l^I$, in the zero-momentum scheme and the on-shell scheme. In the zero-momentum scheme, each divergent diagram has its own counter term, and has nothing to do with the other diagrams. Another important peculiarity is that the renormalized result is absolutely decoupled.

In the on-shell scheme, the counter term formulas for the SM vertexes in the MSSM and SM are similar. Almost all the renormalization constants are deduced from the one-loop self-energies of the corresponding particles. In the SM, all the renormalization constants in the counter term must be taken into account. At the same time, in the MSSM we can not always renormalize the one-triangle diagram by the counter term made up of renormalization constants. After careful study, both the characters of the on-shell scheme are discovered. One character is that all the triangle diagrams belonging to one type of a vertex are essential. Only the sum of the amplitude can be renormalized completely. The other character is that not all the renormalization constants are always necessary. Which renormalization constant must be considered lies on the idiographic condition.

This work shows that for the SM vertex $Z\overline{l}^I l^I$, the lepton wave function renormalization constants δZ_L^1 and δZ_R^1 are requisite to obtaining the needed counter term. However, the condition of the vertex $W^+\overline{\nu}^I l^I$ is dissimilar. To gain the final finite results, we have to calculate all the renormalization constants in the counter term formula. For the MSSM vertex, the foregoing experience is of reference value. The on-shell scheme for the MSSM vertex $\tilde{L}_i^* \overline{\chi}_\alpha^0 l^I$ shows the property, i.e. only the wave function renormalization constant for the relevant slepton (\tilde{L}) is enough to complete the on-shell scheme.

In the two renormalization schemes, we study the decoupling behavior for the counter terms of these vertexes. Obviously, the counter terms obtained in the two renormalization schemes have the same characters for infinite and large logarithm terms when the SUSY particle masses are equal and very heavy. Because the renormalized results in the zero-momentum scheme are decoupled, the on-shell renormalization scheme can also give decoupled renormalized results.

There are a great many vertexes in the MSSM, so it is hard to make one-loop on-shell renormalization for all of them. The studied vertexes in this work are representative, which can be helpful in building a consistent on-shell renormalization scheme in the MSSM. Though there are a number of questions to deal with, one can be convinced that a perfect on-shell renormalization scheme

can be found in the future. This text is also propitious in studying the on-shell renormalization scheme in

other models, even if the model is more complex than the MSSM.

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