

Quasielastic electron scattering in a derivative coupling model with relativistic random phase approximation^{*}

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Abstract: We apply the derivative coupling model with ZM and ZM3 parameters to investigate the longitudinal response function in quasielastic electron scattering in the relativistic random phase approximation. The non-spectral method is chosen to describe the nucleon Green's function in a finite nucleus. Some remarks have been made in conclusion.

Key words: quasielastic electron scattering, derivative coupling model, relativistic random phase approximation, relativistic mean field theory

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1 Introduction

Relativistic mean field (RMF) theory based on nucleons and mesons as the effective degrees of freedom is successful in describing nuclear matter and finite nuclei [1–5]. It is well known that the original Walecka model based on RMF theory gives a value of nuclear-matter incompressibility at the saturation density that is too large. Many improved ideas have been developed, such as those including nonlinear meson couplings or with density dependent nucleon-meson couplings, etc. In these models, an approach with derivative scalar couplings introduced by Zimányi and Moszkowski [6], i.e., the so-called ZM model and a modified version labeled by ZM3 has been applied to study the properties of nuclear matter, finite nuclei and neutron stars and gives reasonable results [7–10].

The relativistic random phase approximation (RPA) is widely used in studying the many-body effects of the nuclear system (see, e.g., Refs. [11–14] and references therein). In this work, we use the non-spectral method in which the contributions of the continuum spectrum to nuclear excitations are described by the single particle Green's function [15, 16]. The advantages of this method are that it is not only unnecessary to discretize the single particle states in the continuum but also fully consistent within the RMF theory in the sense that the wave function of the nuclear ground state, the meson fields, and the nucleon Green's function are obtained in

the same effective Lagrangian.

Quasielastic electron scattering is a useful approach to probe the properties of nucleons in nuclei and provides a feasible way to examine in detail the theoretical nuclear models. In this paper, we exploit the derivative coupling model in terms of the ZM and ZM3 versions to investigate the quasielastic electron scattering on nuclei, focusing on its longitudinal response function and use the RPA to describe the correlation between nucleons in nuclei. The rest of this paper is organized as follows. In Section 2, a brief review of the models and the basic formulae are given. In Section 3, we present our calculated results and give some discussions. Finally, we summarize our work in Section 4.

2 The formulae

The double-differential electron scattering cross-section is expressed as

$$\frac{d^2\sigma}{d\Omega dE'} = \sigma_M \left\{ \left(\frac{q^2}{|\mathbf{q}|^2} \right)^2 S_L(q) + \left[\frac{-q^2}{2|\mathbf{q}|^2} + \tan^2 \left(\frac{\theta}{2} \right) \right] S_T(q) \right\}, \quad (1)$$

where σ_M is the Mott cross-section, $q = (\omega, |\mathbf{q}|)$ is the four-momentum transfer of the electron to a target nucleus, and S_L and S_T are the longitudinal and transverse re-

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sponse functions, respectively. The electromagnetic current operator is defined as

$$J^\mu(q) = \int d^3x e^{iq \cdot x} \bar{\psi}(\mathbf{x}) \left(F_1(q^2) \gamma^\mu \tau_p + F_2(q^2) \frac{\kappa_\tau}{2M} i \sigma^{\mu\nu} q_\nu \right) \psi(\mathbf{x}), \quad (2)$$

where $\bar{\psi}$ and ψ are the nucleon field operators, M is the nucleon bare mass, and $\sigma_{\mu\nu} \equiv \frac{1}{2} i[\gamma_\mu, \gamma_\nu]$. The proton and neutron anomalous magnetic moments are $\kappa_p = 1.793$ and $\kappa_n = -1.913$. The Dirac and Pauli form factors for nucleons are given by [17]

$$\begin{aligned} F_{1p}(q^2) &= F_{2n}(q^2) \left[\frac{1 - (q^2/4M^2)(1 + \kappa_p)}{1 - q^2/4M^2} \right], \\ F_{2p}(q^2) &= F_{2n}(q^2) \left(\frac{1}{1 - q^2/4M^2} \right), \\ F_{1n}(q^2) &= 0, \\ F_{2n}(q^2) &= \left[1 - \frac{q^2}{0.71 \text{ GeV}^2} \right]^{-2}. \end{aligned} \quad (3)$$

The longitudinal response function S_L is proportional to the imaginary part of the longitudinal polarization function in momentum space:

$$S_L(q) = -\frac{1}{\pi} \text{Im} \Pi_L(\mathbf{q}, \mathbf{q}; \omega) = -\frac{1}{\pi} \text{Im} \Pi_{j_0 j_0}(\mathbf{q}, \mathbf{q}; \omega), \quad (4)$$

where ' j_0 ' stands for the time component of Eq. (2).

To derive the longitudinal polarization function Π_L in a finite nucleus, it is firstly desirable to know the wave function of a nucleon in the ground state of finite nuclei. We then describe the finite nuclei in the derivative coupling model in which nucleons interact with each other via the exchange of isoscalar-scalar σ , isoscalar-vector ω , isovector-vector ρ mesons, and photons. The corresponding Lagrangian density is given by

$$\begin{aligned} \mathcal{L} &= \bar{\psi} \left[\gamma_\mu \left(i \partial^\mu - \Gamma_\omega^* \omega^\mu - \frac{\Gamma_\rho^*}{2} \vec{\tau} \cdot \vec{\rho}^\mu - e \frac{1 + \tau_3}{2} A^\mu \right) - M^* \right] \psi \\ &+ \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu \\ &- \frac{1}{4} \vec{\rho}_{\mu\nu} \cdot \vec{\rho}^{\mu\nu} + \frac{1}{2} m_\rho^2 \vec{\rho}_\mu \cdot \vec{\rho}^\mu - \frac{1}{4} A_{\mu\nu} A^{\mu\nu}, \end{aligned} \quad (5)$$

where ψ stands for the nucleon field, A_μ is the photon field, σ , ω_μ , and $\vec{\rho}_\mu$ are for σ -, ω -, and ρ - meson field. Here $\omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$, $\vec{\rho}_{\mu\nu} = \partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu$, and

$A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The effective nucleon mass is defined as $M^* = M - \Gamma_\sigma^* \sigma$. For ZM and ZM3 models, the effective couplings between mesons and nucleons are listed in Table 1, with the parameter m^* defined as

$$m^* = \frac{1}{1 + \Gamma_\sigma^* \sigma / M}. \quad (6)$$

In the spirit of the RMF theory, the field operators of the mesons and the photon are replaced by their expectation values, which are classical fields. Then the Dirac equations for nucleons are derived from Eq. (5) by the variational principle to give

$$(E_\alpha \gamma_0 + i \vec{\gamma} \cdot \vec{\nabla} - M^*(r) - \gamma_0 V(r)) \psi_\alpha = 0, \quad (7)$$

with the vector potential V defined as

$$V(r) = \Gamma_\omega^* \omega_0(r) + \frac{\Gamma_\rho^*}{2} \tau_3 \rho_{30}(r) + e \frac{1 + \tau_3}{2} A_0(r). \quad (8)$$

Then, coupled with the meson field equations, Eq. (7) can be solved for the ground state of finite nuclei by an iterative procedure. The detailed formulae for ZM and ZM3 models in finite nuclei are given in Ref. [8]. In the spherical nuclei, the wave function of a nucleon can be written as

$$\psi_\alpha = \begin{pmatrix} i[\psi_{l^{(1)}j}^1(r)/r] \mathcal{Y}_{l^{(1)}jm}(\Omega) \\ -[\psi_{l^{(2)}j}^2(r)/r] \mathcal{Y}_{l^{(2)}jm}(\Omega) \end{pmatrix}, \quad (9)$$

where $l^{(2)} = 2j - l^{(1)}$, $\mathcal{Y}_{l^{(1)}jm}$ is a spin spherical harmonic, α stands for a set of quantum numbers, such as the radial quantum number, etc. We should note that the nucleon wave function has a specific energy eigenvalue E_α .

Table 1. Effective couplings between mesons and nucleons in ZM and ZM3 models, with Γ_σ , Γ_ω , and Γ_ρ given in Ref. [8].

model	Γ_σ^*	Γ_ω^*	Γ_ρ^*
ZM	$m^* \Gamma_\sigma$	Γ_ω	Γ_ρ
ZM3	$m^* \Gamma_\sigma$	$m^* \Gamma_\omega$	$m^* \Gamma_\rho$

Next we should know the nucleon Green's function in a finite nucleus. It can be obtained from the 4×4 matrix equations given as

$$(\omega \gamma_0 + i \vec{\gamma} \cdot \vec{\nabla} - M^*(r) - \gamma_0 V(r)) G(\mathbf{x}, \mathbf{y}; \omega) = \delta^3(\mathbf{x} - \mathbf{y}), \quad (10)$$

where $M^*(r)$ and $V(r)$ are the same as those in Eq. (7). Note that ω is not energy eigenvalue E_α any more, it can be any energy value. With the form of the nucleon wave function in Eq. (9), the Green's function can be written as

$$G(\mathbf{x}, \mathbf{y}; \omega) = \frac{1}{xy} \sum_{ljm} \begin{pmatrix} g_{lj}^{11}(x, y; \omega) \mathcal{Y}_{l^{(1)}jm}(\Omega_x) \mathcal{Y}_{l^{(1)}jm}^\dagger(\Omega_y) & ig_{lj}^{12}(x, y; \omega) \mathcal{Y}_{l^{(1)}jm}(\Omega_x) \mathcal{Y}_{l^{(2)}jm}^\dagger(\Omega_y) \\ ig_{lj}^{21}(x, y; \omega) \mathcal{Y}_{l^{(2)}jm}(\Omega_x) \mathcal{Y}_{l^{(1)}jm}^\dagger(\Omega_y) & -g_{lj}^{22}(x, y; \omega) \mathcal{Y}_{l^{(2)}jm}(\Omega_x) \mathcal{Y}_{l^{(2)}jm}^\dagger(\Omega_y) \end{pmatrix}, \quad (11)$$

where x and y stand for the radial distance for \mathbf{x} and \mathbf{y} . Then inserting this form in Eq. (10) yields

$$\begin{pmatrix} -M_N^* - V + \omega & d/dx - \kappa/x \\ d/dx + \kappa/x & -M_N^* + V - \omega \end{pmatrix} \begin{pmatrix} g_{lj}^{11} & g_{lj}^{12} \\ g_{lj}^{21} & g_{lj}^{22} \end{pmatrix} = \delta(x-y) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad (12)$$

where $g_{lj}^{\alpha\beta}$ can be solved as proper combinations of the regular and irregular solutions of Eq. (7) with E_α replaced by ω . The regular solutions ψ_u are normalizable at the origin, while the irregular ones ψ_v are at infinity. It can be proven that ψ_u/r are power functions in radius at the origin and ψ_v/r approach asymptotically spherical Hankel functions at infinity. Then $g_{lj}^{\alpha\beta}$ can be given in terms of ψ_u and ψ_v by

$$g_{lj}^{\alpha\beta}(x, y; \omega) = \frac{1}{\Delta_{lj}} \begin{cases} \psi_{u;l(\alpha)j}^\alpha(x) \psi_{v;l(\beta)j}^\beta(y), & x \leq y, \\ \psi_{v;l(\alpha)j}^\alpha(x) \psi_{u;l(\beta)j}^\beta(y), & x > y, \end{cases} \quad (13)$$

with the Wronskian determinant $\Delta_{lj} = \psi_{u;l(1)j}^1 \psi_{v;l(2)j}^2 - \psi_{v;l(1)j}^1 \psi_{u;l(2)j}^2$, which is constant in radius.

The longitudinal polarization in momentum space is related to that in coordinate space by Fourier transformation:

$$\Pi_L(\mathbf{q}, \mathbf{p}; \omega) = \int d^3 \mathbf{x} e^{-i\mathbf{q} \cdot \mathbf{x}} \int d^3 \mathbf{y} e^{-i\mathbf{p} \cdot \mathbf{y}} \Pi_L(\mathbf{x}, \mathbf{y}; \omega). \quad (14)$$

The relation of the polarization function and nucleon Green's function is given by

$$\begin{aligned} \Pi_{ab}(\mathbf{x}, \mathbf{y}; \omega) = & \sum_h \{ \bar{\Psi}_h(\mathbf{x}) \Gamma_a G(\mathbf{x}, \mathbf{y}; E_h + \omega) \Gamma_b \Psi_h(\mathbf{y}) \\ & + [\bar{\Psi}_h(\mathbf{x}) \Gamma_a G(\mathbf{x}, \mathbf{y}; E_h - \omega) \Gamma_b \Psi_h(\mathbf{y})]^* \}. \end{aligned} \quad (15)$$

Then, by solving Dyson's equation, the RPA correlations between nucleons can be included in the polarization function which is given by

$$\begin{aligned} & \Pi_L^{\text{RPA}}(\mathbf{q}, \mathbf{q}; \omega) \\ = & \Pi_L(\mathbf{q}, \mathbf{q}; \omega) + \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \Pi_{0a}(\mathbf{q}, \mathbf{k}; \omega) D^{ab}(\mathbf{k}, \omega) \Pi_{b0}^{\text{RPA}}(\mathbf{k}, \mathbf{q}; \omega), \end{aligned} \quad (16)$$

where D^{ab} are the free meson propagators that can be seen in any book on quantum field theory. Using the polarization functions of Eqs. (14) and (16), we can obtain the longitudinal response function S_L in the Hartree and RPA approximation, respectively.

For calculating the polarization function Π_{ab} , the meson-nucleon vertices need to be known (i.e., $\Gamma_{a(b)}$ in Eq. (15)). In the Lagrangian density of the derivative coupling model, we note that the coupling parameters

are

$$\begin{aligned} \text{ZM: } & \Gamma_\sigma \rightarrow m^* \Gamma_\sigma, \Gamma_{\omega, \rho} \rightarrow \Gamma_{\omega, \rho}, \\ \text{ZM3: } & \Gamma_\sigma \rightarrow m^* \Gamma_\sigma, \Gamma_{\omega, \rho} \rightarrow m^* \Gamma_{\omega, \rho}. \end{aligned} \quad (17)$$

However, if we directly use these vertices, the calculated results will be incorrect, as will be discussed in Section 3. In fact, we can expand Eq. (6) in $\Gamma_\sigma \sigma / M$ to give

$$m^* = 1 + \sum_{i=1}^{\infty} (-1)^i \left(\frac{\Gamma_\sigma \sigma}{M} \right)^i, \quad (18)$$

where the σ -field is replaced by its classical expectation value in the spirit of the RMF approximation. In the RPA, one σ is absorbed in the σ meson propagator, so the σ terms drop one power. Moreover, we note that every σ term can participate in the interaction of nucleons. Therefore, the σ -meson-nucleon vertex in Eq. (17) should be modified accordingly by $(\sigma dm^*/d\sigma + m^*) \Gamma_\sigma$, i.e., the vertices in Eq. (15) should be replaced by

$$\text{ZM and ZM3: } \Gamma_\sigma \rightarrow m^{*2} \Gamma_\sigma, \quad (19)$$

and other meson-nucleon vertices are the same as in Eq. (17). These forms of vertices have also been used in Ref. [18], which is devoted to the study of the propagation of mesons in a medium.

3 Results and discussion

In this work, we will discuss the effects of the different forms of vertices of Eqs. (17) and (19) on the longitudinal response function. In comparison with the non-spectral method, the calculations of local-density approximation (LDA) for finite nuclei, which treat the response function as a proportion of the space integral of the imaginary part of the polarization function in infinity nuclear matter [19], are also presented below.

Figure 1 illustrates the calculated results of the longitudinal response function S_L versus the transferred energy at $|\mathbf{q}| = 400$ MeV for ^{12}C in the derivative coupling model with the so-called ZM parameters. The empirical data are also shown for comparison. The left panel shows the result in the non-spectral representation. The right panel plots the LDA results. The dashed curves refer to the result without considering the effects of RPA correlation between nucleons (i.e., the so-called Hartree approximation). The solid and dotted curves display the cases in the RPA, but the vertices used in them correspond to Eqs. (19) and (17), respectively. Firstly, we compare the RPA results with two different forms of vertices. It is observed that, whatever the case, the peak of S_L is reduced compared with the Hartree approximation when the RPA correlations between nucleons are considered, that has been pointed out in the earlier literatures within Walecka models [21–23]. However, in both panels, we note that the curve labeled by RPA[Y] shows

a divergence at low energy transfer. It indicates that the case of RPA in which the vertices of Eq. (17) are used is unrealistic in comparison with that with the vertices of Eq. (19). Furthermore, comparing the two panels in Fig. 1, we can see that the calculated peak values in the non-spectral representation are lower and wider

than those in the LDA, and the peaks in the left panel all move to higher transferred energy. The reason is that the non-spectral calculation considers the effects of the nuclear binding energy in nuclei more completely than the LDA method. All these make the non-spectral RPA calculation somewhat closer to the empirical data.

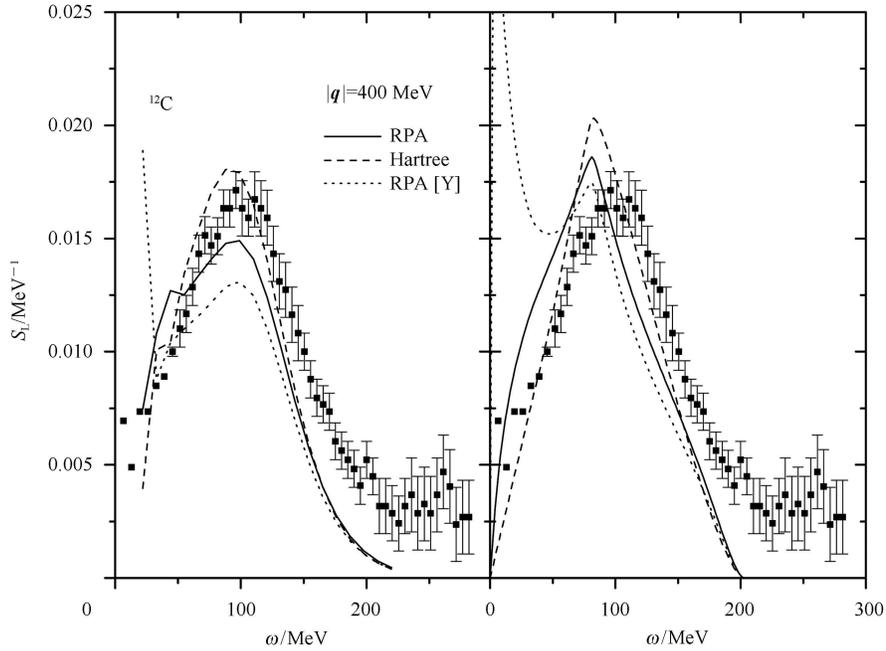


Fig. 1. Longitudinal response function versus transferred energy for ^{12}C in the ZM model. The left panel corresponds to the calculated results with the non-spectral method, and the right panel refers to the case in local-density approximation. Experimental data are taken from Ref. [20].

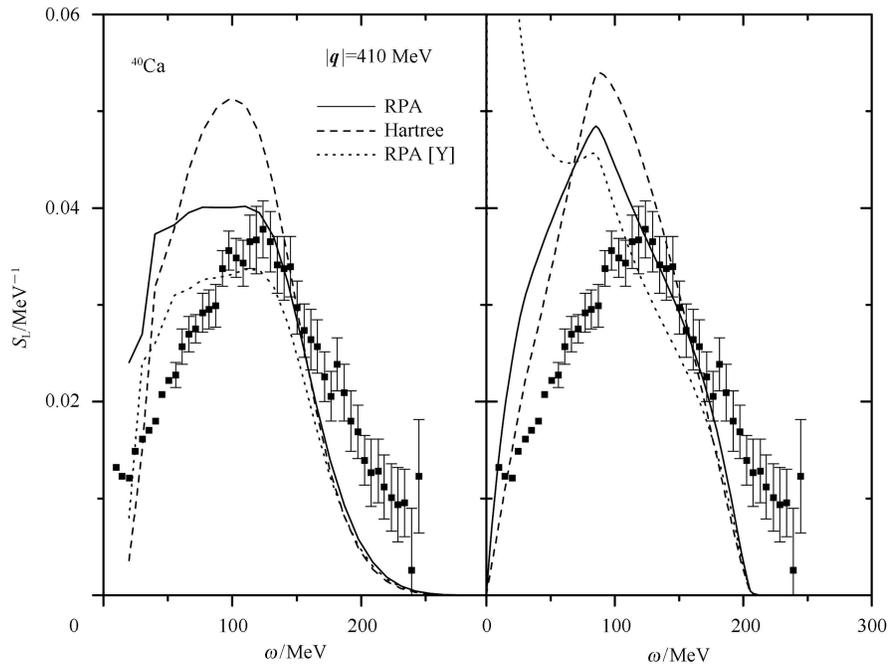


Fig. 2. Same as Fig. 1, but at $|q|=410$ MeV for ^{40}Ca .

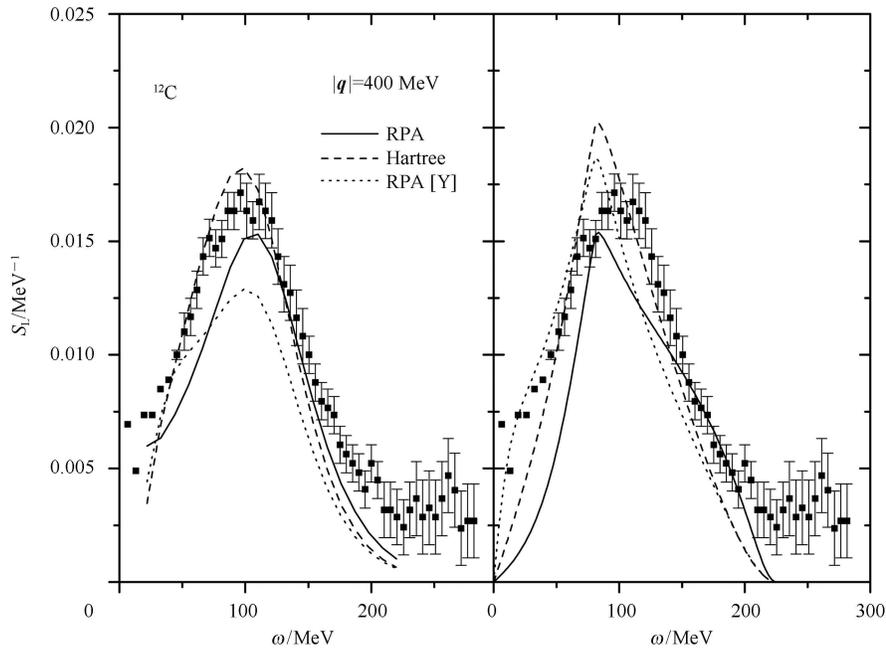


Fig. 3. Same as Fig. 1, but in the ZM3 model.

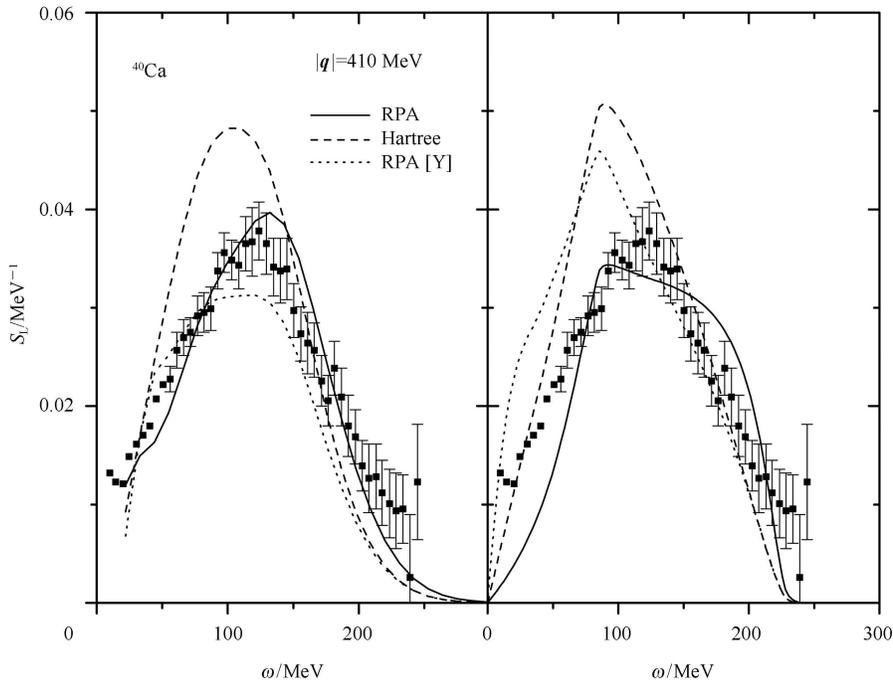


Fig. 4. Same as Fig. 2, but in the ZM3 model.

Figure 2 is the same as Fig. 1 but at $|q| = 410$ MeV for ^{40}Ca . We see that in the right panel the curve with RPA[Y] in the LDA shows a divergence at low energy transfer, and in the left panel the curve with RPA[Y] in the non-spectral representation has a strange bump at a similar position. Not only that, it is surprisingly seen from the left panel that the curve with RPA also presents

an unrealistic bump.

Actually, besides in the left panel in Fig. 2, it can be seen that the curves labeled by RPA in Figs. 1 and 2 all present a bump, although insignificantly, and do not reproduce the empirical data well. Therefore, due to the similarity to the phenomenon found in the nonlinear Walecka model in our earlier work [19], we conclude in

this paper that the derivative coupling model with the the so-called ZM parameters is not so good to use in investigating quasielastic electron scattering. However, with the ZM3 parameters, in Figs. 3 and 4, it is seen that although the curves with RPA[Y] still present an unrealistic bump, those labeled by RPA are close to the empirical data and there is no unrealistic phenomenon in them, especially for the non-spectral calculations plotted in the left panels in Figs. 3 and 4.

4 Summary

In summary, we apply the derivative coupling model with the ZM and ZM3 parameters to study the quasielastic electron scattering and use the RPA to describe the correlation between nucleons in nuclei. In finite nuclei, the calculations in terms of the non-spectral method with the single particle Green's function technique are compared with those in the LDA.

From the present work, three conclusions may be drawn, as follows.

1) For studying quasielastic electron scattering in finite nuclei, two approaches are adopted, i.e., the non-spectral method and the LDA (in fact, there is the third approach, the so-called spectral method [24], which is not considered in the present paper), but the former is somewhat more precise than the latter. It is due to the fact that in a finite nucleus, the non-spectral method describes the polarization or nucleon Green's function more exactly.

2) The vertex terms in the Lagrangian density should not be simply adopted to use in the RPA calculation, but need some corresponding self-consistent modifications, e.g., in this paper, we should use Eq. (19) rather than (17) as the vertices in the RPA calculation.

3) The ZM3 model is more suitable for studying the quasielastic electron scattering than the original ZM version of the derivative coupling model.

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