Electron capture of nuclides ^{52,53,54,55,56}Fe in magnetars *

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Abstract: Based on the theory of relativity in superstrong magnetic fields (SMFs), we have carried out an estimation on electron capture (EC) rates of nuclides ^{52,53,54,55,56}Fe in the SMFs in magnetars. The rates of change of electronic fraction (RCEF) in the EC process are also discussed. The results show that the EC rates increase greatly and even exceeds by 4 orders of magnitude (e.g. ^{54}Fe, ^{55}Fe and ^{56}Fe) in SMF. On the contrary, the RCEF decreases largely and even exceeds by 5 orders of magnitude in the SMF.

Key words: superstrong magnetic field, electron capture, magnetars

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1 Introduction

Supernova explosions break because of unstable nuclear burning and iron nucleus collapse. The electron capture (EC) is very important and plays a key role in this process. Under the supernova explosion conditions, Fuller et al. [1] and Aufderheide et al. [2, 3] investigated the EC rates of many an iron group nucleuses. Dean et al. [4] calculated the EC rates of the iron group nucleus ^{52,53,54,55,56}Fe and found them to be very abundant and their weak interaction reactions are believed to play important roles at the late stages of stellar evolution. Nabi et al. [5] also discussed the weak interaction on ^{54,55,56}Fe; Aufderheide et al. [2, 3] placed ^{53,54,55,56}Fe among the top ten most important capture nuclei during the presupernova evolution. Liu and Lou [6–8] also discussed the weak interaction reactions on ^{52,53,54,55,56}Fe in the presupernova condition. However they discussed the EC in the case of no superstrong magnetic field (SMF). Luo and Peng [9] analyzed the EC rates at nonzero temperatures. However, in their work, they focused only on the ground state transition and paid no attention to the Gamow-Teller (GT) transition.

It is generally known that on the surface of most neutron stars the strengths of magnetic fields are from 10^8 to 10^{13} G [10]. But the strengths of magnetic fields are from 10^{13} to 10^{15} G [11] for some magnetars. How would the SMF effect the EC process? How would the SMF affect the Fermi energy and the electron chemical potential? These are very important and interesting reasons for us to discuss the EC rates in SMF.

Previous research [12–14] shows that an ultrastrong magnetic field affects the electron capture rate and NEL rates greatly and with the increase of the strength of magnetic field, the EC rates and NEL rates decrease. Recent studies [11, 15] have found that strengthening the magnetic field will make the Fermi surface elongate from a spherical surface to a Landau surface along the magnetic field direction, its level is perpendicular to the magnetic field direction and quantized. Thus, we should revise the theory of non-relativistic Landau level.

Based on the p-f shell model and the theory of relativity in an SMF [11, 15], in this paper, we focus on ^{52,53,54,55,56}Fe and investigate the EC rates in an SMF.

2 The EC in an SMF

An SMF B is considered along the z-axis according to the theory of relativity in superstrong magnetic fields. And the Dirac equation can be solved exactly. The positive energy levels of an electron in an SMF are given by [16]

$$\frac{E_n}{m_{\rm e}c^2} = \left[\left(\frac{p_z}{m_{\rm e}c}\right)^2+1+2\left(n+\frac{1}{2}+\sigma\right)b\right]^{\frac{1}{2}},$$

$$(n = 0,\ 1,\ 2,\ 3\cdots), \tag{1}$$

where $b=\frac{B}{B_{cr}}=0.02266B_{12};\ B=10^{12}B_{12};\ B_{cr}=\frac{m_{\rm e}^2c^3}{\rm eh}=4.414{\times}10^{13}$ G [17], and p_z is the electron momentum along the field, σ is the spin quantum number of an electron, $\sigma=-\frac{1}{2}$ when $n=0$; $\sigma=\pm\frac{1}{2}$ when $n\geqslant1$.

In an extremely strong magnetic field ($B\gg B_{cr}$), the Landau column becomes a very long and very narrow cylinder along the magnetic field, the electron chemical

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potential is found by inverting the expression for the lepton number density [12, 13, 14]

$$n_{\rm e}=\rho Y_{\rm e}=\frac{b}{2\pi^2\lambda_{\rm e}^3}\sum_{n=0}^{\infty}q_{n0}\int_0^{\infty}(f_{-\rm e}-f_{+\rm e}){\rm d}p_z, \qquad (2)$$

where $Y_{\rm e}=\dfrac{Z}{A}$ is the electron fraction; ρ is the mass density in g/cm^3. $\lambda_{\rm e}=h/m_{\rm e}c$ is the Compton wavelength, $m_{\rm e}$ is the electron mass and c is the light speed; $q_{n0}=2-\delta_{n0}$ is the electron degenerate number,

$$f_{-\rm e}=\left[1+\exp\left(\frac{\varepsilon_n-U_{\rm F}-1}{kT}\right)\right]^{-1}$$

and

$$f_{+\rm e}=\left[1+\exp\left(\frac{\varepsilon_n+U_{\rm F}+1}{kT}\right)\right]^{-1}$$

are the electron and positron distribution functions, respectively, k is the Boltzmann constant, T is the electron temperature and $U_{\rm F}$ is the electron chemical potential.

The electron capture rates for the kth nucleus (Z,A) in thermal equilibrium at temperature T are given by a sum over the initial parent states i and the final daughter states f [1]

$$\lambda_k=\sum_i\frac{(2J_i+1){\rm e}^{-E_i/kT}}{G(Z,A,T)}\sum_f\lambda_{if}, \qquad (3)$$

where J_i and E_i are the spin and excitation energies of the parent states, $G(Z,A,T)$ is the nuclear partition function. The EC rate from one of the initial states to all possible final states is λ_{if},

$$\lambda_{if}=\frac{\ln2}{(ft)_{if}}f_{if}$$

with the relation

$$\frac{1}{(ft)_{if}}=\frac{1}{(ft)_{if}^{\rm F}}+\frac{1}{(ft)_{if}^{\rm GT}}.$$

The ft-values and the corresponding GT or Fermi transition matrix elements are related by the following expression

$$\frac{1}{(ft)_{if}^{\rm GT}}=\frac{10^{3.596}}{|M_{\rm GT}|_{if}^2},\ \text{and}\ \frac{1}{(ft)_{if}^{\rm F}}=\frac{10^{3.79}}{|M_{\rm F}|_{if}^2}. \qquad (4)$$

The Fermi matrix element and the GT matrix element are given as follows respectively [1]

$$|M_{\rm F}|^2=\frac{1}{2J_i+1}\sum_{m_i}\sum_{m_f}\left|\langle\psi_f m_f|\sum_N\tau_N^-|\psi_i m_i\rangle\right|^2 = T(T+1)-T_Z^i\left(T_Z^i-1\right), \qquad (5)$$

$$|M_{\rm GT}|^2=\frac{1}{2J_i+1}\sum_{m_i}\sum_{m_f}\left|\langle\psi_f m_f|\sum_N\tau_N^-\sigma_N|\psi_i m_i\rangle\right|^2, \qquad (6)$$

where T is the nuclear isospin and $T_z=T_Z^i=(Z-N)/2$ is its projection for the parent or daughter nucleus. $|\psi_i m_i\rangle$ is the initial parent state, $\langle\psi_f m_f|$ is the finial daughter state, and the Fermi matrix element is averaged over the initial and summed over the final nuclear spins. $\sum\limits_N\tau_N^-$ is the minus component of isovector, spatial scalar operator T^- which commutes with the total isospin T^2. σ is the Pauli spin operator and $\sum\limits_N\tau_N^-\sigma_N$ is a spatial vector and an isovector.

The phase space factor in an SMF is defined as [13, 14]

$$f_{if}^{\rm B}=\frac{b}{2}\sum_{n=0}^{\infty}\theta_n, \qquad (7)$$

$$\theta_n=q_{n0}\int_{q_n}^{\infty}(Q_{if}+\varepsilon_n)^2\frac{F(z,\varepsilon_n)}{1+\exp\left(\dfrac{\varepsilon_n-U_F-1}{kT}\right)}{\rm d}p_z, \qquad (8)$$

where $Q_{if}=Q_{00}+E_i-E_f$ is the EC threshold energy; $Q_{00}=M_{\rm p}c^2-M_{\rm d}c^2$, with $M_{\rm p}$ and $M_{\rm d}$ being the masses of the parent nucleus and the daughter nucleus respectively; E_i and E_f are the excitation energies of the i th states and fth state of the nucleus respectively; the ε_n is the total rest mass and kinetic energies; $F(Z,\varepsilon_n)$ is the Coulomb wave correction which is the ratio of the square of the electron wave function distorted by the coulomb scattering potential to the square of wave function of the free electron. The q_n is defined as

$$p_0=\begin{cases}\left(Q_{if}^2-\xi\right)^{\frac{1}{2}}, & \left(Q_{if}\leqslant-\xi^{\frac{1}{2}}\right)\\ 0 & (\text{otherwise})\end{cases}, \qquad (9)$$

where $\xi=1+2\left(n+\dfrac{1}{2}+\sigma\right)b$. On the other hand, the rates of the change of electron fraction (RCEF) are caused by each nucleus. The RCEF plays a key role in stellar evolution and presupernovae outburst. In order to understand how the SMF effects the RCEF, the RCEF due to an EC reaction on the k th nucleus in an SMF is defined as

$$\frac{{\rm d}Y_{\rm e}}{{\rm d}t}({\rm EC})=\dot{Y}_{\rm e}^{\rm EC}=-\frac{X_k}{A_k}\lambda_k^{\rm EC}, \qquad (10)$$

where X_k is the mass fraction of the k th nucleus and A_k is the mass number of the k th nucleus.

3 The study of EC rates in an SMF and discussion

Figures 1 and 2 show the EC rates of nuclides ^{52,53,54,55,56}Fe as a function of $U_{\rm F}$ at $\rho Y_{\rm e}=2.43\times10^5$ g/cm^3; 4.48×10^6 g/cm^3; $\rho Y_{\rm e}=5.86\times10^8$ g/cm^3; 3.3×10^{10} g/cm^3 and $T_9=5$; 9 respectively (T_9 is the temperature in units

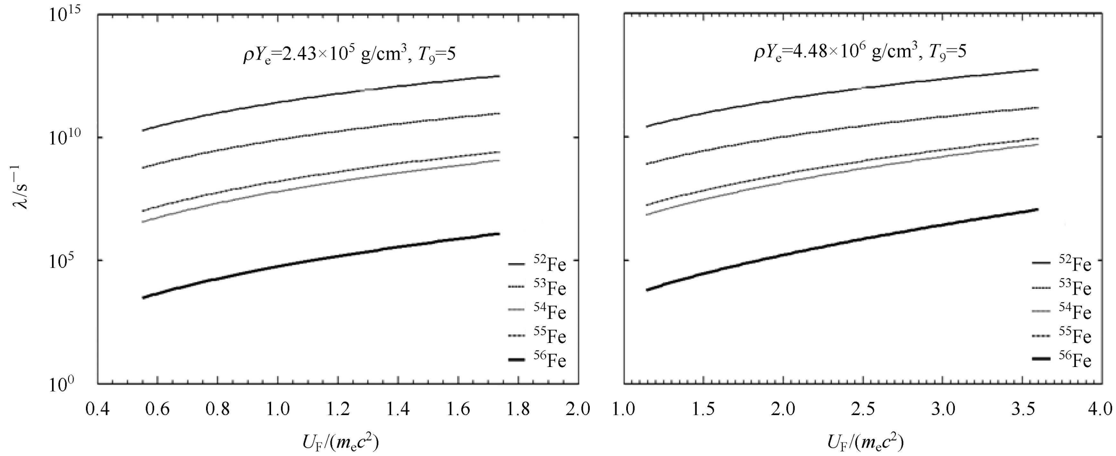

Fig. 1. The EC rates as a function of the electron chemical potential at density $\rho Y_{\rm e}=2.43\times10^5$ g/cm^3; 4.48×10^6 g/cm^3, $T_9=5$ and the strength of the SMF is 10^{13} G$\leqslant B\leqslant10^{18}$ G.

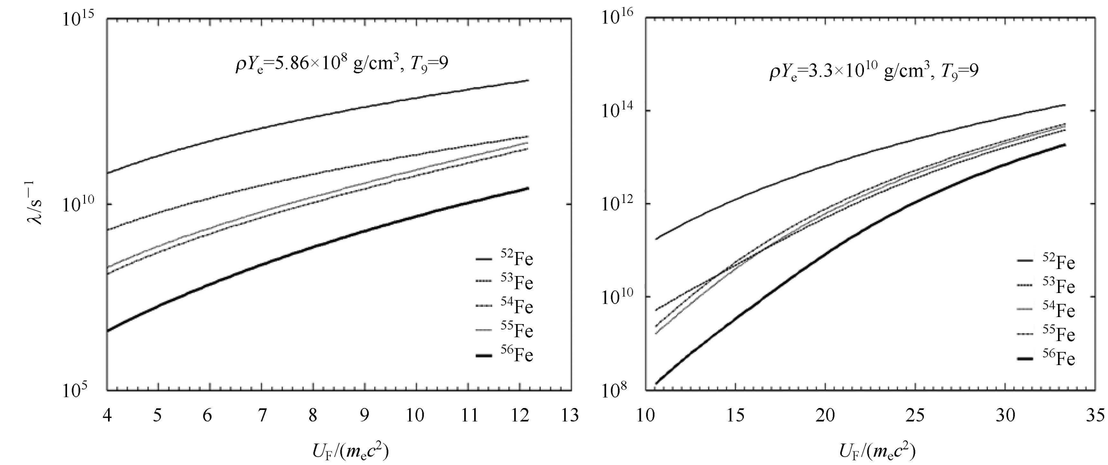

Fig. 2. The EC rates as a function of the electron chemical potential at density $\rho Y_{\rm e}=5.86\times10^8$ g/cm^3; 3.3×10^{10} g/cm^3 $T_9=9$ and the strength of the SMF is 10^{13} G$\leqslant B\leqslant10^{18}$ G.

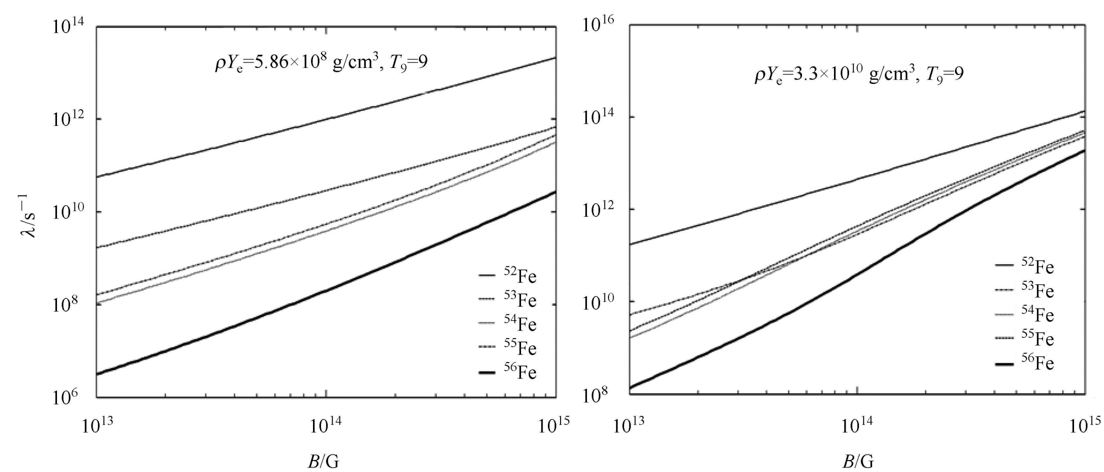

Fig. 3. The EC rates as a function of the magnetic field at density $\rho Y_{\rm e}=5.86\times10^8$ g/cm^3; 3.3×10^{10} g/cm^3 and temperature of $T_9=9$.

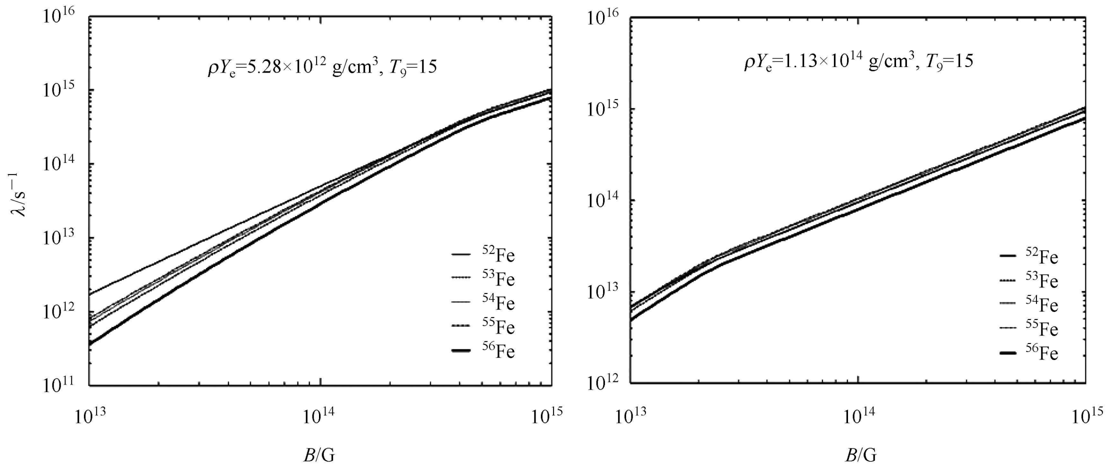

Fig. 4. The EC rates as a function of the magnetic field at density $\rho Y_e = 5.28\times10^{12}$ g/cm^3; 1.13×10^{14} g/cm^3 and temperature of $T_9 = 15$.

of 10^9 K). One can see that the U_F has only a slight effect on the EC rates at relative lower density. However, at relative higher density, the influence is increased. The EC rates increase greatly and even exceed by 5 orders of magnitude (e.g. ^{56}Fe). We find that the SMF has a minor effect on the EC rates for most nuclides at relatively higher temperature in Figs. 3 and 4. However the EC rates are influenced greatly at relatively lower temperatures and are increased by more than 4 orders of magnitude (e.g. ^{54}Fe, ^{55}Fe and ^{56}Fe).

It is known that the RCEF is an important parameter in presupernova explosions and numerical simulations. Electronic abundance variation is one of the fatal parameters in a supernova. Figs. 5 and 6 show the RCEF of ^{52,53,54,55,56}Fe as a function of an SMF at $\rho Y_e = 5.86\times10^7$ g/cm^3, 1.45×10^8 g/cm^3 and $T_9 = 3.40$, 3.80

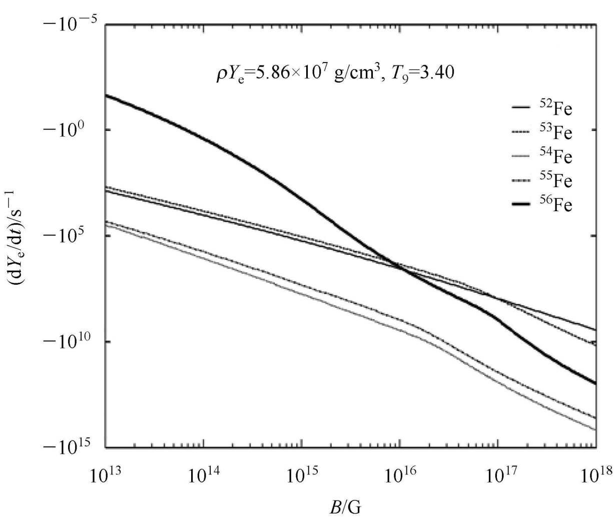

Fig. 5. The $\dot{Y}_e$ as a function of the magnetic field at density $\rho Y_e = 5.86\times10^7$ g/cm^3 and temperature of $T_9 = 3.40$.

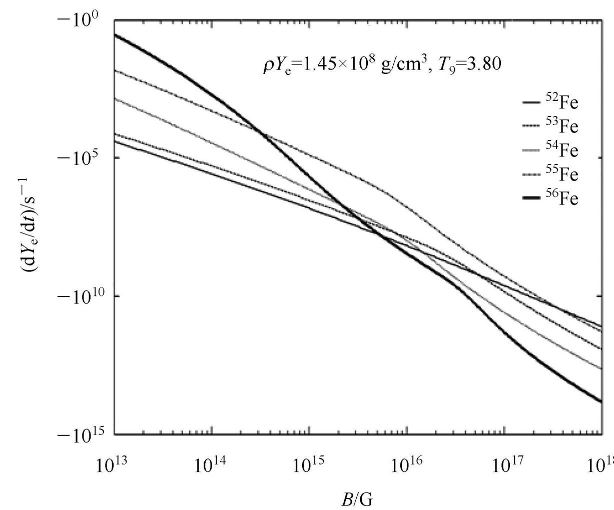

Fig. 6. The $\dot{Y}_e$ as a function of the magnetic field at density $\rho Y_e = 1.45\times10^8$ g/cm^3 and temperature of $T_9 = 3.80$.

respectively. The results show that the EC rates are greatly affected by the SMF. The RCEF decreases greatly and even exceeds by 7 orders of magnitude in SMF (e.g. for ^{56}Fe in Fig. 6).

In summary, by analyzing the effect on EC rates in an SMF for the different nuclides, one can see that the SMF has different effects on EC rates for different nuclides due to the different threshold energy and transition orbits in the EC reaction. On the other hand, the electron chemical potential has different effects on EC rates in different surroundings. This is because the higher the density, the larger the influence on EC rate and the lower the electron energy at low temperature, the higher the electron chemical potential. From Figs. 3 and 4, we find that the SMF has different effects on EC rates under different conditions. The lower the temperature, the larger the influence on EC rate, because the electron energy is so low at lower temperatures that the SMF can strongly affect the EC rates.

The RCEF is a very sensitive parameter in the EC reaction. From Figs. 5 and 6, one can see that the RCEF is influenced largely and decreased greatly in SMF. It is because the EC rates are increased greatly by SMF. With the increase of the density and the magnetic field, the electron chemical potential will be increased greatly. Thus it will create a large amount of electrons whose energies are greater than the Q-values, and join in the EC reactions.

4 Conclusions

In this paper, based on the shell model and the theory of relativity in an SMF, the EC rates of nuclides ^{52,53,54,55,56}Fe in an SMF are investigated. We draw the conclusion that the electron chemical potential affects the EC rates greatly in different astrophysical surroundings. The higher the density, the larger the influence on EC rate by chemical potential is. The EC rates increase and even exceed by 5 orders of magnitude (e.g. ^{56}Fe). On the other hand, the SMF also affects the EC rates greatly at different densities and temperatures. The EC rates (e.g. ^{54}Fe, ^{55}Fe and ^{56}Fe) increase greatly and even exceed by 4 orders of magnitude in the SMF. On the contrary, the RCEF due to EC decreases and even exceeds by 5 orders of magnitude in the SMF.

As everyone knows, weak interaction mediated rates (e.g. EC and beta decay) are the key roles in nuclear physics input of number simulation codes and also the principal factors in the process of supernova explosion and the evolutions of some magnetars. The above conclusions, we concluded possiblly have a significant influence on the further research on nuclear astrophysics, especially on the study of late evolution of magnetars and on r-process nucleosynthesis in neutron star.

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