Mixed symmetry states and electromagnetic transition in the $N=Z$ nucleus ^{28}Si *

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Abstract: The interacting boson model with isospin (IBM-3) has been used to study mixed symmetry states and electromagnetic transitions at low-lying states for a ^{28}Si nucleus. The theoretical calculations show that the 2_4^+ state is the lowest mixed symmetry state in ^{28}Si and the 4_3^+ state is also a mixed symmetry state.

Key words: IBM-3, mixed symmetry states, electromagnetic transitions

PACS: 21.60.FW, 27.30.+t **DOI:** 10.1088/1674-1137/38/10/104102

1 Introduction

The study on the nuclear physics is in the forward position of matter science. Nuclear theories and experiments are very useful for the medical, military, particle physics and nuclear astrophysics [1] and are important for exploring the matter world.

Nuclear is a limited multi-body system. It is difficult to solve the complex multi-body system even though we use modern computers. At present, the human understanding of nuclear is at phenomenological level. The nuclear model theory is developed through researching nuclear phenomenon [2–4]. Based on these models, the nuclear structure is well described, the characteristics of nuclear structure are explained and some experiment results of nuclear are predicted [5–13]. Different models are supplements each other, but one can not replace another.

The interacting boson model IBM [14–16] of nuclei, introduced by Arim a and Iachello, is an algebra model that describes the collective nuclei motion. IBM is a simple model, which stresses the dynamical symmetries of the nuclei system and describes the motion of several collective nuclei. It is also convenient to calculate nuclei properties and spectrum at the collective excitation cross area. In the Bohr and Mottelson theory, the nucleus is an entity with a geometric shape and the nucleus collective motion is studied using dynamic variables of the transformation parameters. This model describes vibration and rotation separately and explains successfully a lot of experimental results. Because of different limit parameters of vibration and rotation, it can not describe the transition from one limit to another. The IBM model gives us the geometric conception and relative dynamical parameters. Based on the methods of the Li group and Li algebra, the IBM reveals the new dynamic symmetric characters and establishes the wave functions. The model successfully describes vibration, rotation and γ-unstable nuclei.

The IBM is proposed originally for researching even-even nuclei. Considering strong pair correlation in nuclei, the IBM model treats pairs of valence nucleons(particles/holes) as bosons with angular momentum $l=0$ (s bosons) or $l=2$ (d bosons). The $U(6)$ symmetry is introduced considering the same standing between the scalar pair correlation and quadrupole pair correlation. Thus even-even nuclei is seen as the system including n of d bosons and s bosons. In the original version (IBM-1), only one kind of boson is considered, and it has been successful in describing various properties of medium and heavy even-even nuclei [17–23]. The proton and the neutron in the heavy nuclei lie in different major shells and the bosons are further classified into proton-boson and neutron-boson in IBM-2. IBM-2 is effective for nuclear states of valence proton and valence neutrons are in the different single particle orbits [24–27]. For lighter nuclei, the valence protons and valence neutrons are filling the same major shell and the isospin should be taken into account, so the IBM has been extended to the interacting boson model with isospin (IBM-3) [28, 29]. The interacting boson model with isospin (IBM-3) has been used to study mixed symmetry states and electromagnetic transitions at low-lying states for a ^{28}Si nucleus.

2 The IBM-3 Hamiltonian for the interacting boson model with isospin

In IBM-3, to take into account the isospin conservation in the framework of boson models, besides proton-

Received 15 November 2013
∗ Supported by NSFC (11165001) and Natural Science Foundation of Inner Mongolia of China (2013MS0117, 2011MS0109)
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proton and neutron-neutron pairs (π, ν bosons), a proton-neutron pair (δ bosons) is also introduced. Thus there are three type of bosons including proton-proton bosons (s_π, d_π), neutron-neutron bosons (s_υ, d_υ) and proton-neutron bosons (s_δ, d_δ). The three s-boson and three d-boson form the isospin $T=1$ triplet. $T_Z=1, 0, -1$ correspond to pp, pn and nn pairs respectively. The wave function has also to be classified by the $U_C(3)\supset SU(2)_T$ group chain, where $SU(2)_T$ is the usual isospin group. The corresponding creation and destruction operators of the bosons are

$$b^+_{lm_l,1m_T}, b_{lm_l,1m_T}, \qquad (1)$$

where $l=0,2$ and $-l\leqslant m_l\leqslant l$, $-1\leqslant m_T\leqslant 1$. The 324 bilinear combination of $b^+_{lm_l,Tm_T}$, b_{lm_l,Tm_T}, generate the unitary group $U(18)$ of IBM-3. In the coupled tensor form, the operators can be written as

$$\left(b^+_{l1}\times\tilde{b}_{l'1}\right)^{(L,T)}_{M_L,M_T} = \sum_{mm'\mu\mu'}\langle lml'm'|LM_L\rangle\langle 1\mu 1\mu'|TM_T\rangle b^+_{lm,1\mu}\tilde{b}_{l'm',1\mu'}, \qquad (2)$$

where the symbol $\langle|\rangle$ is the Clebsch-Gordan coefficient.

The dynamical symmetry group for IBM-3 is $U(18)$, which starts with $U_{sd}(6)\times U_c(3)$ and must contain $SU_T(2)$ and $O(3)$ as subgroups because the isospin and the angular momentum are good quantum numbers. The natural chains of IBM-3 group $U(18)$ are the following [30]

$$\begin{aligned}&U(18)\supset(U_c(3)\supset SU_T(2))\times(U_{sd}(6)\supset U_d(5)\supset O_d(5)\\&\supset O_d(3)),\\&U(18)\supset(U_c(3)\supset SU_T(2))\times(U_{sd}(6)\supset O_{sd}(6)\supset O_d(5)\\&\supset O_d(3)),\\&U(18)\supset(U_c(3)\supset SU_T(2))\times(U_{sd}(6)\supset SU_{sd}(3)\\&\supset O_d(3)).\end{aligned} \qquad (3)$$

The subgroups $U_d(5)$, $O_{sd}(6)$ and $SU_{sd}(3)$ describe vibrational, γ-unstable and rotational nuclei respectively. The isospin-invariant IBM-3 Hamiltonian can be written as [28]

$$H=\varepsilon_s\hat{n}_s+\varepsilon_d\hat{n}_d+H_2, \qquad (4)$$

where

$$\begin{aligned}H_2 =& \frac{1}{2}\sum_{L_2T_2}C_{L_2T_2}((d^+d^+)^{L_2T_2}\cdot(\tilde{d}\tilde{d})^{L_2T_2})\\&+\frac{1}{2}\sum_{T_2}B_{0T_2}((s^+s^+)^{0T_2}\cdot(\tilde{s}\tilde{s})^{0T_2})\\&+\sum_{T_2}A_{2T_2}((s^+d^+)^{2T_2}\cdot(\tilde{d}\tilde{s})^{2T_2})\\&+\frac{1}{\sqrt{2}}\sum_{T_2}D_{2T_2}((s^+d^+)^{2T_2}\cdot(\tilde{d}\tilde{d})^{2T_2})\\&+\frac{1}{2}\sum_{T_2}G_{0T_2}((s^+s^+)^{0T_2}\cdot(\tilde{d}\tilde{d})^{0T_2}),\end{aligned} \qquad (5)$$

and

$$\begin{aligned}&(b^+_1b^+_2)^{L_2T_2}\cdot\left(\tilde{b}_3\tilde{b}_4\right)^{L_2T_2}\\&=(-1)^{(L_2+T_2)}\sqrt{(2L_2+1)(2T_2+1)}\\&\times\left[(b^+_1b^+_2)^{(L_2T_2)}\times\left(\tilde{b}_3\tilde{b}_4\right)^{(L_2T_2)}\right]^{00},\end{aligned} \qquad (6)$$

is the dot product in both angular momentum and isospin. The tilted quantity is defined as

$$\tilde{b}_{lm,m_Z}=(-1)^{(l+m+1+m_Z)}b_{l-m-m_Z}. \qquad (7)$$

There T_2 and L_2 represent the two-boson system isospin and angular momentum. The parameters A, B, C, D and G are the two-body matrix elements by $A_{T_2}=\langle sd20|H_2|sd20\rangle$, with T_2=0, 1, 2; $B_{T_2}=\langle s^20T_2|H_2|s^20T_2\rangle$, $G_{T_2}=\langle s^20T_2|H_2|d^20T_2\rangle$, $D_{T_2}=\langle sd2T_2|H_2|d^22T_2\rangle$ and $C_{L_2T_2}=\langle d^2L_2T_2|H_2|d^2L_2T_2\rangle$, with T_2=0,2, and L_2=0,2,4 and $C_{L_21}=\langle d^2L_21|H_2|d^2L_21\rangle$ with L_2=1, 3. The parameters A_{21}, C_{11}, C_{31} are Majorana parameters which are similar to those in the IBM-2, and they are important to shift the states with mixed symmetry with respect to the holohedral symmetric ones. Microscopic studies of the IBM-3 parameters [31, 32] show that the IBM-3 Hamiltonian depends not only on the boson number but also on the isospin value.

Another important aspect of nuclear structure is its transition properties. The general one-boson E2 operator in IBM3 consists of isovector and isoscalar parts. So the quadrupole operator was expressed as [33]:

$$T(\mathrm{E2})=T^0(\mathrm{E2})+T^1(\mathrm{E2}), \qquad (8)$$

where

$$T^0(\mathrm{E2}) = \alpha_0\sqrt{3}[(s^+\tilde{d})^{20}+(d^+\tilde{s})^{20}]+\beta_0\sqrt{3}[(d^+\tilde{d})]^{20}, \quad (9)$$

$$T^1(\mathrm{E2}) = \alpha_1\sqrt{2}[(s^+\tilde{d})^{21}+(d^+\tilde{s})^{21}]+\beta_1\sqrt{2}[(d^+\tilde{d})]^{21}. \quad (10)$$

The M1 transition is also a one-boson operator with an isoscalar part and an isovector part

$$T(\mathrm{M1})=T^0(\mathrm{M1})+T^1(\mathrm{M1}), \qquad (11)$$

where

$$T^0(\mathrm{M1}) = g_0\sqrt{3}(d^+\tilde{d})^{10}=g_0L/\sqrt{10}, \qquad (12)$$
$$T^1(\mathrm{M1}) = g_1\sqrt{2}(d^+\tilde{d})^{11}, \qquad (13)$$

where g_0 and g_1 are the isoscalar and isovector g-factors respectively, and L is the angular momentum operator. In order to analyze the contribution from the isoscalar and isovector parts in the M1 and E2 transitions, we note the terms in the zero isospin Z component of the transition operators as [31]

$$T^0_{\rm sd}({\rm E2}) = [(s^+\tilde{d})^2+(d^+\tilde{s})^2]_\pi+[(s^+\tilde{d})^2+(d^+\tilde{s})^2]_\delta +[(s^+\tilde{d})^2+(d^+\tilde{s})^2]_\nu, \quad (14)$$

$$T^0_{\rm dd}({\rm E2}) = [(d^+\tilde{d})^2]_\pi+[(d^+\tilde{d})^2]_\delta+[(d^+\tilde{d})^2]_\nu, \quad (15)$$

$$T^1_{\rm sd}({\rm E2}) = [(s^+\tilde{d})^2+(d^+\tilde{s})^2]_\pi-[(s^+\tilde{d})^2+(d^+\tilde{s})^2]_\nu, \quad (16)$$

$$T^1_{\rm dd}({\rm E2}) = [(d^+\tilde{d})^2]_\pi-[(d^+\tilde{d})^2]_\nu, \quad (17)$$

$$T^0_{\rm dd}({\rm M1}) = [(d^+\tilde{d})^1]_\pi+[(d^+\tilde{d})^1]_\delta+[(d^+\tilde{d})^1]_\nu, \quad (18)$$

$$T^1_{\rm dd}({\rm M1}) = [(d^+\tilde{d})^1]_\pi-[(d^+\tilde{d})^1]_\nu. \quad (19)$$

The transition operators are as follows:

$$T({\rm E2}) = \alpha_0 T^0_{\rm sd}({\rm E2})+\beta_0 T^0_{\rm dd}({\rm E2})+\alpha_1 T^1_{\rm sd}({\rm E2}) +\beta_1 T^1_{\rm dd}({\rm E2}), \quad (20)$$

$$T({\rm M1}) = \sqrt{\frac{3}{4\pi}}\{g_0 T^0_{\rm dd}({\rm M1})+g_1 T^1_{\rm dd}({\rm M1})\}. \quad (21)$$

We will use these results in our investigation.

3 Symmetry state and mixed symmetry state

The mixed symmetry state is a new kind of dynamical symmetry nuclei state that the Neutron-Proton Interacting Boson Model predicts. The symmetry states correspond to the holohedral symmetry of neutron boson and proton boson. The mixed symmetry state corresponds to the part symmetry and the part antisymmetry of the neutron boson and proton boson. There are orbit magnetic dipole excitations from light nuclei ^{46}Ti to heavy nuclei ^{238}U since Bohel proved mixed symmetry with experiments in 1984. Majorana interacting is very important for mixed symmetry states [25, 26]. The parameters A_{21}, C_{11}, C_{31} are Majorana parameters which are similar to those in the IBM-2.

The agreement between the model calculation and experimental data is reasonably consistent by adjusting the Hamiltonian parameters. The parameters of the Hamiltonian are shown in Table 1.

The Majorana parameters are $C_{11}=-6.00$, $C_{31}=-5.10$ and $A_{21}=-4.54$. The variation of the parameters greatly affects the mixed symmetry states and the influence is small for the symmetry state. The states energy are calculated around the Majorana parameter $A_{21}=-4.54$. Fig. 1 shows the variations of the energy of these states with the parameters.

Figure 1 shows that the variation of the Majorana parameter A_{21} greatly affect the 1^+_1, 2^+_4, 3^+_2 and 4^+_3 state. The 1^+_1, 2^+_2, $3^+_1$and 4^+_2 states are the mixed symmetry states. The 3^+_1 state with 6.276 MeV is the symmetry state and it is the same with that of Ref. [34].

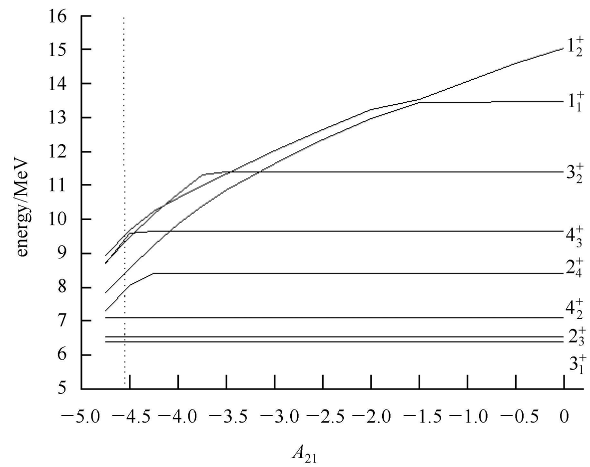

Fig. 1. The variations of the energy of the states with the Majorana parameter A_{21}

The mixed symmetry wave functions are

$$|1^+_1\rangle = 0.3815|d^1_\nu d^1_\pi d^1_\delta s^3_\delta\rangle+0.2205|d^1_\nu d^1_\pi d^3_\delta s^1_\delta\rangle +0.3115\{|d^1_\nu s^2_\nu d^2_\pi s^1_\pi\rangle-|d^2_\nu s^1_\nu d^1_\pi s^2_\pi\rangle+|s^2_\nu d^1_\pi s^1_\pi d^2_\delta\rangle +|d^1_\nu s^1_\nu d^2_\delta s^2_\pi\rangle-|d^1_\nu s^1_\nu d^1_\pi s^1_\pi d^1_\delta s^1_\delta\rangle\}+\cdots,$$

$$|2^+_4\rangle = 0.3014|d^1_\nu d^1_\pi d^1_\delta s^3_\delta\rangle+0.2091|d^1_\nu d^1_\pi d^3_\delta s^1_\delta\rangle -0.2534\{|s^2_\nu d^1_\pi s^1_\pi d^2_\delta\rangle+|d^1_\nu s^1_\nu s^2_\pi d^2_\delta\rangle\} -0.1964\{|d^2_\nu s^1_\nu d^1_\pi s^2_\pi\rangle+|d^1_\nu s^2_\nu d^2_\pi s^1_\pi\rangle +|d^1_\nu s^1_\nu d^1_\pi s^1_\pi d^1_\delta s^1_\delta\rangle\}+\cdots,$$

$$|3^+_2\rangle = 0.3224|d^1_\nu d^1_\pi d^1_\delta s^3_\delta\rangle+0.2632\{|d^1_\nu s^2_\nu d^2_\pi s^1_\pi\rangle-|d^2_\nu s^1_\nu d^1_\pi s^2_\pi\rangle +|s^2_\nu d^1_\pi s^1_\pi d^2_\delta\rangle+|d^1_\nu s^1_\nu s^2_\pi d^2_\delta\rangle-|d^1_\nu s^1_\nu d^1_\pi s^1_\pi d^1_\delta s^1_\delta\rangle\} +0.1864|d^1_\nu d^1_\pi d^3_\delta s^1_\delta\rangle+\cdots,$$

$$|4^+_3\rangle = -0.3042|s^2_\nu s^2_\pi d^2_\delta\rangle+0.2634|d^1_\nu d^1_\pi s^4_\delta\rangle +0.2522|d^1_\nu d^1_\pi d^2_\delta s^2_\delta\rangle-0.2371|d^1_\nu s^1_\nu d^1_\pi s^1_\pi d^2_\delta\rangle -0.2282|d^1_\nu d^1_\pi s^1_\nu s^1_\pi s^2_\delta\rangle+0.2151|d^1_\nu s^2_\nu d^1_\pi s^2_\pi\rangle+\cdots.$$

The symmetry state functions are

$$|0^+_1\rangle = -0.3938|s^2_\nu s^2_\pi d^1_\nu d^1_\pi\rangle-0.3194|s^3_\nu s^3_\pi\rangle +0.2842|s^4_\delta(d^2_\delta)_0\rangle+0.2258|s^2_\nu s^2_\pi s^2_\delta\rangle -0.2273\{|s^3_\nu s^1_\pi(d^2_\pi)_0\rangle+|s^1_\nu s^3_\pi(d^2_\nu)_0\rangle\} -0.1956\{|s^1_\nu s^1_\pi s^4_\delta\rangle+|s^1_\nu s^1_\pi s^2_\delta(d^2_\delta)_0\rangle\} +0.1856\{|s^2_\nu s^1_\pi s^1_\delta d^1_\pi d^1_\delta\rangle+|s^1_\nu s^2_\pi s^1_\delta d^1_\nu d^1_\delta\rangle +|s^1_\nu s^1_\pi s^2_\delta d^1_\nu d^1_\pi\rangle\}+0.1785|s^6_\delta\rangle+0.1712|s^2_\delta(d^4_\delta)_0\rangle$$

Table 1. The parameters of Hamiltonian of nuclei ^{28}Si.

$A_i(i=0,1,2)$	$C_{i0}(i=0,2,4)$	$C_{i2}(i=0,2,4)$	$C_{i1}(i=1,3)$	$B_i(i=0,2)$	$D_i(i=0,2)$	$G_i(i=0,2)$
−10.46	−12.08	2.92	−6.00	−10.12	0.00	−0.76
−4.54	−10.44	4.56	−5.10	4.88	0.00	−0.76
4.54	−9.18	5.82				

$$\begin{aligned}
&\quad -0.1608\{|s_\nu^1 s_\delta^3 d_\pi^1 d_\delta^1\rangle+|s_\pi^1 s_\delta^3 d_\nu^1 d_\delta^1\rangle\}+\cdots,\\
|2_1^+\rangle &= 0.3100\{|s_\nu^3 s_\pi^2 d_\pi^1\rangle+|s_\nu^2 s_\pi^3 d_\nu^1\rangle\}-0.2694|s_\delta^3(d_\delta^3)_2\rangle\\
&\quad -0.2450|s_\delta^5 d_\delta^1\rangle+0.2208\{|s_\nu^1 s_\pi^2(d_\nu^2)_0 d_\pi^1\rangle\\
&\quad +|s_\nu^2 s_\pi^1 d_\nu^1(d_\pi^2)_0\rangle\}+0.2192|s_\nu^1 s_\pi^1 s_\delta^3 d_\delta^1\rangle\\
&\quad +0.1893\{|s_\nu^2 s_\pi^1 d_\nu^1(d_\pi^2)_4\rangle+|s_\nu^1 s_\pi^2(d_\nu^2)_4 d_\pi^1\rangle\}\\
&\quad +0.1790\{|s_\nu^2 s_\pi^2 s_\delta^1 d_\delta^1\rangle+|s_\nu^2 s_\pi^1 s_\delta^2 d_\pi^1\rangle_+|s_\nu^1 s_\pi^2 s_\delta^2 d_\nu^1\rangle\}\\
&\quad -0.1472|s_\nu^1 s_\pi^1 s_\delta^1 d_\nu^1 d_\pi^1 d_\delta^1\rangle+0.1320|s_\nu^1 s_\pi^1 s_\delta^1(d_\delta^3)_2\rangle+\cdots,\\
|4_1^+\rangle &= 0.4158|s_\nu^2 s_\pi^2 d_\nu^1 d_\pi^1\rangle+0.3001|s_\delta^4(d_\delta^2)_4\rangle\\
&\quad -0.2401\{|s_\nu^3 s_\pi^1(d_\pi^2)_4\rangle+|s_\nu^1 s_\pi^3(d_\nu^2)_4\rangle\}\\
&\quad +0.2255|s_\delta^2(d_\delta^4)_4\rangle-0.2079|s_\nu^1 s_\pi^1 s_\delta^2(d_\delta^2)_4\rangle\\
&\quad +0.1960\{|s_\nu^2 s_\pi^1 s_\delta^1 d_\pi^1 d_\delta^1\rangle+|s_\nu^1 s_\pi^2 s_\delta^1 d_\nu^1 d_\delta^1\rangle\\
&\quad +|s_\nu^1 s_\pi^1 s_\delta^2 d_\nu^1 d_\pi^1\rangle\}-0.1712\{|s_\nu^1 s_\pi^1(d_\nu^2)_0(d_\pi^2)_4\rangle\\
&\quad +|s_\nu^1 s_\pi^1(d_\nu^2)_4(d_\pi^2)_0\rangle\}-0.1697\{|s_\nu^1 s_\delta^3 d_\pi^1 d_\delta^1\rangle\\
&\quad +|s_\pi^1 s_\delta^3 d_\nu^1 d_\delta^1\rangle\}-0.1447\{|s_\nu^2 d_\nu^1(d_\pi^3)_2\rangle\\
&\quad +|s_\pi^2(d_\nu^3)_2 d_\pi^1\rangle\}-0.1274\{|s_\nu^1 s_\pi^1(d_\nu^2)_2(d_\pi^2)_4\rangle\\
&\quad +|s_\nu^1 s_\pi^1(d_\nu^2)_4(d_\pi^2)_2\rangle\}+\cdots,\\
|6_1^+\rangle &= -0.4087\{|s_\nu^2 s_\pi^1 d_\nu^1(d_\pi^2)_4\rangle+|s_\nu^1 s_\pi^2(d_\nu^2)_4 d_\pi^1\rangle\}\\
&\quad +0.3406|s_\delta^3(d_\delta^3)_6\rangle+0.2725|s_\nu^1 s_\pi^1 s_\delta^1 d_\nu^1 d_\pi^1 d_\delta^1\rangle\\
&\quad -0.2044\{|s_\nu^1 s_\delta^2 d_\pi^1(d_\delta^2)_4\rangle+|s_\pi^1 s_\delta^2 d_\nu^1(d_\delta^2)_4\rangle\}\\
&\quad -0.1669\{|s_\nu^1 s_\pi^1 s_\delta^1(d_\delta^3)_6\rangle+|s_\delta^3 d_\nu^1 d_\pi^1 d_\delta^1\rangle\}\\
&\quad +0.1639|s_\delta^1(d_\delta^5)_6\rangle-0.1423\{|s_\pi^1(d_\nu^3)_2(d_\pi^2)_4\rangle\\
&\quad +|s_\nu^1(d_\nu^2)_4(d_\pi^3)_2\rangle\}-0.1362\{|s_\nu^3(d_\pi^3)_6\rangle+|s_\pi^3(d_\nu^3)_6\rangle\\
&\quad +|s_\nu^2 s_\pi^1 d_\pi^1(d_\delta^2)_4\rangle_+|s_\nu^2 s_\delta^1(d_\pi^2)_4 d_\delta^1\rangle+|s_\nu^1 s_\pi^2 d_\nu^1(d_\delta^2)_4\rangle\}\\
&\quad -0.1362\{|s_\nu^1 s_\delta^2 d_\nu^1(d_\pi^2)_4\rangle+|s_\pi^2 s_\delta^1(d_\nu^2)_4 d_\delta^1\rangle\\
&\quad +|s_\pi^1 s_\delta^2 d_\pi^1(d_\nu^2)_4\rangle\}+\cdots.\\
|2_2^+\rangle &= 0.4158|s_\nu^2 s_\pi^2 d_\nu^1 d_\pi^1\rangle-0.3001|s_\delta^4(d_\delta^2)_2\rangle\\
&\quad +0.2401\{|s_\nu^3 s_\pi^1(d_\pi^2)_2\rangle+|s_\nu^1 s_\pi^3(d_\nu^2)_2\rangle\}\\
&\quad -0.2255|s_\delta^2(d_\delta^4)_2\rangle+0.2079|s_\nu^1 s_\pi^1 s_\delta^2(d_\delta^2)_2\rangle\\
&\quad -0.1960\{|s_\nu^2 s_\pi^1 s_\delta^1 d_\pi^1 d_\delta^1\rangle+|s_\nu^1 s_\pi^2 s_\delta^1 d_\nu^1 d_\delta^1\rangle\\
&\quad +|s_\nu^1 s_\pi^1 s_\delta^2 d_\nu^1 d_\pi^1\rangle\}+0.1712\{|s_\nu^1 s_\pi^1(d_\nu^2)_0(d_\pi^2)_2\rangle\\
&\quad +|s_\nu^1 s_\pi^1(d_\nu^2)_2(d_\pi^2)_0\rangle\}+0.1710|s_\nu^1 s_\pi^1(d_\nu^2)_4(d_\pi^2)_4\rangle\\
&\quad +0.1697\{|s_\nu^1 s_\delta^3 d_\pi^1 d_\delta^1\rangle+|s_\pi^1 s_\delta^3 d_\nu^1 d_\delta^1\rangle\}\\
&\quad +0.1447\{|s_\nu^2 d_\nu^1(d_\pi^3)_2\rangle+|s_\pi^2(d_\nu^3)_2 d_\pi^1\rangle\}+\cdots.\\
|4_2^+\rangle &= -0.3406|s_\delta^3(d_\delta^3)_4\rangle+0.2958\{|s_\nu^2 s_\pi^1 d_\nu^1(d_\pi^2)_2\rangle\\
&\quad +|s_\nu^1 s_\pi^2(d_\nu^2)_2 d_\pi^1\rangle\}+0.2820\{|s_\nu^2 s_\pi^1 d_\nu^1(d_\pi^2)_4\rangle\\
&\quad +|s_\nu^1 s_\pi^2(d_\nu^2)_4 d_\pi^1\rangle\}-0.1972|s_\nu^1 s_\pi^1 s_\delta^1 d_\nu^1 d_\pi^1 d_\delta^1\rangle\\
&\quad -0.1880|s_\nu^1 s_\pi^1 s_\delta^1 d_\nu^1 d_\pi^1 d_\delta^1\rangle+0.1669|s_\nu^1 s_\pi^1 s_\delta^1(d_\delta^3)_4\rangle\\
&\quad -0.1639|s_\delta^1(d_\delta^5)_4\rangle+0.1479\{|s_\nu^1 s_\delta^2 d_\pi^1(d_\delta^2)_2\rangle\\
&\quad +|s_\pi^1 s_\delta^2 d_\nu^1(d_\delta^2)_2\rangle\}+0.1410\{|s_\nu^1 s_\delta^2 d_\pi^1(d_\delta^2)_4\rangle\\
&\quad +|s_\pi^1 s_\delta^2 d_\nu^1(d_\delta^2)_4\rangle\}+0.1362\{|s_\nu^3(d_\pi^3)_4\rangle\\
&\quad +|s_\pi^3(d_\nu^3)_4\rangle\}+\cdots.\\
|2_3^+\rangle &= -0.3347\{|s_\nu^3 s_\pi^2 d_\nu^1\rangle+|s_\nu^2 s_\pi^3 d_\nu^1\rangle\}+0.2646|s_\delta^5 d_\delta^1\rangle\\
&\quad -0.2367|s_\nu^1 s_\pi^1 s_\delta^3 d_\delta^1\rangle-0.2184|s_\delta^1(d_\delta^5)_2\rangle
\end{aligned}$$

$$\begin{aligned}
&\quad +0.1932\{|s_\nu^2 s_\pi^2 s_\delta^1 d_\delta^1\rangle+|s_\nu^2 s_\pi^1 s_\delta^2 d_\pi^1\rangle+|s_\nu^1 s_\pi^2 s_\delta^2 d_\nu^1\rangle\}\\
&\quad +0.1657\{|s_\nu^1(d_\nu^2)_0(d_\pi^3)_2\rangle+|s_\pi^1(d_\nu^3)_2(d_\pi^2)_0\rangle\}\\
&\quad +0.1586|s_\delta^3(d_\delta^3)_2\rangle+0.1421\{|s_\pi^1(d_\nu^3)_2(d_\pi^2)_4\rangle\\
&\quad +|s_\nu^1(d_\nu^2)_4(d_\pi^3)_2\rangle\}+0.1300\{|s_\nu^1 s_\pi^2(d_\nu^2)_0 d_\pi^1\rangle\\
&\quad +|s_\nu^2 s_\pi^1 d_\nu^1(d_\pi^2)_0\rangle\}+\cdots.\\
|3_1^+\rangle &= 0.3454\{|s_\nu^2 s_\pi^1 d_\nu^1(d_\pi^2)_2\rangle-|s_\nu^1 s_\pi^2(d_\nu^2)_2 d_\pi^1\rangle\}\\
&\quad +0.3406|s_\delta^3(d_\delta^3)_3\rangle+0.2303|s_\nu^1 s_\pi^1 s_\delta^1 d_\nu^1 d_\pi^1 d_\delta^1\rangle\\
&\quad -0.1285\{|s_\nu^2 s_\pi^1 d_\nu^1(d_\pi^2)_4\rangle-|s_\nu^1 s_\pi^2(d_\nu^2)_4 d_\pi^1\rangle\}\\
&\quad +0.1712\{|s_\nu^1 s_\delta^2 d_\pi^1(d_\delta^2)_2\rangle+|s_\pi^1 s_\delta^2 d_\nu^1(d_\delta^2)_2\rangle\}\\
&\quad -0.1669|s_\nu^1 s_\pi^1 s_\delta^1(d_\delta^3)_3\rangle+0.1639|s_\delta^1(d_\delta^5)_3\rangle\\
&\quad -0.1456|s_\nu^1 s_\pi^1 s_\delta^1 d_\nu^1 d_\pi^1 d_\delta^1\rangle-0.1410|s_\delta^3 d_\nu^1 d_\pi^1 d_\delta^1\rangle\\
&\quad -0.1362\{|s_\nu^3(d_\pi^3)_3\rangle+|s_\pi^3(d_\nu^3)_3\rangle\}\\
&\quad -0.1203\{|s_\nu^1(d_\nu^2)_2(d_\pi^3)_2\rangle-|s_\pi^1(d_\nu^3)_2(d_\pi^2)_2\rangle\}+\cdots.
\end{aligned}$$

"……" represents some smaller component. From the above wave function expressions, we can see that every state is a δ boson. The δ boson is very important.

4 Electromagnetic transition

The electromagnetic transition of ^{28}Si is studied based on a calculation of the energy level and the wave function of ^{28}Si. The parameters of the quadrupole transition (E2) and magnetic dipole (M1) transition are determined by the experimental data of electromagnetic transition of ^{28}Si. They are $\alpha_0=\beta_0=0.0233$ eb, $\alpha_1=\beta_1=0.0122$ eb, g_0= 0 μN, $g_1=0.2335$ μN. The results are shown in Table 2.

Table 2 shows that the theoretical calculation results accord with experimental data. Ref. [14] shows that the high spin mixed symmetry state can be the yrast state under certain conditions. The transition probability from the yrast state to its lower energy level is small. If there is the yrast state, it will be a isomeric state. We also calculate the electric quadrupole moment of ^{28}Si. The electric quadrupole moment of 2_1^+, 2_2^+, 4_1^+ are separately $Q(2_1^+)=0.05415$ eb, $Q(2_2^+)=-0.02018$ eb, and $Q(4_1^+)$=0.06924 eb.

5 Conclusion

The interacting boson model with isospin (IBM-3) has been used to study mixed symmetry states and electromagnetic transitions at low-lying states for a ^{28}Si nucleus. The theoretical calculations show that the 2_4^+ state is the lowest mixed symmetry state in ^{28}Si and the 1_1^+, 2_4^+, 3_2^+ and 4_3^+ states are also mixed symmetry states. The wave functions of the mixed symmetry state and holohedral symmetry state are given. The theoretical results of spectra and electromagnetic values accord with experimental data. The results show that the interacting boson model with isospin (IBM-3) can greatly describe the low-lying level properties of ^{28}Si.

Table 2. Experimental and calculated $B(\mathrm{E2})(\mathrm{e}^2\mathrm{b}^2)$ and $B(\mathrm{M1})$ (μ_{N}^2) for the nuclei ^{28}Si.

$J_i^+\to J_f^+$	T^0_{sd}(E2)	T^0_{dd}(E2)	T^1_{sd}(E2)	T^1_{dd}(E2)	T(E2)	$B(\mathrm{E2})_{\mathrm{cal}}$	$B(\mathrm{E2})_{\mathrm{exp}}$	T^1_{dd}(M1)	T(M1)	$B(\mathrm{M1})_{\mathrm{cal}}$	$B(\mathrm{M1})_{\mathrm{exp}}$
$2_1^+\to 0_1^+$	−7.7388	−0.439E-05	0.989E-03	−0.525E-07	−0.180302	0.00650	0.00667				
$2_2^+\to 2_1^+$	3.9576	−0.301E-05	−0.372E-02	−0.479E-07	0.923E-01	0.00849		−0.134E-07	−0.996E-09	0.00000	
$2_2^+\to 0_1^+$	0.963E-05	−2.0982	0.163E-06	0.303E-02	−0.488E-01	0.00048					
$2_2^+\to 0_2^+$	0.428E-05	1.4179	−0.135E-07	−0.498E-02	0.329E-01	0.00022					
$2_3^+\to 0_2^+$	5.6415	0.315E-05	−0.339E-02	0.304E-06	0.131405	0.00345					
$2_3^+\to 0_1^+$	−0.07395	−0.718E-06	0.213E-02	−0.409E-06	−0.169E-02	0.00000					
$2_3^+\to 2_1^+$	0.223E-05	0.3464	0.382E-06	−0.138E-02	0.805E-02	0.00006		0.382E-06	0.284 E-07	0.00000	
$2_3^+\to 2_2^+$	0.805E-01	−0.125E-06	−0.238E-02	−0.995E-07	0.185E-02	0.00000		0.215E-06	0.159 E-07	0.00000	
$2_4^+\to 2_1^+$	−0.103E-05	−0.814E-03	−0.117E-06	−0.814E-03	−0.289E-04	0.00000		0.216E-06	0.161 E-07	0.00000	
$2_4^+\to 2_2^+$	−0.260E-02	−0.718E-06	−0.260E-02	0.242E-07	−0.924E-04	0.00000		−0.429E-07	−0.319 E-08	0.00000	
$2_4^+\to 2_3^+$	−0.523E-05	−0.176E-02	−0.452E-06	−0.175E-02	−0.623E-04	0.00000		−0.159E-06	−0.118 E-07	0.00000	
$2_4^+\to 0_1^+$	0.247E-02	0.546E-06	0.247E-02	0.726E-08	0.877E-04	0.00000					
$0_2^+\to 2_1^+$	0.660E-01	0.227E-05	−0.703E-03	−0.219E-07	0.153E-02	0.00001	0.00434				
$0_2^+\to 2_2^+$	0.191E-05	0.634E+00	−0.604E-08	−0.223E-02	0.147E-01	0.00109					
$1_1^+\to 0_1^+$								−0.665E-07	−0.494 E-08	0.00000	0.00014
$1_1^+\to 0_2^+$								0.130E-06	0.966 E-08	0.0000	
$1_1^+\to 2_1^+$	−0.561E-06	−0.517E-03	−0.249E-06	−0.517E-03	−0.184E-04	0.00000		−0.285E-07	−0.212 E-08	0.0000	
$1_1^+\to 2_2^+$	0.686E-02	−0.589E-07	0.685E-02	−0.623E-07	0.243E-03	0.00000		−0.101E-06	−0.751 E-08	0.0000	
$1_1^+\to 2_3^+$	0.362E-06	−0.112E-02	−0.370E-06	−0.112E-02	−0.396E-04	0.00000		−0.969E-07	−0.721 E-08	0.0000	
$3_1^+\to 2_1^+$	0.852E-05	−0.108E+01	−0.664E-07	−0.9320E-03	−0.252E-01	0.00045	0.6565E-6	0.144E-06	0.107 E-07	0.00000	0.00046
$3_1^+\to 2_2^+$	4.00947	0.146E-05	0.117E-01	−0.164E-07	0.936E-01	0.00625		0.384E-07	0.286 E-08	0.00000	
$3_1^+\to 2_3^+$	−0.116E-05	−1.0099	−0.484E-06	−0.201E-02	−0.235E-01	0.00040		0.961E-07	0.715 E-08	0.00000	
$3_1^+\to 4_1^+$	1.88479	0.187E-07	0.217E-03	−0.649E-08	0.439E-01	0.00248		0.914E-09	0.679 E-10	0.00000	
$4_1^+\to 2_1^+$	−5.31405	−0.179E-05	0.693E-03	−0.200E-07	−0.123808	0.00852	0.00697				
$4_1^+\to 2_2^+$	0.671E-05	−0.89301	−0.107E-06	0.429E-02	−0.207E-01	0.00024					
$4_2^+\to 2_1^+$	0.755E-05	1.04494	0.436E-06	−0.238E-02	0.243E-01	0.00033					
$4_2^+\to 2_2^+$	3.86858	−0.184E-05	−0.133E-01	0.162E-06	0.899E-01	0.00450					
$4_2^+\to 2_3^+$	0.502E-05	0.97355	−0.475E-06	−0.514E-02	0.226E-01	0.00028					
$4_2^+\to 4_1^+$	−0.275E+01	−0.487E-06	0.390E-02	−0.191E-06	−0.641E-01	0.00411			−0.259 E-07	0.0000	

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