Systematic analysis of fusion barrier heights and positions for proton projectiles using the single folding model*

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Abstract: The nuclear potentials between protons and different target nuclei are calculated by using the single folding model with the density-dependent nucleon-nucleon interaction. The fusion barrier heights and positions for proton projectiles fusing with different target nuclei with masses from 51 amu to 139 amu are systematically shown, with charge numbers and root-mean-square radii of the interacting nuclei. The parameterized formulas for the fusion barrier height and position are obtained for proton projectile fusing with the different nuclei. The calculated results of parameterized formulas are compared to empirical values, as well as those of the proximity potential and Akyüz-Winther (AW) potential. It is shown that the calculated results agree perfectly with theirs. The parameterized formulas can reproduce the exact barrier heights and positions for proton fusion systems.

Key words: fusion reaction, fusion barrier height and position, parametrization

PACS: 25.70.Jj, 24.10.-i **DOI:** 10.1088/1674-1137/38/3/034101

1 Introduction

With the development of heavy-ion accelerators, people can study the characteristics of nuclear matter from nuclear reactions. Fusion reaction has become one of the most important research methods. Due to some new phenomena observed in fusion reactions over recent years, many new models have been developed to try to give a reasonable theoretical explanation.

Experimental analysis of fusion reactions is often done by calculating the fusion excitation function and extracting the barrier height. So the barrier height and position are two important parameters of the fusion reaction and can provide valuable information for studying reaction mechanisms. The nuclear potential and Coulomb potential traditionally contribute to the barrier height and position. The Coulomb potential can be accurately obtained. However, in the calculation of nuclear potential, there have been many methods and models for fusion reactions [1–4]. Among these models, the double-folding model (DFM) has been used for many cases. Through the integral of the distribution of nuclear matter, the interaction between the nuclei can be obtained. What is more, it can also be successfully applied to the nuclear reactions and nuclei decay, including elastic scattering [5, 6], fusion reaction [3, 4], and $\alpha({}^4_2\mathrm{He})$ decay [7–9]. In the double-folding model, the nuclei involved in the reaction system have a certain density distribution. But a proton can usually be regarded as a point charge, so that only the density distribution of the target nucleus is considered in the single-folding model. The single-folding model has previously been used to study the proton decays of nuclei [10].

When protons are chosen as the projectile fusing with the different nuclei, the barrier height and position can be obtained using the single-folding model. For heavy ion systems, including α, the parameterized formulas of the barrier height and position have been given in Refs. [11, 12]. Due to the different density distributions for protons, α and other heavy ions, in this article we will explore an ideal parameterized formula of barrier height and position for proton fusion systems using the DFPD2 program [13], which is combined with the single folding model.

2 Calculation method

In the single-folding model the nuclear potential V_{N} or the Coulomb potential V_{C} is given by [10, 14]

$$V_{\mathrm{NorC}}(R)=\int\rho(\vec{r})v(|\vec{r}-\vec{R}|)\mathrm{d}^3r, \tag{1}$$

Received 8 April 2013, Revised 24 October 2013

* Supported by National Nature Science Foundation of China (11035007, 11235002, 11175011) and Special Joint Project of Beijing Municipal Education Commission

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where $\vec{R}$ and $\vec{r}$ represent the vectors of the incident proton and target nuclei, respectively. $v|\vec{r}-\vec{R}|$ denotes the effective nucleon-nucleon (NN) interaction. Here the projectile is regarded as a point charge, and the integral is required only for the target nucleus. For the density distribution function of the target nucleus $\rho(\vec{r})$, the spherically symmetric two parameter Fermi model (2pF) is adopted [15]

$$\rho(r)=\rho_0/[1+\exp((r-c)/a)], \tag{2}$$

where c stands for the half- density radius. The specific form is [11]

$$c=r_\rho(1-\pi^2a^2/3r_\rho^2), \tag{3}$$

where $r_\rho=1.13A_i^{1/3}$, and A_i is the mass number. The diffusion coefficient $a\approx0.54$ fm. ρ_0 is obtained by normalization of the following formula

$$\int\rho(\boldsymbol{r})\mathrm{d}\boldsymbol{r}=A_i. \tag{4}$$

In Eq. (1) $v(|\vec{r}-\vec{R}|)$ is selected to use the M3Y type *NN* interaction. In order to correspond with experimental results and explain the phenomenon that the interactions become weaker with increase in the surrounding medium, density-dependent factor must be taken into account. Assuming that the density has nothing to do with energy and radius, a density-dependent factor is shown as follows

$$F(\rho)=C[1+\alpha e^{-\beta\rho}-\gamma\rho]. \tag{5}$$

An additional energy-dependent factor needs to be introduced, based on the analysis of many scattering data [13]. The linear function can be expressed as the following

$$g(E)=1-\gamma'(E/A). \tag{6}$$

In particular, for the Paris interaction, $\gamma'=0.003\ \mathrm{MeV}^{-1}$. After considering the above factors, the density-dependent nucleon-nucleon interaction is given by

$$v(s,\rho,E)=g(E)F(\rho)v^{(\mathrm{M3Y})}(s,E), \tag{7}$$

where $v^{(\mathrm{M3Y})}$ $(s,\ E)$ corresponds to the density-independent nucleon-nucleon interaction. In addition, the Paris interaction includes both a direct part and an exchange part. The direct part is of the form:

$$v_\mathrm{D}(r)=11061.625\frac{e^{-4r}}{4r}-2537.5\frac{e^{-2.5r}}{2.5r}, \tag{8}$$

and the exchange part is of the form:

$$v_\mathrm{EX}(r)=-1524.25\frac{e^{-4r}}{4r}-518.75\frac{e^{-2.5r}}{2.5r}-7.8474\frac{e^{-0.7072r}}{0.7072r}. \tag{9}$$

The Coulomb potential $V_\mathrm{C}(R)$ between projectile and target nuclei can also be obtained by Eq. (1) using the single folding model.The total interaction potential is obtained by

$$V(R)=V_\mathrm{N}(R)+V_\mathrm{C}(R). \tag{10}$$

Then the fusion barrier can be inferred using the following conditions

$$\left.\frac{\mathrm{d}V(R)}{\mathrm{d}R}\right|_{R=R_\mathrm{B}}=0,\quad \left.\frac{\mathrm{d}^2V(R)}{\mathrm{d}R^2}\right|_{R=R_\mathrm{B}}\leqslant0. \tag{11}$$

The barrier height and position are marked as V_B and R_B, respectively.

3 Results and discussion

Here we mainly discuss the fusion reaction between the incident proton and different target nuclei with masses from 51 amu to 139 amu. Ref. [11] shows a parameterized formula of the fusion barrier heights for α fusing with different target nuclei. The results of using this formula to calculate the barrier height of protons with different target nuclei are shown in Table 1 and compared with the empirical values of the same systems. From Table 1 it can be observed that the results of this parameterized formula are good when the mass number of the target nuclei is in the range 51 to 90. However, when the mass numbers of the target nuclei increase further, there is a large deviation between the calculated results and the empirical values.

Based on the DFPD2 program [13], the single-folding model is then used to calculate the nuclear potential between the incident proton and the different target nuclei. In the calculation, the incident proton is assumed to be a point charge and the shape of the target nuclei is spherical. Using a density-dependent NN interaction [13], a series of the fusion barrier height V_B and position R_B can be obtained. Fig. 1 depicts the variation of V_B with the root-mean-square radius of the target nuclei. The formula for the root-mean-square radius can be expressed as

$$\langle r_\mathrm{T}^2\rangle^{1/2}=\left[\frac{\int_0^\infty\rho_\mathrm{T}(r)r^4\mathrm{d}r}{\int_0^\infty\rho_\mathrm{T}(r)r^2\mathrm{d}r}\right]^{1/2}. \tag{12}$$

The Coulomb potential is one of the main sources of the fusion barrier heights. The heavier the target nuclei is, the higher the barrier height is. As can be seen from Fig. 1, the values of V_B have a linear relationship with the root-mean-square radius. A linear function can fit this trend well. According to the parameters obtained, the final parameterized formula for calculating the barrier heights is

$$V_\mathrm{B}=3.583\langle r_\mathrm{T}^2\rangle^{1/2}-9.143. \tag{13}$$

The above formula gives a simple and direct method to calculate the fusion barrier height when the incident particle is a proton. Only the root-mean-square radii of the target nuclei need to be known.

Table 1. Comparison between the barrier heights obtained by the parameterized formula [11] and the empirical values [16] for proton fusion systems.

reactions	parameterized/MeV	empirical/MeV
p+^{51}V	4.43	4.22
p+^{59}Co	5.10	4.97
p+^{63}Cu	5.42	5.01
p+^{65}Cu	5.39	4.85
p+^{92}Zr	7.02	6.44
p+^{94}Zr	6.99	6.19
p+^{93}Nb	7.18	6.52
p+^{95}Mo	7.32	6.65
p+^{98}Mo	7.29	6.43
p+^{103}Rh	7.74	6.74
p+^{110}Pd	7.82	6.83
p+^{107}Ag	8.03	7.01
p+^{109}Mo	8.01	7.04
p+^{111}Cd	8.15	7.10
p+^{116}Cd	8.08	6.99
p+^{115}In	8.26	7.26
p+^{117}Sn	8.40	7.51
p+^{118}Sn	8.39	7.49
p+^{119}Sn	8.38	7.48
p+^{120}Sn	8.36	7.32
p+^{122}Sn	8.33	7.26
p+^{124}Sn	8.31	7.27
p+^{130}Te	8.56	7.26

Figure 2 shows the relationship between fusion barrier heights $V_{\rm B}^{\rm par}$ and $Z_{\rm P}Z_{\rm T}/R_{\rm B}^{\rm SF}$. The linear characteristic can be obtained by fitting

$$V_{\rm B}^{\rm par}=1.3550\times\frac{Z_{\rm P}Z_{\rm T}}{R_{\rm B}^{\rm SF}}-0.0515. \qquad (14)$$

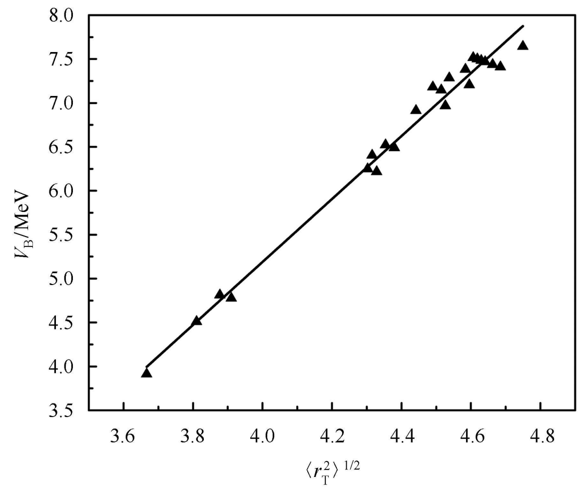

Fig. 1. The relationship between the fusion barrier heights ($V_{\rm B}$) and the root-mean-square radii (rms) of target nuclei. The points and solid line represent the results obtained by the single-folding model calculation and fitting, respectively.

Table 2. Comparison of the barrier heights obtained from empirical data, proximity potential, AW potential [16] and our parameterized values.

reactions	parameterized/MeV	empirical/MeV	proximity/MeV	AW potential/MeV
p+^{51}V	3.99	4.22	4.01	4.03
p+^{59}Co	4.51	4.97	4.62	4.63
p+^{63}Cu	4.75	5.01	4.91	4.93
p+^{65}Cu	4.87	4.85	4.88	4.90
p+^{92}Zr	6.27	6.44	6.36	6.37
p+^{94}Zr	6.37	6.19	6.33	6.34
p+^{93}Nb	6.32	6.52	6.52	6.52
p+^{95}Mo	6.46	6.65	6.65	6.65
p+^{98}Mo	6.55	6.43	6.61	6.61
p+^{103}Rh	6.77	6.74	7.03	7.02
p+^{110}Pd	7.08	6.83	7.09	7.09
p+^{107}Ag	6.95	7.01	7.29	7.29
p+^{109}Mo	7.04	7.04	7.26	7.26
p+^{111}Cd	7.12	7.10	7.40	7.39
p+^{116}Cd	7.32	6.99	7.32	7.33
p+^{115}In	7.28	7.26	7.50	7.49
p+^{117}Sn	7.36	7.51	7.63	7.62
p+^{118}Sn	7.41	7.49	7.61	7.61
p+^{119}Sn	7.44	7.48	7.60	7.60
p+^{120}Sn	7.48	7.32	7.59	7.58
p+^{122}Sn	7.56	7.26	7.56	7.56
p+^{124}Sn	7.64	7.27	7.53	7.53
p+^{130}Te	7.87	7.26	7.76	7.76

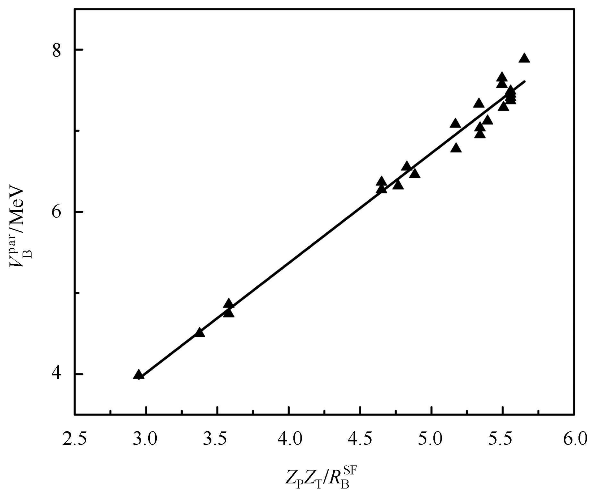

Fig. 2. The relationship between fusion barrier heights V_B^{par} and $Z_P Z_T/R_B^{SF}$. The solid line represents the fitting results.

When V_B can be obtained by Eq. (13), using Eq. (14), R_B can be obtained. So Eqs. (13) and (14) can be regarded as the parameterized formulas of the barrier height and barrier position for proton fusion systems. In order to test the accuracy of the presently parameterized formulas, comparisons of the results from the parameterized formula (13), experimental data, the proximity potential and AW potential are shown in Table 2. From Table 2, the calculation results of Eq. (13) can agree well with the different models, especially in the region of relatively heavy nuclei.This indicates that the obtained parametrization formulas (13) and (14) can be used to calculate the barrier heights and positions of proton fusion systems.

4 Conclusion

We use the single-folding model with the density-dependent NN interaction to calculate the nuclear potential, and then get the parameterized formulas for the barrier heights and positions for proton fusion with different target nuclei. The parameterized results are in good agreement with the different models, especially in the region where the mass numbers are larger than 90. It is shown that the presently obtained parametrization formulas can be used to directly estimate the barrier parameters of proton fusion with different target nuclei, and provide some necessary information for such reactions.

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