# Study of pre-scission particle emissions and fission probability of the $^{178}W$ produced in fusion reactions

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Abstract: A dynamical model based on one-dimensional Langevin equations was used to calculate the average pre-fission multiplicities of neutrons, light charged particles, and the fission probability for compound nucleus <sup>178</sup>W produced in fusion reactions. The pre-scission multiplicities of particles and fission probability are calculated and compared with the experimental data over a wide range of excitation energy. A modified wall and window dissipation with a reduction coefficient,  $k_s$ , has been used in the Langevin equations for reproducing experimental data. It was shown that the results of the calculations are in good agreement with the experimental data by using values of  $k_s$  in the range  $0.24 \leq k_s \leq 0.47$ .

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## 1 Introduction

The emission of light particles during the pre-scission stage of a fissioning nucleus is a useful source of information for the dynamics of nuclear fission. The first description of the fission process was given by Bohr and Wheeler [1]. But, calculating the average pre-fission neutron multiplicities based on the statistical model of Bohr and Wheeler has proved that the estimated fission lifetime of a hot nucleus is shorter than the experimental data. Therefore, it is natural to expect that a dissipative dynamical model would provide an appropriate description of nuclear fission at high excitation energies. In the stochastic approach, the dynamics of nuclear fission can be considered as the motion of a Brownian particle floating in a viscous heat bath [2, 3]. The fission dynamics of a hot nucleus can be considered based on the Fokker Planck equation or the Langevin equations. It should be mentioned that the application of the Langevin equations is more convenient. One of the important parameters in the dynamical study of nuclear fission based on Langevin equations is the nuclear dissipation coefficient. At present, although there are several models for dissipation, they give dependences which are very different from each other. For example, the model of two body dissipation [4] predicts a decrease of dissipation with temperature as  $T^{-2}$ , whereas the linear response theory [5, 6 predicts that dissipation increases with temperature. On the other hand, many authors who have analysed the different aspects of nuclear fission have used the wall

formula for nuclear dissipation, which was developed by Blocki et al. [7] in a simple classical picture of one-body dissipation. One crucial assumption of the wall formula concerns the randomization of the nucleon motion due to the successive collisions that it suffers at the nuclear surface. It was earlier understood that any deviation from this full randomization assumption would give rise to a reduction in the strength of the wall formula friction [7, 8]. Furthermore, many authors used a constant nuclear dissipation to describe different features of the nuclear fission [9–12]. In this paper, we want to use a modified wall and window dissipation with a reduction coefficient [13, 14] in one-dimensional Langevin equations to simulate the dynamics of the nuclear fission of <sup>178</sup>W formed in <sup>19</sup>F+<sup>159</sup>Tb reactions and reproduce experimental data on the average pre-fission multiplicities of neutrons, light charged particles, and the fission probabilities. It should be stressed that in our calculations we want to consider the magnitude of the reduction coefficient as a free parameter. The present paper has been arranged as follows. In Section 2, we describe the model and basic equations. The results of the calculations are presented in Section 3. Finally, the concluding remarks are given in Section 4.

#### 2 Details of the model

In the present study, the average pre-fission multiplicities of neutrons, light charged particles, and the fission probabilities are calculated for  $^{178}$ W. In order to describe the fission dynamics of  $^{178}$ W, we use a stochastic

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approach based on one-dimensional Langevin equations [15]

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{p}{m(r)}, \quad \frac{\mathrm{d}p}{\mathrm{d}t} = \frac{1}{2} \left(\frac{p}{m(r)}\right)^2 \frac{\mathrm{d}m}{\mathrm{d}r} - \frac{\mathrm{d}F}{\mathrm{d}r} - \eta \dot{r} + R(t), \quad (1)$$

where r and p are the distance between the centers of masses of the future fission fragments and its conjugate momentum, respectively. R(t) is a random force with the properties  $\langle R(t) \rangle = 0$  and  $\langle R(t)R(t') \rangle = 2\eta T \delta(t-t')$ , and F is the free energy of the system. In the Fermi gas model, F is related to the level density parameter  $F(r,T) = V(r) - a(r)T^2$ , where T is the temperature of the system and a(r) is the level density parameter. In our calculations, we use a coordinate dependent level density parameter as  $a(r) = a_{\rm v}A + a_{\rm s}A^{2/3}B_{\rm s}(r)$ , where A is the mass number of the compound nucleus and  $B_s$ is the dimensionless functional of the surface energy in the liquid drop model. The values of the parameters  $a_{\rm v}{=}0.073~{\rm MeV^{-1}}$  and  $a_{\rm s}{=}0.095~{\rm MeV^{-1}}$  are taken from the work of Ignatyuk et al. [16]. The potential energy V(r) is obtained from the liquid drop model [17]. In our calculations, the Langevin trajectories are simulated starting from the ground state of the compound nucleus with the excitation energy  $E^*$ . The initial conditions for Langevin equations can be chosen by the Neumann method with generating function

$$\Phi(r_0, p_0, l_0, t=0) \propto \exp\left[-\frac{V(r_0) + E_{\text{coll}}(r_0, p_0)}{T}\right] \times \delta(r_0 - r_{\text{gs}}) \frac{\mathrm{d}\sigma(l)}{\mathrm{d}l}.$$
(2)

The initial state is assumed to be characterized by thermal equilibrium momentum distribution and by spin distribution of compound nuclei  $d\sigma(l)/dl$  according to scaled prescription [18], which reproduces to a certain extent the dynamical results of the surface friction model [19] for the fusion of two heavy ions.

The collective inertia, m, is calculated in the frame of the Werner-Wheeler approach and the nuclear temperature is defined as  $T = \sqrt{E_{\text{int}}/a(r)}$  with

$$E_{\rm int} = E^* - p^2 / (2m) - V(r) - E_{\rm rot} - E_{\rm evap}(t), \qquad (3)$$

where  $E_{\rm rot}$  and  $E_{\rm evap}$  are the rotational energy and the nucleus excitation energy that light particles have carried away by the instant t, respectively.

We use the following expression to calculate the modified wall and window dissipation formula friction [13, 14]

$$\eta = \frac{1}{2}\rho_{\rm m}\bar{v}\left\{\left(\frac{\partial r}{\partial c}\right)^2\Delta\sigma + k_{\rm s}\pi\left[\int_{z_{\rm min}}^{z_{\rm N}}\left(\frac{\partial\rho^2}{\partial c} + \frac{\partial\rho^2}{\partial z}\frac{\partial D_1}{\partial c}\right)^2\right\}\right\}$$

$$\times \left(\rho^{2} + \left(\frac{1}{2}\frac{\partial\rho^{2}}{\partial z}\right)^{2}\right)^{-1/2} \mathrm{d}z + \int_{z_{N}}^{z_{\max}} \left(\frac{\partial\rho^{2}}{\partial c} + \frac{\partial\rho^{2}}{\partial z}\frac{\partial D_{2}}{\partial c}\right)^{2} \times \left(\rho^{2} + \left(\frac{1}{2}\frac{\partial\rho^{2}}{\partial z}\right)^{2}\right)^{-1/2} \mathrm{d}z \right] \right\}, \tag{4}$$

where  $\rho_{\rm m}$  is the mass density of the nucleus,  $\bar{v}$  is the average nucleon speed inside the nucleus, r is the distance between the centers of masses of the future fission fragments,  $\Delta\sigma$  is the area of the window between the two parts of the system,  $\rho^2$  is the surface of the nucleus,  $D_1$ ,  $D_2$  are the positions of the centers of mass of the two parts of the fissioning system relative to the center of mass of the whole system,  $z_{\rm min}$  and  $z_{\rm max}$  are the two extreme ends of the nuclear shape along the z axis, and  $z_N$  is the position of the neck plane.

The surface of a nucleus of mass number A with elongation c can be defined as

$$\rho^{2}(z) = \left(1 - \frac{z^{2}}{c_{0}^{2}}\right) (Ac_{0}^{2} + Bz^{2}), \qquad (5)$$

where  $c_0 = cR$ ,  $R = 1.16A^{1/3}$  and the coefficients A and B are expressed as

$$A = \frac{1}{c^3} - \frac{B}{5}, \quad B = \frac{c-1}{2}.$$
 (6)

The decay widths for emission n, p,  $\alpha$ ,  $\gamma$  are calculated at each Langevin time step  $\Delta t$ . The emission of a particle is allowed by asking, at each time step along the trajectory, whether the ratio of the Langevin time step  $\Delta t$  to the particle decay time  $\tau_{\text{part}}$  is larger than a random number  $\xi$ , where  $\tau_{\text{part}} = \hbar/\Gamma_{\text{tot}}$  and  $\Gamma_{\text{tot}} = \sum_{\gamma} \Gamma_{\gamma}$ .

The probabilities of decay via different channels can be calculated by using a standard Monte Carlo cascade procedure where the kind of decay selected with the weights  $\Gamma_{\nu}/\Gamma_{\text{tot}}$  with ( $\nu$ =n, p,  $\alpha$ ,  $\gamma$ ). After the emission act of a particle of kind  $\nu$ , the kinetic energy  $\varepsilon_{\nu}$  of the emitted particle is calculated by the hit and miss Monte Carlo procedure. Then the intrinsic excitation energy of the residual mass and spin of the compound nucleus are recalculated and the dynamics are continued. The loss of angular momentum is taken into account by assuming that each neutron, proton, or a  $\gamma$  quanta carries away  $1\hbar$ while the  $\alpha$  particle carries away  $2\hbar$ .

Figure 1 shows several typical Langevin trajectories calculated by Langevin equations.

The particle emission width of a particle of kind  $\nu$  is given by Ref. [20]

$$\Gamma_{\nu} = (2s_{\nu}+1) \frac{m_{\nu}}{\pi^2 \hbar^2 \rho_{\rm c}(E_{\rm int})} \\ \times \int_{0}^{E_{\rm int}-B_{\nu}} \mathrm{d}\varepsilon_{\nu} \rho_{\rm R}(E_{\rm int}-B_{\nu}-\varepsilon_{\nu})\varepsilon_{\nu}\sigma_{\rm inv}(\varepsilon_{\nu}), \quad (7)$$

where  $s_{\gamma}$  is the spin of the emitted particle  $\gamma$  and  $m_{\gamma}$ 

is its reduced mass with respect to the residual nucleus.  $\rho_{\rm c}$  and  $\rho_{\rm R}$  are the level densities of the compound and residual nuclei. The variable  $\varepsilon_{\nu}$  is the kinetic energy of the evaporated particle  $\nu$ . The intrinsic energy and the separation energy of particle  $\nu$  are denoted by  $E_{\rm int}$  and  $B_{\nu}$ . The inverse cross sections,  $\sigma_{\rm in\nu}$ , can be calculated as in Ref. [20].



Fig. 1. Typical Langevin trajectories reach the scission point (solid lines), and terminates in the potential well (dotted line).  $R_0$  is the radius of the spherical nucleus.

In our calculations, a Langevin trajectory either reaches the scission point or counts as an evaporation residue event if the intrinsic excitation energy becomes smaller than either the binding energy of a neutron or the fission barrier height. If the Langevin trajectory has not been counted as an evaporation residue event and has not fissioned after a delay time, when the stationary flux over the saddle point is reached we stop the dynamical calculation and switch over to the statistical description with a Kramers type fission decay [21].

#### 3 Results and discussion

The dynamical model based on one-dimensional Langevin equations was used to calculate the average pre-fission multiplicities of neutrons, light charged particles, and the fission probability for compound nucleus  $^{178}$ W produced in heavy ion-induced fusion reactions  $^{19}$ F+ $^{159}$ Tb Figs. 2, 3 and 4 show the results of the average pre-fission multiplicities of neutrons, protons and alpha particles for  $^{178}$ W.

It is clear from Fig. 2 that, at lower excitation energies, the values of the average pre-fission multiplicities of neutrons calculated with different values of the reduction coefficient are very close together and are also close to the experimental data; however, at higher excitation energies the experimental data can be reproduced by considering values of  $k_{\rm s}$  in the range  $0.24 \leq k_{\rm s} \leq 0.47$ . This can be understood as follows, a compound nucleus

at higher excitation energy is formed with a larger value of spin. Thus, the fission barrier height will be reduced (see Fig. 5) and, therefore, the neutron widths are comparable to the fission width. Consequently, the value of the reduction coefficient is a very important parameter to reproduce pre-scission neutron multiplicities. On the other hand, at lower excitation energy, a compound nucleus is formed with a lower value of spin and then the height of the fission barrier is large (see Fig. 5), and so the neutron widths are considerably larger than the fission width. Consequently, if we use different values of the reduction coefficient, the neutrons have enough time to be emitted before fission.

Similar arguments can be considered for interpretation of Figs. 3 and 4.



Fig. 2. Pre-scission neutron multiplicity as a function of excitation energy for  $^{178}$ W calculated with different values of  $k_s$  (open symbols). The experimental data (filled circles) are taken from Ref. [22].



Fig. 3. Pre-scission proton multiplicity as a function of excitation energy for  $^{178}$ W calculated with different values of  $k_s$  (open symbols). The experimental data (filled circles) are taken from Ref. [23].



Fig. 4. Pre-scission alpha multiplicity as a function of excitation energy for  $^{178}$ W calculated with different values of  $k_{\rm s}$  (open symbols). The experimental data (filled circles) are taken from Ref. [23].



Fig. 5. Potential energy surfaces at J=0, 30, 40, 50,  $60\hbar$ .  $R_0$  is the radius of the spherical nucleus.

The calculated and experimental values of the fission probability are shown in Fig. 6 for  $^{178}W$ .

As can be seen from Fig. 6, at lower excitation energies the experimental data can be reproduced by using  $k_{\rm s}$  in the range  $0.24 \leq k_{\rm s} \leq 0.47$ , but at higher excitation energies the fission probability calculated with the different values of  $k_{\rm s}$  are very close and in agreement with the experimental data. It can also be seen that, at higher excitation energies, the fission probability reaches a stationary value. This can be explained as follows, with an increasing the excitation energy pre-fission multiplicities of neutrons and light charged particles increase and each emission of a light particle carries away angular momentum and excitation energy. Consequently, the fission barrier height of the residual nucleus increases and the fission event then becomes less and less probable. Moreover, at a lower excitation energy a compound nucleus forms with a lower value of spin and then the fission barrier height and the fission time are increased. Therefore, the value of  $k_s$  is a very important parameter in calculating the fission probability.



Fig. 6. Fission probability as a function of excitation energy for  $^{178}$ W calculated with different values of  $k_s$  (open symbols). The experimental data (filled circles) are taken from Ref. [24].

It should be mentioned that authors in Ref. [25] analyzed the pre-scission neutron multiplicities for the excited nuclei <sup>197</sup>Tl in the framework of a Langevin equation coupled with a statistical model. In their calculations, they assumed that the nuclear dissipation coefficient is constant up to the saddle point and it would sharply increase linearly between saddle and scission points up to the value  $30 \times 10^{21} \text{s}^{-1}$ . The authors in Ref. [25], when analyzing the pre-scission neutron multiplicities, considered the pre-saddle nuclear dissipation coefficient as an adjustable parameter and, by reproducing experimental data, obtained information about the value of the pre-saddle nuclear dissipation coefficient. However, in the present research we used a modified wall and window dissipation with a reduction coefficient,  $k_{\rm s}$ , in the Langevin equations to reproduce experimental data.

# 4 Conclusions

The dynamical model based on one-dimensional Langevin equations was used to calculate the average pre-fission multiplicities of neutrons, light charged particles and the fission probability for the compound nucleus  $^{178}$ W produced in heavy ion-induced fusion reactions  $^{19}$ F+ $^{159}$ Tb. In our calculations, we used a modified wall and window dissipation with a reduction coefficient in the Langevin equations and assumed the magnitude of the reduction coefficient as a free parameter. It was shown that the experimental data of the pre-scission particle multiplicities and the fission probability for the

compound nucleus <sup>178</sup>W can be reproduced by using a reduction coefficient in the range  $0.24 \leq k_{\rm s} \leq 0.47$ . It should be stressed that our result for  $k_{\rm s}$  is consistent with the other studies [26, 27]. In Refs. [26, 27], the authors have performed a systematic study of many different systems and showed that to reproduce the measure of the variance of the fission fragment mass energy distribution, the dependence of the pre-scission neutron multiplicity on the fragment mass asymmetry neutron multiplicities, the total kinetic energy, and the neutron multiplicities, the reduced coefficient of the contribution from a wall formula has to be decreased by at least half of the one body dissipation strength  $(0.25 \leq k_s \leq 0.5)$ .

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