An analysis of undulator optimization in self-seeding free electron lasers*

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Abstract: A simple analysis is given for the optimum length of undulator in a self-seeding free electron laser (FEL). The obtained relations show the correlation between the undulator length and the system parameters. The power required for the seeding in the second part of the undulator and the overall efficiency of monochromatizating the seeding determine the length of the first part of the undulator; the magnitude of seeding power dominates the length of the second part of the undulator; the whole length of the undulator in a self-seeding FEL is determined by the overall efficiency for getting coherent seed, and is about half as long again as that of SASE, not including the dispersion section. The requirement of the dispersion section strength is also analyzed.

Key words: free electron laser, self-seeding, undulator

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1 Introduction

Self-amplified spontaneous emission (SASE) free-electron lasers (FELs) in the X-ray region of the spectrum are currently opening up new frontiers across science. Although SASE FELs have brightness of up to 10^8 times greater than laboratory sources, their full potential is limited by relatively poor temporal coherence. To improve the temporal coherence of SASE FELs is an important area of research, with many different schemes proposed. One attractive scheme is a self-seeding FEL, an idea which was proposed at DESY to generate a narrow spectrum soft and hard X-FEL [1, 2]. In this scheme, the undulator is divided into two parts by a magnet chicane. The first part of the undulator is operated in the SASE linear regime. The radiation output then passes a monochromator to generate a narrow spectrum seeding laser, which is input into the second part of the undulator, in which it is amplified to saturation. The magnetic chicane delays the electron bunch to match the light delay caused by the monochromator. In these schemes a long (gentle) chicane is needed to depress the electron energy spread due to synchrotron radiation, which is inconvenient. Improved schemes are proposed to avoid this long chicane, one of which uses two separate electron bunches [3], in which the delayed seed from the first electron bunch is amplified by the second electron bunch; another new approach using a particular type of monochromatization for hard X-FEL is proposed [4] at DESY, so that a compact magnetic chicane can be adopted. Using this approach, the self-seeding X-FEL experiment has been successfully demonstrated at SLAC [5], with encouraging results, and significantly reduced X-FEL bandwidth.

The basic conditions for a self-seeding FEL are analyzed in Ref. [1]. In this paper, we give more detailed analysis of the optimization of undulator length. A disadvantage of the self-seeding FEL is that the total length of the undulator system is longer than that of a SASE scheme. For a self-seeding FEL scheme to get the optimum performance and make the system length as short as possible, it is crucial to optimize the length of the undulators and determine where the monochromator and the electron by-pass chicane should be inserted.

2 Analysis

The first part of the undulator is operated in the SASE exponential gain regime, but not to saturation regime; correspondingly its length should be long enough, but shorter than the saturation length. The typical SASE saturation length is about twenty gain lengths, so it has $L_1 < L_s \sim 20L_g$. The output power of the first part of the undulator is $P_1 = P_{ef} e^{L_1/L_g}/9$, where P_{ef} is the effective SASE start up power (equivalent starting noise power). After monochromatizating, the input power of the second undulator i.e. the self-seeding power is

$$P_{20} = P_1\eta = \frac{1}{9}P_{ef}e^{\frac{L_1}{L_g}}\eta, \quad (1)$$

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where $\eta=\eta_m\times r_b$. η_m is the transmission efficiency of the monochromator, and r_b is the fraction of total power in the required narrow bandwidth; $r_b=\Delta\lambda_m/\Delta\lambda_1$. $\Delta\lambda_1$ is the SASE bandwidth at the exit of the first part of the undulator; $\Delta\lambda_1(\mathrm{rms})\approx\lambda\sqrt{\rho/N_1}$, where ρ is the FEL parameter, N_1 is the number of periods of the first part of the undulator, and $\Delta\lambda_m$ is the resolution of the monochromator, which should satisfy the condition [1]:

$$\lambda/\pi\sigma_z\leqslant\Delta\lambda_m/\lambda\ll\Delta\lambda_1/\lambda, \quad (2)$$

where σ_z is the length of the electron bunch. Due to noise start-up, SASE is chaotic light with M_L spikes in the spectral profile on average, while the average bandwidth of spikes is about $\lambda/\pi\sigma_z$, so $M_L=(\Delta\lambda_1/\lambda)/(\lambda/\pi\sigma_z)=\pi\sigma_z\sqrt{\rho/N_1}/\lambda$. We therefore have

$$1/M_L\leqslant r_b\ll 1. \quad (3)$$

The self-seeding power should be much larger than the effective SASE start up power P_{ef}. We denote the ratio of the self-seeding power to the effective SASE start up power to be α, namely we want $\alpha=P_{20}/P_{ef}=\eta e^{L_1/L_g}/9\gg1$. Thus the length of the first part of the undulator should be

$$L_1=\left(\ln\frac{9\alpha}{\eta}\right)L_g\gg\ln\left(\frac{9}{\eta}\right)L_g. \quad (4)$$

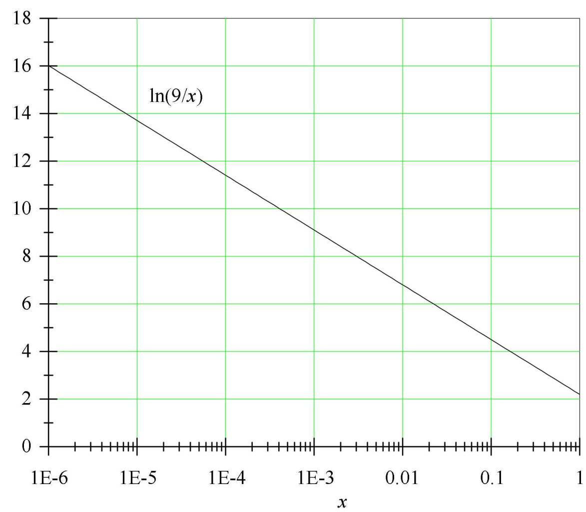

Fig. 1. The logarithmic relationship for the length of the undulator (Eq. (4) and Eq.(10)).

For example, if $\eta_m\sim10^{-1}-10^{-2}$, $b>\sim10^{-2}$, namely $\eta\sim10^{-3}-10^{-4}$, and we require $\alpha\sim10-10^2$, then we have $L_1\sim(12–16)L_g$ (Fig. 1). A longer length of the first part of the undulator can provide a higher seeding power, but it induces a larger energy spread that will suppress the FEL gain. The electron energy modulation induced by SASE in the first part of the undulator can be given from the electron energy equation

$$\frac{d\gamma^2}{dz}\approx-2a_u[JJ]k_s\mathrm{Re}(\tilde{a}_{s1}e^{i\phi}), \quad (5)$$

where $\tilde{a}_s=a_s e^{i\varphi_s}$, $a_s=eE_s/mc^2k_s$ and $a_u=eB_u/mc^2k_u$ are dimensionless vector potentials of the rms radiation field E_s and undulator field B_u, respectively; $k_s=2\pi/\lambda_s$ and $k_u=2\pi/\lambda_u$ are the corresponding wave numbers; and $[JJ]$ is the Bessel function factor. With $\tilde{a}_{s1}\approx a_{ef}e^{\mu_1 z}/3$, $\mu_1=k_u\rho(i+\sqrt{3})$, a_{ef} is the effective start up shot noise field, so we get the maximum electron energy modulation induced by SASE in exponential gain regime:

$$\Delta\gamma_m/\gamma=\frac{k_s a_{s1}a_u[JJ]}{\gamma^2}\sqrt{3}L_g=2\rho\sqrt{P_1/\rho P_e}. \quad (6)$$

This should be smaller than ρ, so the power of the first part of the undulator is then

$$P_1<\rho P_e/4. \quad (7)$$

For the second part of the undulator, the optical power should reach saturation:

$$\frac{1}{9}P_{20}e^{L_2/L_g}=P_s. \quad (8)$$

Where the saturation power P_s is the same as for SASE, $P_s=P_{ef}e^{L_s/L_g}/9$, where L_s is the SASE saturation length . Substituting the expression for P_s and $P_{20}=\alpha P_{ef}$ into Eq. (8), we get the length of the second part of the undulator as

$$L_2=L_s-(\ln\alpha)L_g. \quad (9)$$

Thus the length of the second part of the undulator is dominated by the magnitude of the seeding power. From Eqs. (4) and (9), the total length of the undulator is

$$L_1+L_2=L_s+\left(\ln\frac{9}{\eta}\right)L_g. \quad (10)$$

Therefore the whole length of the undulator in a self-seeding FEL is determined by the overall efficiency for getting coherent seed, and is longer than that of the SASE case. For example, for $\eta\sim10^{-3}-10^{-4}$, $\Delta L\sim(9-12)L_g$ (Fig. 1) is about half as long again as that of SASE, not including the magnetic chicane section. It should be noted that we assume that the gain length of the second part of the undulator is the same as that of the first part of the undulator. If the effect of the energy modulation induced by SASE in the first part undulator as an energy spread is considered, the gain length of the second part of the undulator will be longer than that of the first part of the undulator:

$$L_{g2}/L_{g1}\approx1\bigg/\left[1-\frac{1}{9}\left(\frac{\delta\gamma/\gamma}{\rho}\right)^2\right]. \quad (11)$$

The related formulae should then be corrected correspondingly.

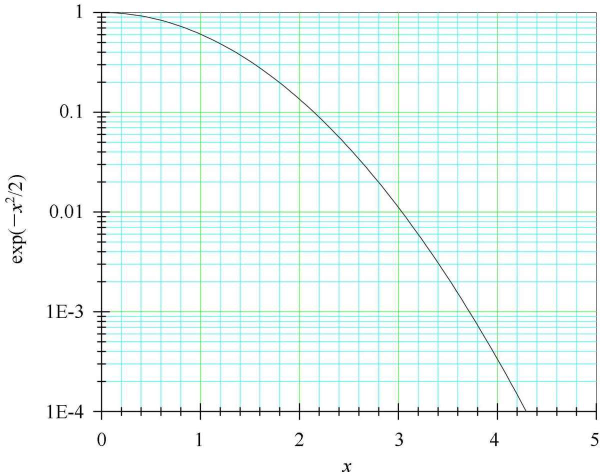

Fig. 2. The exponential term in b_{20}.

Now we consider the requirement of the dispersion intensity of the magnetic chicane. Besides delaying the electron bunch and diverting the electron beam around the monochromator, the dispersion field of the magnetic chicane must wash out the electron microbunching induced by SASE in the first part of the undulator. The bunching factor at the end of the first undulator is[1)]

$$b_1 = \frac{b_{10}}{3} e^{L_1/2L_g} = \sqrt{\frac{P_1}{\rho P_e}}, \tag{12}$$

where b_{10} is the initial bunching factor of SASE. After the chicane and at the entrance of the second undulator it becomes

$$b_{20} \approx J_1\left(D\frac{\Delta\gamma_m}{\gamma}\right)\exp\left[-\frac{1}{2}\left(D\frac{\sigma_\gamma}{\gamma}\right)^2\right], \tag{13}$$

where $D \approx 4\pi(N_d + N_1/2)$, N_1 is the number of periods of the first part of the undulator, N_d is the dispersive parameter of the dispersion section, and $\Delta\gamma_m/\gamma$ is the electron beam energy modulation induced by SASE in the first part of the undulator and given by Eq. (6). The bunching factor at the entrance of the second undulator (Eq. (12)) must not be larger than the initial bunching factor of SASE, which typically is about 10^{-4}–10^{-3}. Because the maximum of the first order Bessel function $J_1 < 0.6$, the exponential term in the right-hand-side of Eq. (12) should have $\exp[-(D\sigma_\gamma/\gamma)^2/2] \leqslant 10^{-3}$, so one can take $D > \sim 4\gamma/\sigma_\gamma$ (Fig. 2), namely $R_{56} > \sim 2\gamma\lambda_s/\pi\sigma_\gamma$. In practical design of the magnetic chicane, an appropriate electron trajectory offset and delay to match the seed laser delay also should be considered.

3 Summary

In summary, the optimum length of an undulator in a self-seeding FEL is analyzed. The relationship between the undulator length and the system parameters are obtained. The power required for the seeding in the second part of the undulator and overall efficiency of monochromatizating the seeding determine the length of the first part of the undulator; the length of the second part of the undulator is dominated by the magnitude of the seeding power; and the whole length of the undulator in a self-seeding FEL is determined by the overall efficiency to get coherent seed, and is typically about half as long again as that of SASE, not including the dispersion section. The needed strength of the dispersion section is also analyzed. This work can help in the design and optimization of a self-seeding FEL. Comparisons with simulation or the experimental results are needed in future work.

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