# Deuteron form factors in a phenomenological approach＊ 

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#### Abstract

The electromagnetic form factors of the deuteron，particularly its quadrupole form factor，are studied with the help of a phenomenological Lagrangian approach where the vertex of the deuteron－proton－neutron with $D$－state contribution is explicitly taken into account．The results show the importance of this contribution to the deuteron quadrupole form factor in the approach．


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## 1 Introduction

The study of the electromagnetic form factors of nu－ cleons and light nuclei，like the deuteron and $\mathrm{He}-3$ ，is crucial for the understanding of the nucleon structures． The deuteron，as the simplest nucleus，has been a sub－ ject of such study for many years（for some recent reviews see $[1-4]$ ）．Since it is a weakly bound state of the proton and neutron，the study of the deuteron can shed light on the study of the nucleon as well as of the nuclear effects． Moreover，as a spin－1 particle，deuteron structures are different from those of spin－ $1 / 2$ nucleons and $\mathrm{He}-3$ ，and from the spinless pion meson．There are many discus－ sions in the literature of the deuteron structures，such as its wave functions，binding energy，electromagnetic form factors，and parton distributions．Those works are usu－ ally based on phenomenological potential models with quark，meson，and nucleon degrees of freedom and based on some effective field theories［1－12］．Realistic deuteron wave functions，with the help of the meson exchange po－ tential model，have been explicitly given by Refs．［13－15］．

In our previous works $[16,17]$ ，a phenomenological Lagrangian approach was applied to the electromagnetic form factors of the deuteron，where the deuteron is re－ garded as a loosely bound state of a proton and a neu－ tron，and the two constituents are in relative $S$－wave for simplicity．The coupling of the deuteron to its two composite particles was determined by the known com－ positeness condition from Weinberg［18］，Salam［19］and others［20，21］．In fact，our phenomenological effective Lagrangian approach has been proven to be successful in the study of weakly bound state problems，like the new
resonances of $\mathrm{X}(3872)$ and $\Lambda_{\mathrm{c}}^{+}(2940)$ ，and the EM form factors of the pion，as well as some other observables ［22，23］．

It should be stressed that since only the contribution from the one－body $S$－wave operator is considered in our previous study［16］，the estimated quadrupole moment of the deuteron is much smaller than the experimental data．According to the non－relativistic potential model calculation［13］，one sees that the deuteron quadrupole moment is very sensitive to the $D$－wave component of the deuteron．Therefore，the $S$－state contribution is not sufficient．In order to avoid this discrepancy，several two－body arbitrary and phenomenological Lagrangians were introduced，by hand，to compensate for the dis－ crepancy［16］．

The purpose of this work is to re－study the deuteron electromagnetic form factors with this phenomenological approach．Here both the $S$－and $D$－state contributions to the vertex of the deuteron－proton－neutron are simul－ taneously taken into account．It is expected that by ex－ plicitly considering the $D$－state contribution in the ver－ tex，the estimated deuteron quadrupole could be sizeably improved．This paper is organized as follows．Section 2 briefly shows our theoretical framework，particularly the $D$－state contribution to the vertex．Numerical results and some discussions are given in Section 3.

## 2 Theoretical framework

The deuteron，as a spin－ 1 particle，has three inde－ pendent form factors．The matrix element for electron－ deuteron（ED）elastic scattering，as shown in Fig．1，can

[^0]be written as
\[

$$
\begin{equation*}
\mathcal{M}=\frac{e^{2}}{Q^{2}} \bar{u}_{\mathrm{e}}\left(k^{\prime}\right) \gamma^{\mu} u_{\mathrm{e}}(k) \mathcal{J}_{\mu}^{D}\left(P, P^{\prime}\right), \tag{1}
\end{equation*}
$$

\]

under the one-photon exchange approximation. In Eq. (1), $k$ and $k^{\prime}$ are the four-momenta of the initial and final electrons and $\mathcal{J}_{\mu}^{D}\left(P, P^{\prime}\right)$ stands for the deuteron EM current. Its general form is

$$
\begin{align*}
\mathcal{J}_{\mu}^{D}\left(P, P^{\prime}\right)= & -\left(G_{1}\left(Q^{2}\right) \epsilon^{\prime *} \cdot \epsilon-\frac{G_{3}\left(Q^{2}\right)}{2 M_{\mathrm{d}}^{2}} \epsilon \cdot q \epsilon^{\prime *} \cdot q\right)\left(P+P^{\prime}\right)_{\mu} \\
& -G_{2}\left(Q^{2}\right)\left(\epsilon_{\mu} \epsilon^{\prime *} \cdot q-\epsilon_{\mu}^{\prime *} \epsilon \cdot q\right) \tag{2}
\end{align*}
$$

where $M_{\mathrm{d}}$ is the deuteron mass, $\epsilon\left(\epsilon^{\prime}\right)$ and $P\left(P^{\prime}\right)$ are the polarization and four-momentum of the initial (final) deuteron, and $Q^{2}=-q^{2}$ is momentum transfer squared, with $q=P^{\prime}-P$. The three EM form factors $G_{1,2,3}$ of the deuteron are related to the charge $G_{\mathrm{C}}$, magnetic $G_{\mathrm{M}}$, and quadrupole $G_{\mathrm{Q}}$ form factors by

$$
\begin{align*}
& G_{\mathrm{C}}=G_{1}+\frac{2}{3} \tau G_{\mathrm{Q}}, \quad G_{\mathrm{M}}=G_{2} \\
& G_{\mathrm{Q}}=G_{1}-G_{2}+(1+\tau) G_{3} \tag{3}
\end{align*}
$$

with the factor $\tau=Q^{2} / 4 M_{\mathrm{d}}^{2}$. They are normalized at zero recoil $\left(Q^{2}=0\right)$ as

$$
\begin{align*}
G_{\mathrm{C}}(0) & =1, \quad G_{\mathrm{Q}}(0)=M_{\mathrm{d}}^{2} \mathcal{Q}_{\mathrm{d}}=25.83 \\
G_{\mathrm{M}}(0) & =\frac{M_{\mathrm{d}}}{M_{\mathrm{N}}} \mu_{\mathrm{d}}=1.714 \tag{4}
\end{align*}
$$

where $M_{\mathrm{N}}$ is the nucleon mass, and $\mathcal{Q}_{\mathrm{d}}$ and $\mu_{\mathrm{d}}$ are the quadrupole and magnetic moments of the deuteron.


Fig. 1. Electron-deuteron scattering.
The unpolarized differential cross section for the eD elastic scattering can be expressed by the two structure functions, $A\left(Q^{2}\right)$ and $B\left(Q^{2}\right)$, as

$$
\begin{equation*}
\frac{\mathrm{d} \sigma}{\mathrm{~d} \Omega}=\sigma_{\mathrm{M}}\left[A\left(Q^{2}\right)+B\left(Q^{2}\right) \tan ^{2}\left(\frac{\theta}{2}\right)\right] \tag{5}
\end{equation*}
$$

where $\sigma_{\mathrm{M}}=\alpha^{2} E^{\prime} \cos ^{2}(\theta / 2) /\left[4 E^{3} \sin ^{4}(\theta / 2)\right]$ is the Mott cross section for point-like particles, $E$ and $E^{\prime}$ are the incident and final electron energies, $\theta$ is the electron scattering angle, $Q^{2}=4 E E^{\prime} \sin ^{2}(\theta / 2)$, and $\alpha=e^{2} / 4 \pi=1 / 137$
is the fine-structure constant. The two form factors $A\left(Q^{2}\right)$ and $B\left(Q^{2}\right)$ are related to the three EM form factors of the deuteron as

$$
\begin{align*}
& A\left(Q^{2}\right)=G_{\mathrm{C}}^{2}\left(Q^{2}\right)+\frac{8}{9} \tau^{2} G_{\mathrm{Q}}^{2}\left(Q^{2}\right)+\frac{2}{3} \tau G_{\mathrm{M}}^{2}\left(Q^{2}\right) \\
& B\left(Q^{2}\right)=\frac{4}{3} \tau(1+\tau) G_{\mathrm{M}}^{2}\left(Q^{2}\right) \tag{6}
\end{align*}
$$

Clearly, the three form factors $G_{\mathrm{C}, \mathrm{M}, \mathrm{Q}}$ cannot be simply determined by measuring the unpolarized elastic eD differential cross section. To uniquely determine the three form factors of the deuteron, one additional polarization variable is necessary. For example, one may take the polarization of $T_{20}[1]$.

Making an assumption that the deuteron, as a hadronic molecule, is a weakly bound state of the proton and neutron, one may simply write a phenomenological effective Lagrangian for the coupling of the deuteron to its two constituents of proton and neutron as

$$
\begin{align*}
\mathcal{L}_{D}(x)= & g_{D} D_{\mu}(x) \int \mathrm{d} y \Phi_{D}\left(y^{2}\right) \bar{p}(x+y / 2) \Gamma_{D}^{\mu} C \bar{n}^{T}(x-y / 2) \\
& + \text { H.c. }, \tag{7}
\end{align*}
$$

where $D_{\mu}$ is the deuteron field, $C \bar{n}^{\mathrm{T}}(x)=n^{\mathrm{c}}(x), C=\mathrm{i} \gamma^{2} \gamma^{0}$ denotes the matrix of charge conjugation, and $x$ is the centre-of-mass (C. M.) coordinate. In the above equation $\Gamma_{D}^{\mu}$ is the vertex for the deuteron-proton-neutron coupling and the correlation function $\Phi_{D}\left(y^{2}\right)$ characterizes the finite size of the deuteron as a $p n$ bound state. The correlation function $\Phi_{D}\left(y^{2}\right)$ depends on the relative Jacobi coordinate $y$.

If only the $S$-wave contribution is considered, the simplest form of the vertex is $\Gamma_{D}^{\mu} \sim \gamma^{\mu}$ which has been employed before [16]. When both the $S$ - and $D$-state contributions are considered, then the vertex becomes more complicated. According to the work of Blankenbecler, Gloderber, and Halpern [24] the vertex of the deuteron-proton-neutron is

$$
\begin{equation*}
\Gamma_{D}^{\mu}=\Gamma_{D}^{1, \mu}+\Gamma_{D}^{2, \mu} \tag{8}
\end{equation*}
$$

where the first and second terms stand for the contributions from $S$ - and $D$-states, respectively. They are

$$
\begin{equation*}
\Gamma_{D}^{1, \mu}=\frac{1}{2 \sqrt{2}}\left(1+\frac{P}{M_{\mathrm{d}}}\right) \gamma^{\mu} \tag{9}
\end{equation*}
$$

and

$$
\begin{equation*}
\Gamma_{D}^{2, \mu}=\frac{\rho}{16}\left(1+\frac{P}{M_{\mathrm{d}}}\right)\left(\gamma^{\mu}-\frac{3}{k^{2}} \not k \gamma^{\mu} \not k\right) \tag{10}
\end{equation*}
$$

with $\rho$ being a measure of the $D$-state admixture, $k$ the relative momentum between the proton and neutron, and $k^{2}=M_{\mathrm{N}} \delta$ with $\delta$ being the binding energy of the deuteron. Here, it should be mentioned that in the rest frame of the deuteron, the non-relativistic reduction
gives

$$
\epsilon_{i} \Gamma_{D}^{1, i} C=-\frac{\mathrm{i}}{\sqrt{2}} \vec{\sigma} \cdot \vec{\epsilon} \sigma_{2}=\left(\begin{array}{cc}
\epsilon_{-1} & -\frac{1}{\sqrt{2}} \epsilon_{z}  \tag{11}\\
-\frac{1}{\sqrt{2}} \epsilon_{z} & \epsilon_{+1}
\end{array}\right)
$$

This means that a combination of two spin- $1 / 2$ states, proton and neutron, forms a spin triplet state. Similarly, in the non-relativistic limit,

$$
\left(\gamma^{\mu}-\frac{3}{k^{2}} \not k \gamma^{\mu} / k\right)
$$

means the proton and neutron couple to a spin triplet state and this spin triplet state re-couples $Y_{2 m_{l}}(\hat{k})$ to form a state with the same quantum numbers as the deuteron.

The coupling of the deuteron to its two constituents, $g_{D}$ in Eq. (7), is determined by the known compositeness condition $Z=0$ proposed by Weinberg, Salam and others [18-21]. This condition implies that the probability to find a proton and neutron system inside the deuteron is unity. Thus, the coupling of $g_{D}$ is determined according to $Z_{D}=1-\Sigma_{D}^{\prime}\left(M_{D}^{2}\right)=0$, with

$$
\begin{equation*}
\Sigma_{D}^{\prime}\left(M_{D}^{2}\right)=g_{D}^{2} \Sigma_{D \perp}^{\prime}\left(M_{D}^{2}\right) \tag{12}
\end{equation*}
$$

being the derivative of the transverse part of the mass operator (see Fig. 2). Usually, the mass operator splits into the transverse and longitudinal parts of $\Sigma_{D}^{\alpha \beta}(k)=$ $g_{\perp}^{\alpha \beta} \Sigma_{D \perp}\left(k^{2}\right)+\frac{k^{\alpha} k^{\beta}}{k^{2}} \Sigma_{D \|}\left(k^{2}\right)$, with $g_{\perp}^{\alpha \beta}=g^{\alpha \beta}-k^{\alpha} k^{\beta} / k^{2}$ and $g_{\perp}^{\alpha \beta} k_{\alpha}=0$. We see that the coupling of the deuteron to its constituents of the proton and neutron, $g_{D}$, is well determined by the compositeness condition.


Fig. 2. The mass operator of the deuteron.
A basic requirement for the choice of an explicit form of this correlation function is that its Fourier transform vanishes sufficiently fast in the ultraviolet region of Euclidean space to render the Feynman diagrams ultraviolet finite. Usually a Gaussian-type function is selected as the correlation for simplicity. One may choose $\tilde{\Phi}_{D}\left(k^{2}\right)=$ $\exp \left(-k_{\mathrm{E}}^{2} / \Lambda^{2}\right)$ for the Fourier transform of the correlation function, where $k_{\mathrm{E}}$ is the Euclidean Jacobi relative momentum and $\Lambda$ stands for the free size-parameter, which represents the distribution of the constituents in the deuteron.

Here the analytical expression for the coupling is

$$
\begin{equation*}
\frac{1}{g_{D}^{2}}=\Sigma_{D \perp, 1}^{\prime}+\rho \Sigma_{D \perp, 2}^{\prime}, \tag{13}
\end{equation*}
$$

where $\Sigma_{D \perp, 1}^{\prime}$ and $\Sigma_{D \perp, 2}^{\prime}$ stand for the derivatives of the transverse parts of the mass operator from the contributions of the $S-S$ and $D-D$ states, respectively. The explicit expressions are

$$
\begin{align*}
\Sigma_{D \perp, 1}^{\prime}= & \frac{1}{32 \pi^{2}} \int_{0}^{\infty} \frac{\mathrm{d} \alpha \mathrm{~d} \beta}{Z_{0}^{3}} \times\left\{\frac{A(\alpha, \beta)}{Z_{0}}\left[1+\frac{\Lambda_{S}^{2}}{4 M_{\mathrm{d}}^{2} Z_{0}}\right]\right. \\
& +\frac{B(\alpha, \beta)}{2} \times\left[\mu_{\mathrm{d}}^{2}\left(1+\frac{A(\alpha, \beta)}{Z_{0}^{2}}\left(1+\frac{\Lambda_{s}^{2}}{4 M_{\mathrm{d}}^{2} Z_{0}}\right)\right)\right. \\
& \left.\left.+\frac{3 \Lambda_{S}^{2}}{2 M_{\mathrm{d}}^{2} Z_{0}^{2}}-\frac{1}{4 Z_{0}}\right]\right\} \\
& \times \exp \left[-2(\alpha+\beta) \mu_{\mathrm{N}}^{2}+\frac{A(\alpha, \beta)}{2 Z_{0}} \mu_{\mathrm{d}}^{2}\right] \tag{14}
\end{align*}
$$

where $\mu_{\mathrm{N}, \mathrm{d}}=M_{\mathrm{N}, \mathrm{d}}^{2} / \Lambda_{S}^{2}$ and

$$
\begin{align*}
A(\alpha, \beta) & =(1+2 \alpha)(1+2 \beta) \\
B(\alpha, \beta) & =\alpha+\beta+4 \alpha \beta \\
Z_{0} & =1+\alpha+\beta \tag{15}
\end{align*}
$$

and

$$
\begin{align*}
\Sigma_{D \perp, 2}^{\prime}= & \int_{0}^{\infty} \frac{\mathrm{d} \alpha \mathrm{~d} \beta}{16 \sqrt{2} \pi^{2} Z_{1}^{3}} \times\left\{\frac{A^{\prime}(\alpha, \beta)}{Z_{1}}\left[1+\frac{3 \Lambda_{S}^{2}}{8 \epsilon M_{D} Z_{1}}\right]\right. \\
& +\frac{B^{\prime}(\alpha, \beta)}{2}\left[\mu_{\mathrm{d}}^{2}\left(1+\frac{A^{\prime}(\alpha, \beta)}{Z_{1}^{2}}\left(1+\frac{3 \Lambda_{S}^{2}}{8 \delta M_{\mathrm{d}} Z_{1}}\right)\right)\right. \\
& \left.\left.+\frac{1}{2 Z_{1}^{2}}\left(1-\frac{15 M_{\mathrm{d}}}{8 \delta}+\frac{9 \Lambda_{S}^{2}}{2 \delta M_{\mathrm{d}}}\right)\right]\right\} \\
& \times \exp \left[-2(\alpha+\beta) \mu_{\mathrm{N}}^{2}+\frac{A^{\prime}(\alpha, \beta)}{2 Z_{1}} \mu_{\mathrm{d}}^{2}\right] \tag{16}
\end{align*}
$$

with

$$
\begin{align*}
A^{\prime}(\alpha, \beta) & =\left(\frac{1+a_{S D}}{2}+2 \alpha\right)\left(\frac{1+a_{S D}}{2}+2 \beta\right) \\
B^{\prime}(\alpha, \beta) & =\frac{1+a_{S D}}{2}(\alpha+\beta)+4 \alpha \beta \\
Z_{0} & =\frac{1+a_{S D}}{2}+\alpha+\beta \tag{17}
\end{align*}
$$

and $a_{S D}=\Lambda_{S}^{2} / \Lambda_{D}^{2}$. Here we simply ignore the $\rho^{2}$ - dependent term since $\rho$ is expected to be small, and we consider the $S$ - and $D$ - interference. Since the correlation functions of the $S$ - and $D$-states may not necessarily be the same, we have a total of three parameters $\Lambda_{S}, \Lambda_{D}$ and $\rho$ in this calculation.

Then, we can calculate the matrix element of photondeuteron interaction as shown in Fig. 3 and we have

$$
\begin{align*}
\mathcal{M}^{\mu}= & \sum_{(\mathrm{N}=\mathrm{p}, \mathrm{n})} \sum_{(i, j=1,2)} \int \frac{\mathrm{d}^{4} k}{(2 \pi)^{4} i} g_{D}^{2} \epsilon_{\alpha}^{\prime *} \epsilon_{\beta} \\
& \times \operatorname{Tr}\left[\frac{\Gamma_{D}^{i, \alpha}\left(\not k+\not q+\not p / 2+M_{\mathrm{N}}\right)}{(k+q+p / 2)^{2}-M_{\mathrm{N}}^{2}}\right. \\
& \left.\cdot \frac{\Gamma_{\gamma \mathrm{N}}^{\mu}\left(\not k+\not p / 2+M_{\mathrm{N}}\right)}{(k+p / 2)^{2}-M_{\mathrm{N}}^{2}} \cdot \frac{\Gamma_{D}^{j, \beta}\left(\not k-\not p / 2-M_{\mathrm{N}}\right)}{(k-p / 2)^{2}-M_{\mathrm{N}}^{2}}\right] \\
& \times \exp \left[-k_{\mathrm{E}}^{2} / \Lambda_{j}^{2}-(k+q / 2)_{\mathrm{E}}^{2} / \Lambda_{i}^{2}\right] \tag{18}
\end{align*}
$$

where the photon-nucleon current of

$$
\begin{equation*}
\Gamma_{\gamma \mathrm{N}}^{\mu}=F_{1, \mathrm{~N}}\left(Q^{2}\right) \gamma^{\mu}+F_{2, \mathrm{~N}}\left(Q^{2}\right) \frac{\mathrm{i} \sigma^{\mu \nu}}{2 M_{\mathrm{N}}} q_{\nu} \tag{19}
\end{equation*}
$$

is employed with $F_{1, \mathrm{~N}}$ and $F_{2, \mathrm{~N}}$ being the known nucleon Dirac and Pauli form factors, and $\mathrm{N}=\mathrm{p}, \mathrm{n}$ stands for the proton and neutron, respectively.


Fig. 3. Photon-deuteron interaction.

## 3 Numerical results and discussions

We calculate the matrix element of Eq. (18) and consider the one-photon exchange approximation for the photon-deuteron current as shown in Eq. (1). Thus we can get the three corresponding deuteron form factors $G_{1,2,3}$ as well as the deuteron charge $G_{\mathrm{c}}\left(Q^{2}\right)$, magnetic $G_{\mathrm{M}}\left(Q^{2}\right)$ and quadrupole $G_{\mathrm{Q}}\left(Q^{2}\right)$ form factors. There are some parameterizations for the nucleon form factors of $F_{1,2}\left(Q^{2}\right)$ in the literature in Refs. [25-27]. In the present calculation, we employ the parameterizations used by Blunden [27]. The three model-dependent parameters, $\Lambda_{S}=0.10 \mathrm{GeV}, \Lambda_{D}=0.08 \mathrm{GeV}$ and $\rho=0.03$, are fixed by fitting to the experimental data. The obtained charge, magnetic and quadrupole form factors are shown in Figs. 4-6. The experimental data in the figures are from Refs. [29-39]. In the figures, the solid and dotted curves stand for our calculations and the parameterizations of Ref. [28], respectively. In order to explicitly see the contribution of the $D$-wave to the deuteron quadrupole form factors, we also show the results with only the $S$-wave contribution (see the dashed curve in Fig. 6) for a comparison.


Fig. 4. Estimated deuteron charge form factor $G_{\mathrm{c}}\left(Q^{2}\right)$. The solid and dotted curves are the results of our calculations and of the phenomenological parameterization [28]. The data are open circle [29], open square [30], open diamond [31], plus [32], triangle [33], filled circle [34], and filled square [35], respectively.


Fig. 5. Estimated deuteron magnetic form factor $G_{\mathrm{M}}\left(Q^{2}\right)$. The solid and dotted curves are the results of our calculations and of the phenomenological parameterization [28]. The data are circle [36], square [37], diamond [38], and triangle [39], respectively.

It should be stressed that in this work, according to the discussions of Ref. [24], we explicitly include the $D$ state contribution to the deuteron-proton-neutron vertex as shown in Eq. (10). We found that this contribution is very important for the understanding of the quadrupole moment and quadrupole form factors (see the solid and dashed curves in Fig. 6). Here, the estimated $G_{\mathrm{M}}(0)$ and $G_{\mathrm{Q}}(0)$ are about 1.53 and 21.38 , respectively. These two values are reasonable compared to the normalization conditions of 1.714 and 25.83 given in Eq. (4). If we only take the $S$-wave contribution into account, we hardly reproduce the experimental measurement for the quadrupole moment at the zero-recoil limit,
although the estimated charged and magnetic moments are still consistent with the data. In Fig. 6, the dashed curve shows that the obtained $G_{Q}(0) \sim 4$ is much smaller than the experimental value. Comparing the dashed and solid curves in Fig. 6, we conclude that the very small value of the quadrupole moment, with only the $S$-wave contribution, is remarkably improved due to the inclusion of the $D$-state contribution. Meanwhile, the charge and magnetic moments also remain reasonable.


Fig. 6. Estimated deuteron quadrupole form factor $G_{\mathrm{Q}}\left(Q^{2}\right)$. Notations are the same as Fig. 4. The dashed curve stands for the result with only the $S$-wave contribution considered.

In summary, in this work we explicitly consider the $D$-state contribution to the vertex of the deuteron-
proton-neutron, as well as the $S$-wave contribution, and find that our four-dimensional phenomenological Lagrangian approach can reasonably reproduce the deuteron charge, magnetic, and particularly, quadrupole form factors simultaneously. The estimated quadrupole moment is much improved due to the inclusion of the $D$-state contribution. It should be stressed that our present approach is fully relativistic and it is different from the potential model calculations based on the threedimensional framework.

Of course, the present calculation can be further improved, since we still cannot correctly reproduce the crossing point of the deuteron magnetic form factor as pointed out by Ref. [28]. It is found that the experimental data for $G_{\mathrm{C}}\left(Q^{2}\right)$ or $G_{\mathrm{M}}\left(Q^{2}\right)$ show the existence of a zero (or a crossing) point, at $Q_{0 \mathrm{C}}^{2}=0.7 \mathrm{GeV}^{2}$ or at $Q_{0 \mathrm{M}}^{2}=2 \mathrm{GeV}^{2}$, respectively (see Figs. 4-5). We can reproduce the zero point at $Q^{2} \sim 0.55 \mathrm{GeV}^{2}$ for the charged form factor due to the cancellation among the terms in the trace of Eq. (18). However, the cancellation among the trace terms of the magnetic form factor cannot provide a crossing point in the region of $0 \leqslant Q^{2} \leqslant 2 \mathrm{GeV}^{2}$. The estimated magnetic form factor decreases monotonically in the region. This is probably due to the fact that our selected correlation functions are still simple. Moreover, the explicit form of the $D$-state contribution, as shown in Eq. (10), is not unique [14]. A more sophisticated calculation is in progress. Finally, it is expected that future calculations of the deuteron generalized parton distribution functions with help of this approach could be promising.

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