Transmission calculation by empirical numerical model and Monte Carlo simulation in high energy proton radiography of thick objects *

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Abstract: An empirical numerical model that includes nuclear absorption, multiple Coulomb scattering and energy loss is presented for the calculation of transmission through thick objects in high energy proton radiography. In this numerical model the angular distributions are treated as Gaussians in the laboratory frame. A Monte Carlo program based on the Geant4 toolkit was developed and used for high energy proton radiography experiment simulations and verification of the empirical numerical model. The two models are used to calculate the transmission fraction of carbon and lead step-wedges in proton radiography at 24 GeV/c, and to calculate radial transmission of the French Test Object in proton radiography at 24 GeV/c with different angular cuts. It is shown that the results of the two models agree with each other, and an analysis of the slight differences is given.

Key words: proton radiography, angular distributions, scattering, Gaussians, Geant4, transmission

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1 Introduction

High-energy proton radiography provides a new and quantitative technique for hydrotest experiment diagnoses [1]. Transmission radiographs with high spatial resolution can be made if protons with energies of tens of GeV illuminate a thick dynamic test object which is placed in the object plane of a point-to-point magnetic quadrupole imaging system [2].

The basic principles of proton radiography are the proton-object interaction mechanisms, including energy loss, nuclear interaction, and multiple Coulomb scattering. In proton radiography experiments, the attenuation of protons passing through an object is measured to analyze the material and the shape of the object. Because of multiple Coulomb scattering and elastic scattering, the image will be blurred, and the definition will be reduced. In the design of a proton radiography experiment, the magnetic lens system and the collimators can be used to control the proton beam, reduce the scattering exposure in the image plane, and sort the scattered beam in terms of how it has been scattered [3]. The density distribution and material composition of the object can be obtained by comparing the imaging radiographs from different collimators.

In the present work, an empirical forward model is developed for protons passing through thick objects, taking the essential physics of proton radiography into consideration. Meanwhile, a Monte Carlo approach based on the Geant4 toolkit [4] is used to simulate proton radiography experiments for thick objects. The results of the two methods are compared.

2 The empirical numerical model

As mentioned above, the three most important effects on the protons as they go through an object are absorption, multiple Coulomb scattering, and energy loss. In the simplest form of the present model, a model for multiple Coulomb scattering (MCS) is used with a large-angle, small impact parameter cutoff due to the form factor of the nucleus, plus strong-interaction attenuation. The scattering distributions are treated as Gaussians or sums of Gaussians in the laboratory frame scattering angle.

First, the angular distribution for a slab that is thin relative to the scattering channel is calculated, in order to derive expressions of the angular distributions for thick objects. When protons pass through a slab of material with thickness L (areal density) in the thin limit ($L \ll \lambda$),

$$\lambda=\frac{A}{N_{\mathrm{A}}\sigma}, \tag{1}$$

where λ is the mean free path for a particular channel, A is the atomic weight, N_{A} is Avogadro's number, and σ is the cross section for the reaction channel.

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If only the elastic scattering channel is considered, the angular distribution is

$$I_{\mathrm{E}}(\theta)=\frac{\delta(\theta)+L/\lambda_{\mathrm{E}}f_{\mathrm{E}}(\theta)}{1+L/\lambda_{\mathrm{E}}}, \tag{2}$$

where λ_{E} is the mean free path for elastic scattering, δ is the Dirac delta function, and $f_{\mathrm{E}}(\theta)$ is the angular distribution for elastic scattering.

Second, applying MCS to Eq. (2), the angular distribution is

$$I(\theta)=\frac{f_{\mathrm{MCS}}(\theta)+L/\lambda_{\mathrm{E}}(f_{\mathrm{MCS}}*f_{\mathrm{E}})(\theta)}{1+L/\lambda_{\mathrm{E}}}, \tag{3}$$

where $f_{\mathrm{MCS}}(\theta)$ is the angular distribution for MCS, and $(f_{\mathrm{MCS}}*f_{\mathrm{E}})(\theta)$ is the convolution of the elastic scattering angular distribution and the MCS angular distribution.

Third, taking nuclear attenuation into consideration, the angular distribution becomes

$$I(\theta)=\frac{f_{\mathrm{MCS}}(\theta)+L/\lambda_{\mathrm{E}}(f_{\mathrm{MCS}}*f_{\mathrm{E}})(\theta)}{1+L/\lambda_{\mathrm{E}}}\mathrm{e}^{-L/\lambda_{\mathrm{A}}}, \tag{4}$$

where λ_{A} is the mean free path for nuclear attenuation.

Finally, if the thickness L and the mean free path are comparable, the slab length L can be divided into N segments to satisfy the condition $(L/N \ll \lambda)$. Thus, the elastic scattering angular distribution of the slab can be approximated by the convolution of the elastic scattering angular distribution for one segment L/N with itself N–1 times. The angular distribution can then be written as

$$I(\theta)=\left(\sum_{n=0}\frac{1}{n!}\left(\frac{L}{\lambda_{\mathrm{E}}}\right)^{n}(f_{\mathrm{MCS}}*(f_{\mathrm{E}})^{n})(\theta)\right)\mathrm{e}^{-L/\lambda_{\mathrm{A}}}. \tag{5}$$

In proton radiography, an angular collimator is placed in the Fourier plane of the magnification lens to control the scattering exposure in the image plane. Considering the effect of these angle cuts of the collimator, the transmission in the image plane of the proton passing the thick object is then

$$T=2\pi\int_{0}^{\phi_{\mathrm{cut}}}I(\theta)\mathrm{d}\theta, \tag{6}$$

where ϕ_{cut} is the angle of the collimator.

In the above empirical numerical model, the key is the scattering distribution calculation. In this work, the scattering distributions are treated as Gaussians in the laboratory frame, and the expression of Schiz et al. [5] and Baishev et al. [6] is used for the scattering distribution calculation.

When the empirical numerical model is applied to an object with different materials, such as a multi-sphere, Eq. (5) will be modified as

$$I(\theta)=\prod_{i}\left(\sum_{n=0}\frac{1}{n!}\left(\frac{L_i}{\lambda_{\mathrm{E}i}}\right)^{n}(f_{\mathrm{MCS}}*(f_{\mathrm{E}})^{n})(\theta)\right)\mathrm{e}^{-L_i/\lambda_{\mathrm{A}i}}, \tag{7}$$

where i stands for the different materials of the objects.

3 The Monte Carlo simulation

A program based on the Geant4 toolkit was developed and used for the proton radiography experiment simulation. A beam of 24 GeV/c with 10^{10} incident protons illumined the thick object, which was composed of carbon and lead, with the setup of the simulation as shown in Fig. 1. First, the beam is prepared with a diffuser and monitor lens to meet optics requirements. Next the beam is controlled by the magnification imaging lens and collimator, and detected after it passes through the objected being radiographed.

In the simulation, the diffuser is made of tantalum with thickness 1.2 cm. The magnification imaging lens consists of 4 quadrupoles of diameter 20 cm and length 120 cm. The collimator is a cylinder of tungsten of length 1.2 m, and the collimator approximated multiple scattering angle acceptance cuts of 6.68 mrad. The main parameters of the simulation setup are shown in Table 1.

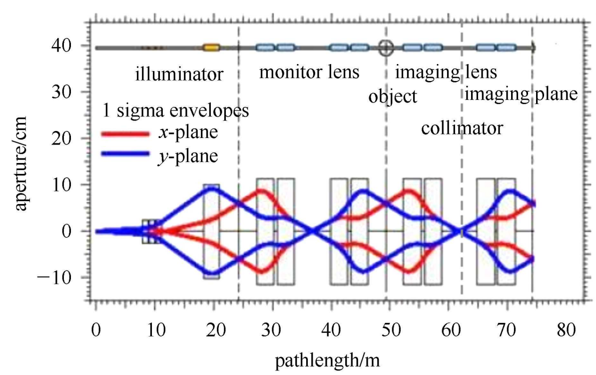

Fig. 1. (color online) The proton radiography setup used for the Monte Carlo simulation.

Table 1. The main parameters of the setup for the present simulation.

simulation parameter	value
proton momentum/GeV·c^{-1}	24
diffuser thickness/cm	1.2
collimator length/m	1.2
angular cut/mrad	6.68
quadroupole aperture/mm	120
quadroupole gradient/T·m^{-1}	8
quadroupole length/m	2
dirft length/m	3.4
chromatic aberration coefficient/m	40.4
field of view/mm	80

In the Monte Carlo simulation, the hadronic shower models provided in Geant4 are used to compute the interaction of the proton and the object, and the Runge-Kutta method is used to track the proton trajectory in the magnetic field.

4 Comparison of the two models with experimental data

The results of the empirical numerical model and the Monte Carlo simulation were compared with the transmission fractions extracted from step-wedge data for various elements from the EA955 experiment at Brookhaven National Laboratory [7]. In this paper, the objects are carbon and lead step-wedges with areal density from 0 to 26 cm. The comparisons of the transmission fraction of the two models and the experimental data are shown in Fig. 2.

The figures show that the results for transmission versus areal density from the two models are in good agreement with the results from experiment EA955 at the AGS proton accelerator at Brookhaven National Laboratory. The figures also show that the transmission calculated by the numerical model is a little larger than the Monte Carlo simulation and the experimental data. There are two main reasons for this difference.

First, the quasi-elastic scattering process in the proton interaction physics is not included in the numerical model. In order to verify this point, the quasi-elastic scattering is considered in the numerical model. That is, the angular distribution will be modified, and the Eq. (5) will be written as

$$I(\theta)=\left(\sum_{n=0}\frac{1}{n!}\left(\frac{L}{\lambda_{\mathrm{E}}}\right)^{n}(f_{\mathrm{MCS}}*(f_{\mathrm{E}}+f_{\mathrm{qes}})^{n})(\theta)\right)\mathrm{e}^{-L/\lambda_{\mathrm{A}}}, \tag{8}$$

where, the f_{que} is the angular distribution for quasi-elastic scattering. The results are also shown in Fig. 2, and signed by the calculation of numerical model (new). From the figures, it can be seen that the calculation of the numerical model is improved.

Second, in the Monte Carlo simulation and the proton radiography experiment, there is also transportation through the quadrupole magnets, while the numerical model just imposed a cut on the particles based on the angle that a given particle has as it left the object.

The empirical numerical model and the Monte Carlo simulation were then applied to the French Test Object (FTO), which is a multi-sphere constituted by vacuum with radius from 0–1.0 cm, uranium with radius from 1.0–4.5 cm, and copper with radius from 4.5–6.0 cm. The radial transmission of the FTO for proton radiography at 24 GeV/c with the collimator angular cut at 4.56 mrad and 6.68 mard was calculated with the two methods, and the results are shown in Fig. 3. From Fig. 3, it can be seen that the results of the two methods agree with each other when the angular cut of the collimator is 6.68 mrad. When the angular cut of the collimator is 4.56 mrad, the transmission calculated by the numerical model is a little larger than the results of the Monte Carlo model. The differences mainly come from the effect of the quasi-elastic scattering process, which has a stronger effect when the imaging plane receives a smaller angular range. Then, we introduced the quasi-elastic scattering effect into the numerical calculation, and modified the Eq. (7) to be

$$I(\theta)=\prod_{i}\left(\sum_{n=0}\frac{1}{n!}\left(\frac{L_i}{\lambda_{\mathrm{E}i}}\right)^{n}(f_{\mathrm{MCS}}*(f_{\mathrm{E}}+f_{\mathrm{eqs}})^{n})(\theta)\right)\times\mathrm{e}^{-L_i/\lambda_{\mathrm{A}i}}. \tag{9}$$

The results are shown in Fig. 3 as calculation by numerical model (new), and the calculation of the numerical model is improved.

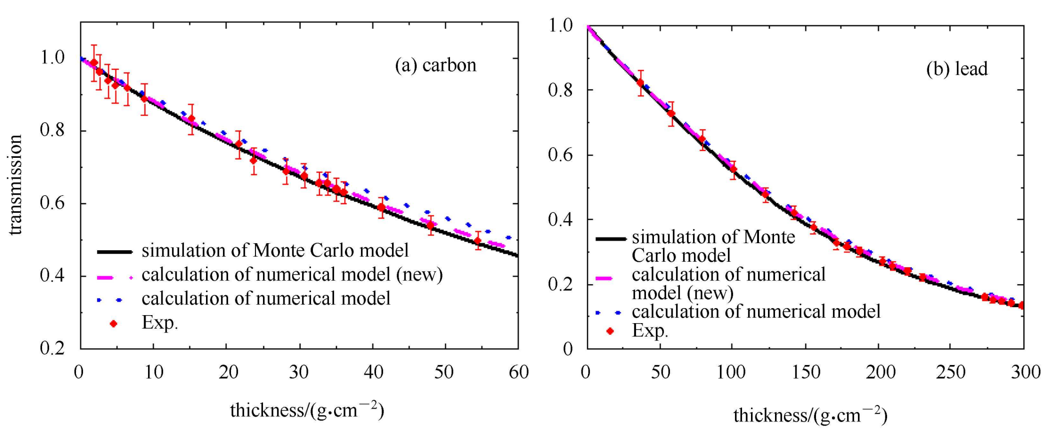

Fig. 2. (color online) Comparison of Monte Carlo simulation and numerical model calculation results with experimental data for carbon (a) and lead (b).

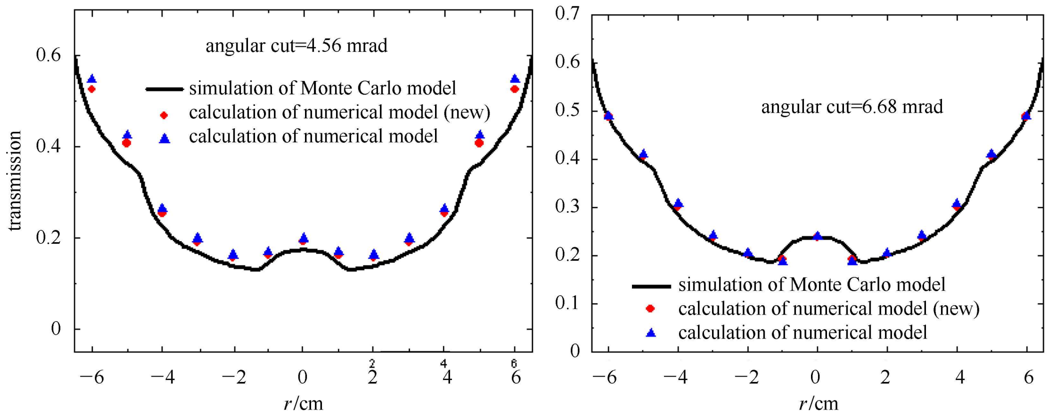

Fig. 3. (color online) Comparison of the Monte Carlo model and numerical model results for the FTO.

5 Conclusions

The present empirical numerical model, which includes the essential physics of all of the elements of the radiographic chain, and a Monte Carlo model based on the Geant4 toolkit, were developed and employed to calculate the transmission of high energy protons in passing through a thick slab of material. The two models agree with experimental step wedge proton transmission data. The calculations also show there are slight differences between the numerical model and the Monte Carlo model, which come from the contribution of the quasi-elastic scattering process and the angle-cut assumption. From transmission calculations of the step-wedge and FTO in high energy proton radiography, the effect of the quasi-elastic scattering progress cannot be ignored. The empirical numerical model can be used to calculate the experimental data with short computation times, and the Monte Carlo simulation can predict the output of proton radiography experiments more accurately with long computation times.

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