Langevin analysis of fission excitation functions induced by protons *

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Abstract: The stochastic Langevin approach to fission is applied to analyze fission excitation functions measured in p+^{206}Pb and p+^{209}Bi systems. A presaddle friction strength of (3–5)$\times 10^{21}$ s^{-1} is extracted by comparing theoretical predictions with experimental data. Furthermore, the small distortion of the formed compound nuclei with respect to the spherical shape under the condition of low angular momentum suggests that experimentally, populating an excited compound system via light-ion induced reactions favors a more accurate determination of presaddle friction with a fission cross section.

Key words: compound nucleus decay, fission excitation function, nuclear dissipation

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1 Introduction

It has been experimentally established [1–6] that measured particle multiplicities emitted in the fission process deviate significantly from that predicted by standard statistical models (SMs), as energy deposited in compound nuclei are increased. This is considered to arise from dissipation effects in fission that are not taken into account in model calculations [7–14].

Probing presaddle friction strength (β) is the current focus of a great number of experimental and theoretical researches, and a lot of studies have been carried out to determine the magnitude of β [4–6]. Light particles can be evaporated along the whole fission path when the fissioning system proceeds from its ground-state configuration up to its scission point, so they are a less-direct signature of presaddle friction (β) due to the interference of postsaddle emission. Various new observables sensitive to β have also been proposed, such as evaporation residue cross section [15] and its spin distribution [16, 17], and the widths of fission-fragment charge distributions [18]. However, the presaddle friction strength is still controversial [19].

Besides that, presently the friction mechanism in fission and its possible dependence on deformation (or on temperature) are not specifically determined, since an adjustable parameter κ_s [11], which is a reduction factor for the strength of the wall formula in the one-body dissipation model, is involved. Furthermore, some assumptions on the characteristics of the populated compound nuclei (CNs), such as a spherical shape of CNs at the ground state, are made in model simulations, which could lead to uncertainties in determining the magnitude of β.

As a direct consequence of dissipation effects, fission is retarded; that is, fission probability is reduced. Therefore, fission cross sections are identified as the most sensitive and fundamental probe of presaddle friction [19, 20].

Previous works [9, 11, 20–23] concerning β employed fission excitation functions provided in heavy-ion reactions, where the formed CNs have a high spin ℓ (up to $\sim 75\hbar$). The high ℓ leads to a distortion of the produced CNs at their ground state. The distortion could affect the transient time and this is not accounted for in earlier and current Langevin calculations [8, 9, 11, 17, 21–24], where a Langevin trajectory starts to simulate fission motion under the assumption that a spherical CN is produced. This neglect causes ambiguity in constraining β when confronting theory with experiment. However, CNs populated by light ions have a smaller ℓ than that by heavy-ion reactions. Thus, employing fission excitation function data induced by light ions can put more severe constraints on β and, correspondingly, give more reliable values of the friction parameter.

For CNs produced by light projectiles bombarding targets, the influence of fusion on the subsequent decay of CNs can be negligible [6]. This is different to the case of heavy projectiles. It is shown [25] that neutron emission in the fusion process of heavy projectiles and heavy targets can affect the formation of superheavy nuclei.

In this context, in the present work, light ions, namely proton-induced fission excitation function data

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of nuclei ^{207}Bi and ^{210}Po available in the EXFOR database [26] are employed to place more stringent constraints on the β. To our knowledge, few used this type of fission data to get information on presaddle dissipation. Apart from that, these data will provide a strict test for the widely accepted stochastic fission model and also shed new light on the magnitude of presaddle friction.

2 Model

A brief description of the combination of the dynamical Langevin equations with a statistical decay model (CDSM) is given. The stochastic approach [27] has been demonstrated to successfully describe a large volume of experimental data on many fission observables for a lot of compound nuclei over a wide range of excitation energy and fissility. The dynamic part of the CDSM is described by the Langevin equation that is expressed by entropy. We employ the following one-dimensional overdamped ($\beta \geqslant 2\times10^{21}$ s^{-1}) [27] Langevin equation to perform the trajectory calculations:

$$\frac{\mathrm{d}q}{\mathrm{d}t}=\frac{T}{M\beta}\frac{\mathrm{d}S}{\mathrm{d}q}+\sqrt{\frac{T}{M\beta}}\Gamma(t). \tag{1}$$

Here q is the dimensionless fission coordinate and is defined as half the distance between the center of mass of the future fission fragments divided by the radius of the compound nucleus, M is the inertia parameter, and β is the dissipation strength. The temperature in Eq. (1) is denoted by T and $\Gamma(t)$ is a fluctuating force with $\langle\Gamma(t)\rangle=0$ and $\langle\Gamma(t)\Gamma(t')\rangle=2\delta(t-t')$. The driving force of the Langevin equation is calculated from the entropy:

$$S(q,E^*,\ell)=2\sqrt{a(q)[E^*-V(q,\ell)]}. \tag{2}$$

The angular momentum ℓ due to rotation is indicated. E^* is the excitation energy of the system. Eq. (2) is constructed from the Fermi-gas expression with a finite-range liquid-drop potential $V(q)$ [28] that includes the q-dependent surface, Coulomb and rotation energy terms.

In constructing the entropy, the deformation-dependent level density parameter is used:

$$a(q)=a_1A+a_2A^{2/3}B_{\mathrm{s}}(q), \tag{3}$$

where A is the mass number, and a_1=0.073 and a_2=0.095 are taken from Ignatyuk et al. [29]. B_{s} is the dimensionless surface area (for a sphere B_{s}=1) which can be parametrized by the analytical expression [30]

$$B_{\mathrm{s}}(q)=\begin{cases}1+2.844(q-0.375)^2, & \text{if } q<0.452,\\ 0.983+0.439(q-0.375), & \text{if } q\geqslant0.452.\end{cases} \tag{4}$$

In the CDSM prescission particle, evaporation along Langevin fission trajectories from their ground state to their scission point has been taken into account using a Monte Carlo simulation technique. Particle emission widths are given by Blann's formula [31].

For starting a trajectory an orbit angular momentum value is sampled from the fusion spin distribution function

$$\sigma_{\mathrm{fus}}(\ell)=\frac{2\pi}{k^2}\frac{2\ell+1}{1+\exp[(\ell-\ell_{\mathrm{c}})/\delta\ell]}. \tag{5}$$

The parameters ℓ_{c} and $\delta\ell$ are the critical angular momenta for fusion and diffuseness, respectively. For proton-induced fusion reactions, they are found to follow an approximate scaling, which is in accordance with the surface friction model [32] that describes the fusion cross sections very well. Namely,

$$\ell_{\mathrm{c}}=\sqrt{4.16(E_{\mathrm{c.m.}}-7.21)-1.7E_{\mathrm{c.m.}}/(\pi\lambda\!\!\!^{-2})}, \tag{6}$$

where $E_{\mathrm{c.m.}}=E_{\mathrm{lab}}A_{\mathrm{T}}/(A_{\mathrm{T}}+A_{\mathrm{P}})$, $\lambda\!\!\!^{-}=\hbar(A_{\mathrm{T}}+A_{\mathrm{P}})/A_{\mathrm{T}}/\sqrt{2A_{\mathrm{P}}\,m_{\mathrm{nuc}}\,E_{\mathrm{lab}}}$. E_{lab} denotes the laboratory energy of the projectile proton, m_{nuc} is the nucleon mass. A_{T} and A_{P} represent the mass number of target and projectile, respectively. The diffuseness δl scales as

$$\delta l=\begin{cases}\left[(A_{\mathrm{P}}A_{\mathrm{T}})^{3/2}\times10^{-5}\right][1.5+0.02(E_{\mathrm{c.m.}}-17.21)]\\ \text{for } E_{\mathrm{c.m.}}>17.21,\\ \left[(A_{\mathrm{P}}A_{\mathrm{T}})^{3/2}\times10^{-5}\right][1.5-0.04(E_{\mathrm{c.m.}}-17.21)]\\ \text{for } E_{\mathrm{c.m.}}<17.21.\end{cases} \tag{7}$$

These scaling values have been widely tested by successfully fitting proton-induced fusion cross sections of various reaction systems [32]. To accumulate sufficient statistics, 10^7 Langevin trajectories are simulated.

3 Results and discussions

A deviation of the experimental fission cross section (σ_{f}) from that given by SMs (which use the traditional Bohr-Wheeler formula for fission width) is a clear signature that dissipation plays a prominent role in the fission process of a hot nucleus decay. We thus carry out the SM calculation that ignores friction effects.

Compared to the fission width given by the Langevin model, Kramers' fission width does not consider the time dependence of fission widths and postsaddle fission dynamics. Thus for an accurate value of β, Langevin fission width is employed in our calculation.

The fission excitation function data of p+^{206}Pb $\longrightarrow$ ^{207}Bi system [26] are used and compared with Langevin simulations.

It is noted from Fig. 1 that SM calculations appreciably overestimate experimental σ_{f}, indicating the necessity of accounting for the dissipation effects in calculation.

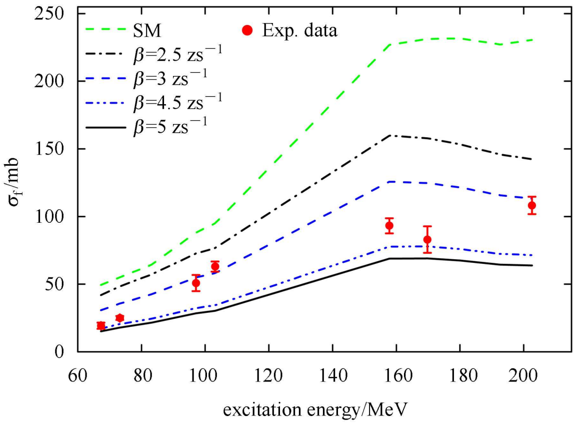

Fig. 1. (color online) Fits to measured excitation function data of fission cross sections (denoted by red circles with error bars) in the p+^{206}Pb system [26]. SM predictions are represented by a dashed green line. Langevin model calculations are carried out at various friction strengths β. The unit of β is zs^{-1}; 1 zs=10^{21}s^{-1}. Note that β represents the friction strength throughout the presaddle fission process.

Nuclear friction hinders fission, thus providing more time for particle evaporation that leads to the enhancement of prescission particles. For the decaying system under consideration, presaddle emission constitutes a major portion of prescission particles as the saddle-to-scission distance is comparatively short, which limits postsaddle emission. Additionally, dissipation decreases fission widths even if the fission probability flow attains its quasistationary value. The two factors lead to a rise of presaddle neutrons in the presence of nuclear friction.

In addition, angular momentum is also a factor that can affect the magnitude of $M_{\rm n}$. This is because fission barriers are a decreasing function of ℓ (Fig. 2). For proton-nucleus collisions, the low spin of the produced excited nuclei yields a high barrier, which protects the decaying system from disentangling quickly. In other words, the system stays longer inside the saddle point, and more time is thus available for evaporating neutrons.

One can see from Fig. 3 that as the friction effects are taken into account in the calculations, the predicted presaddle neutron multiplicity $M_{\rm n}$ increases. For example, at excitation energy E^*=73 MeV, $M_{\rm n}$ given by SM is 1.8, and it is 2.23 at β=2.5 zs^{-1} and it further rises up to 2.48 at a stronger friction β=5 zs^{-1}. Moreover, the friction effects on $M_{\rm n}$ are significant with increasing E^*. At E^*=158 MeV the difference $M_{\rm n}$ calculated by SM and by the Langevin model assuming β=2.5 zs^{-1} reaches 2.4, which is far larger than that at low energy E^*=73 MeV, where the corresponding difference is 0.43. The reason for this is that at higher energy, particle evaporation time becomes shorter. As a result, during the transient time caused by friction, more particles can be emitted that leads to an enhancement of $M_{\rm n}$ at high energy. The stronger the hindrance, the longer the transient time. So, a larger friction strength yields a larger $M_{\rm n}$, as observed in Fig. 3.

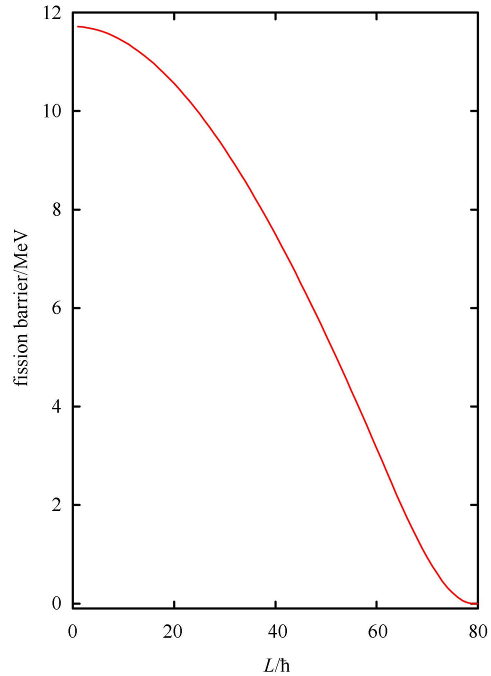

Fig. 2. (color online) Fission barrier of nuclei ^{207}Bi as a function of angular momentum calculated with the method in Refs. [28, 30].

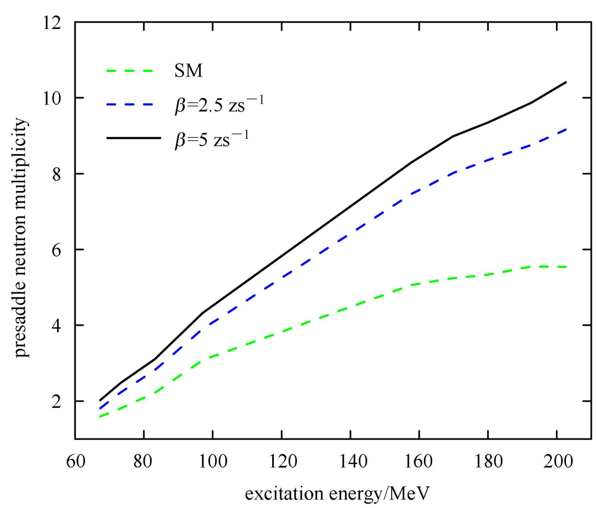

Fig. 3. (color online) Presaddle neutron multiplicity predicted by SM and the Langevin model at β=2.5 zs^{-1} and 5 zs^{-1} for p+^{206}Pb reaction.

When an excited nucleus decays, fission and evaporation are two competitive decay channels. A strong neutron evaporation will reduce the fission probability, since more energy is carried away from the decaying system that suppresses fission. Consequently, relative to the SM estimate, fission cross sections are decreased when friction effects are considered because of more neutron emission prior to fission, and the decrease becomes greater (see Fig. 1) at larger friction due to an increase of $M_{\rm n}$ with increasing β.

In order to better constrain the value of presaddle friction, we made a detailed calculation by taking a number of β values. As can be seen, the estimated $\sigma_{\rm f}$ at β=2.5 zs^{-1} are lower than the SM results but still higher than the data. It means that although introducing friction effects can delay fission, a stronger hindrance is required to fit the data. We find that experimental data lie between the curves calculated at β=3 zs^{-1} and β=4.5 zs^{-1}. A slight increase of β, for example β=5 zs^{-1}, leads to an evident deviation from all data points. This clearly shows the crucial role that friction plays in satisfactorily interpreting the experimental results.

The fission excitation functions measured in p+^{209}Bi are also analyzed and a quite narrow range of β=(3–5) zs^{-1} is obtained; see Fig. 4.

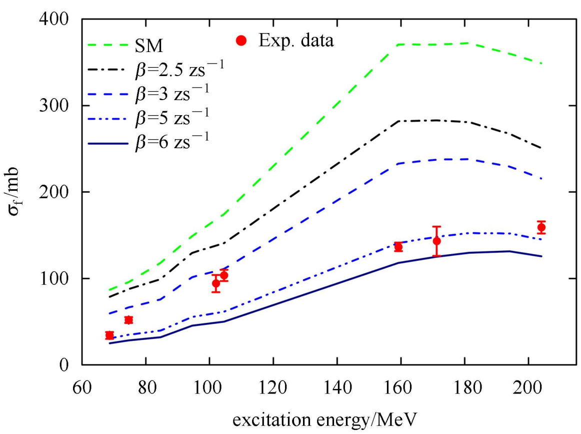

Fig. 4. (color online) Same as Fig. 1, but for the p+^{209}Bi system [26].

We compare the resulting β value with other works, where various presaddle friction strengths were reported. A fit to prescission multiplicity gives different β values, for example, (5–8) zs^{-1} [33], (3–10) zs^{-1} [34], $\sim$5 zs^{-1} [35], etc. The good agreement between theoretical and experimental giant dipole resonance γ rays and evaporation-residue cross sections proposes the friction strength of (4−6) zs^{-1} [36], <8 zs^{-1} [37] and $\leqslant$10 zs^{-1} [15]. Explaining the data of evaporation residue spin distributions requires a friction strength of $\sim$5 zs^{-1} [17]. The measured mass- and kinetic-energy distributions of fission fragments suggests a β value of 5.5 zs^{-1} [38]. Recent measurements for fission-fragment charge-distribution widths found that the magnitude of β is (2–5) zs^{-1} [18, 39]. These friction values are weaker than that given by the one-body dissipation model.

With an increase in model dimensionality, fission rates rise [8, 9, 11, 24]. This increases the fission width, implying that a larger β could be required in order to reproduce the measured $\sigma_{\rm f}$.

While it has been shown [9] that a constant inertia parameter assumed for overdamped motion is quite a good approximation, it is worthwhile to examine the influence of a variable inertia on the value of the extracted β.

In addition to excitation energy and system size that have been known to affect the characteristics of a CN decay, the distribution of CN shapes populated in heavy-ion collisions has been noted [40] to significantly affect evaporation. Neglecting the influence of the CNs shape distribution on decay properties could give rise to uncertainties when deriving the amplitude of the friction parameter. It exhibits the importance of incorporating the shape distribution into the calculation, especially when confronting theoretical predictions with experimental measurements concerning heavy-ion-induced CN decay processes. By contrast, CNs formed in light-ion-induced fusions have a nearly spherical shape due to the low spin involved. The prominent advantage available in the latter type of reaction is favorable for the better determination of β. It means that experimentally, yielding a low-spin CN can improve the accuracy of the value of β deduced from comparing experimental and theoretical fission cross sections.

The present study solves the Langevin equation that is coupled with particle emission along the entire fission path. We note that in a recent work [41], an analytical solution for a multi-dimensional Langevin equation under the condition of large friction has been obtained and the probability of particles passing over a saddle point has been discussed in great detail. Therefore, it is interesting to combine the analytical results and the method used in solving the problem of diffusion [41] with statistical particle evaporation to survey the fission of hot nuclei.

4 Summary

In summary, in the framework of stochastic models we have extracted a presaddle friction value of (3–5) zs^{-1} by confronting theory with proton induced fission excitation function data of ^{207}Bi and ^{210}Po. In addition, light-ion-induced fusion reactions greatly reduce the distortion of the CNs shape at their ground state caused by large angular momentum, simplifying the theoretical description of the fission of hot nuclei, and therefore, favoring a more accurate determination of presaddle friction. This suggests that on the experimental side, to accurately probe information of presaddle dissipation by measuring fission excitation functions, it is optimal to populate a compound system with low spin.

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