Melting temperature of heavy quarkonium with a holographic potential up to sub-leading order *

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Abstract: A calculation of the melting temperatures of heavy quarkonium states with the holographic potential was introduced in a previous work. In this paper, we consider the holographic potential at sub-leading order, which permits finite coupling corrections to be taken into account. It is found that this correction lowers the dissociation temperatures of heavy quarkonium.

Keywords: melting temperature, heavy quarkonium, holographic potential, correction

PACS: 11.25.Tq, 12.39.Pn, 11.15.Tk **DOI:** 10.1088/1674-1137/40/6/064101

1 Introduction

Heavy quarkonium dissociation is an important signal of the formation of Quark-Gluon Plasma (QGP)in heavy ion collisions at RHIC and LHC. However, much experiment data indicates that the QGP is strongly coupled. Thus, the study of heavy quarkonium and its dissociation requires non-peturbative techniques, such as Lattice QCD and potential models [1]. The lattice simulation of the quark-antiquark potential and the spectral density of hadronic correlators yield a consistent picture of quarkonium dissociation as well as the numerical value T_d. On the other hand, heavy quarkonium dissociation can be studied within potential models, for instance the energy levels and the dissociation temperature can be carried out with the aid of a non-relativistic Schrodinger equation with a temperature dependent effective potential when we neglect the velocity($v \ll 1$) of the constituent quarks [2–4], and if we consider the relativistic correction, the two-body Dirac equation can be employed [5]. The holographic potential addressed in this paper is one example of a potential model.

Holographic potential at finite temperature at strong coupling relies on the AdS/CFT duality which can explore the strongly coupled $\mathcal{N}=4$ supersymmetric Yang-Mills (SYM) plasma through the correspondence between the type IIB superstring theory formulated on $AdS_5 \times S^5$ and $\mathcal{N}=4$ SYM in four dimensions [6–10].

In a previous work [11], the melting temperatures of heavy quarkonium states were studied with the holographic potential. In this paper we consider the holographic potential including its sub-leading order and obtain the correction to the melting temperatures.

The paper is organized as follows. In the next section, we will present our strategy for computation. The sub-leading order of the holographic potential and its corrections to the dissociation temperatures will be discussed in Section 3 and Section 4. Section 5 concludes the paper.

2 Setup

Heavy quarkonium, J/ψ or Υ, can be modelled as a non-relativistic bound state of a heavy quark and its antiparticle, and the wave function of their relative motion satisfies the Schrödinger equation

$$[-\frac{1}{2\mu}\nabla^2+U(r,T)]\psi=-E(T)\psi, \tag{1}$$

$E(T)$ is the binding energy of the bound state and $U(r,T)$ is related to the free energy $F(r,T)$ by

$$U(r,T)=-T^2\left[\frac{\partial}{\partial T}\left(\frac{F(r,T)}{T}\right)\right]_r, \tag{2}$$

$F(r,T)$ can be extracted from the Wilson loop operator between a static pair of $q\bar{q}$

$$\mathrm{e}^{-\frac{1}{T}F(r,T)}=\frac{\mathrm{tr}\langle W^\dagger(L_+)W(L_-)\rangle}{\mathrm{tr}\langle W^\dagger(L_+)\rangle\langle W(L_-)\rangle}, \tag{3}$$

Received 17 November 2015, Revised 2 February 2016

∗ Supported by QLPL(QLPL201408) and NSFC (11547204, 11375070, 11135011, 11221504)

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where $L_{\pm}$ denotes the Wilson line running in the Euclidean time direction at spatial coordinates $\left(0,0,\pm\frac{1}{2}r\right)$ and is closed by the periodicity $\beta=\frac{1}{T}$ and

$$W(L_{\pm})=P\mathrm{e}^{-\mathrm{i}\oint_{L_{\pm}}\mathrm{d}x^{\mu}A_{\mu}(x)} \tag{4}$$

with A_{μ} the gauge potential and the symbol P enforcing the path ordering along the loop C. The thermal expectation value $<W(C)>$ can be measured for QCD on a lattice and the heavy quark potential is defined with F-ansatz or U-ansatz.

The holographic principle places $L_{\pm}$ on the boundary ($z\rightarrow 0$)of the Schwarzschild-AdS$_5\times S^5$, whose metric can be written as

$$\mathrm{d}s^2=\pi^2T^2z^2(f\mathrm{d}t^2+\mathrm{d}\vec{x}^2)+\frac{1}{\pi^2T^2z^2f}\mathrm{d}z^2, \tag{5}$$

where $f=1-\frac{1}{z^4}$, $\mathrm{d}\vec{x}^2=\mathrm{d}x_1^2+\mathrm{d}x_2^2+\mathrm{d}x_3^2$ with $x_1=x_2=0$ and x_3 a function of z.

In the case of $\mathcal{N}=4$ SYM, the AdS/CFT duality relates the Wilson loop expectation value to the path integral of the string-sigma action developed in Ref. [12] of the worldsheet in the AdS$_5\times S^5$ bulk.

To leading order of the strong coupling, the path integral is given by its classical limit, which is the minimum area of the worldsheet

$$F(r,T)=-\frac{4\pi^2}{\Gamma^4\left(\frac{1}{4}\right)}\frac{\sqrt{\lambda}}{r}\min[g_0(rT),0], \tag{6}$$

with

$$-\frac{4\pi^2}{\Gamma^4\left(\frac{1}{4}\right)}\frac{\sqrt{\lambda}}{r}g_0(rT)=\frac{1}{\pi\alpha'}\left[\int_0^{z_0}\mathrm{d}z\left(\frac{\sqrt{f}z_0^2}{z^2\sqrt{z_0^4-z^4}}-\frac{1}{z^2}\right)\right.$$
$$\left.-\int_{z_0}^{z_h}\frac{\mathrm{d}z}{z^2}\right], \tag{7}$$

where $g_0(rT)$ is a monotonically decreasing function with $g_0(0)=1$, $g_0(r_0T)=0$ and r_0 is the screening length. If we introduce a dimensionless radial coordinate $\rho=\pi Tr$, we have

$$g_0(\rho)=1-\frac{\rho}{\rho_0} \tag{8}$$

with $\rho_0=0.7359$.

The melting temperatures of heavy quarkonium states with the leading order potential related to (6) were discussed in Ref. [11].

3 The holographic potential model

Now we add the sub-leading order term to the holographic potential and explore its contribution.

As was shown in Ref. [13], the strong coupling expansion of $F(r,T)$ at large λ can be written as

$$F(r,T)=-\frac{4\pi^2}{\Gamma^4\left(\frac{1}{4}\right)}\frac{\sqrt{\lambda}}{r}\min$$
$$\left[g_0(rT)-\frac{1.3346g_1(rT)}{\sqrt{\lambda}}+O\left(\frac{1}{\lambda}\right),0\right] \tag{9}$$

where $g_1(rT)$ is a monotonically decreasing function, which reaches 0.92 at r_0.

Likewise, we use the dimensionless radial coordinate $\rho=\pi Tr$, and we find

$$F(r,T)=-\frac{\alpha}{r}\phi(\rho)\theta(\rho_1-\rho), \tag{10}$$

where $\alpha=\frac{4\pi^2}{\Gamma^4\left(\frac{1}{4}\right)}\sqrt{\lambda}\approx 0.2285\sqrt{\lambda}$, ρ_1 is determined by $\phi(\rho_1)=0$.

The analytical small ρ expansion and numerical results of $\phi(\rho)$ both suggest

$$\phi(\rho)=g_0(\rho)-\frac{1.3346g_1(\rho)}{\sqrt{\lambda}}$$
$$\approx 1-\frac{\rho}{\rho_0}-\frac{1.3346g_1(\rho)}{\sqrt{\lambda}}. \tag{11}$$

As the temperature correction to the sub-leading term of the heavy quark potential is small, or in other words $g_1(\rho)$ decreases monotonically from 1 to 0.92 as $\rho\in(0,\rho_0)$, we can fit $g_1(\rho)=1-0.11\rho$. This yields

$$\phi(\rho)=1-\frac{\rho}{\rho_0}-\frac{1.3346(1-0.11\rho)}{\sqrt{\lambda}}. \tag{12}$$

To proceed, we define the dissociation temperature T_{d}' as the temperature when the bound energy $E(T_{\mathrm{d}}')$ becomes zero, and the corresponding Schrödinger equation reduces to

$$\frac{\mathrm{d}^2R}{\mathrm{d}\rho^2}+\frac{2}{\rho}\frac{\mathrm{d}R}{\mathrm{d}\rho}-\left[\frac{l(l+1)}{\rho^2}+U\right]R=0 \tag{13}$$

with $U=\frac{mV_{\mathrm{eff}}}{\pi^2T^2}$.

Actually, one should consider the exact holographic potential, but it has been found in Ref. [5] that the comparison with the dissociation temperature obtained from (12) is very close to that from the exact holographic potential. So we stay with the truncated Coulomb potential for the rest of the paper. Here we consider the U-ansatz, where we have

$$U=-\frac{\eta^2}{\rho_1\rho}\left[\phi(\rho)-\rho\left(\frac{\mathrm{d}\phi}{\mathrm{d}\rho}\right)\right]\theta(\rho_1-\rho) \tag{14}$$

with

$$\eta=\sqrt{\frac{\alpha\rho_1 m}{\pi T}}. \tag{15}$$

It follows from (12) and (14) that

$$U=-\frac{\eta^2}{\rho_1\rho}\left(1-\frac{1.3346}{\sqrt{\lambda}}\right)\theta(\rho_1-\rho) \tag{16}$$

On writing

$$\eta_x=\eta\sqrt{1-\frac{1.3346}{\sqrt{\lambda}}}, \tag{17}$$

we have

$$U=-\frac{\eta_x^2}{\rho_1\rho}\theta(\rho_1-\rho). \tag{18}$$

Substituting (18) into (13) one gets

$$R(r)=\frac{1}{\sqrt{\rho}}J_{2l+1}\left(2\eta_x\sqrt{\frac{\rho}{\rho_1}}\right),\qquad \rho\leqslant\rho_1,$$
$$R(r)=\text{const.}\rho^{-l-1},\qquad \rho>\rho_1 \tag{19}$$

with $J_n(x)$ the Bessel function. Then the threshold η_x can be related to the matching condition at $\rho=\rho_1$

$$\frac{\mathrm{d}}{\mathrm{d}\rho}(\rho^{l+1}R(r))|_{\rho=\rho_1^-}=0, \tag{20}$$

this yields the correspond secular equation for η_x

$$2l+1-\eta_x\frac{J_{2l+2}(2\eta_x)}{J_{2l+1}(2\eta_x)}=0. \tag{21}$$

Knowing the values of l, η_x can be calculated from (21).

Finally, the melting temperature of heavy quarkonium with a holographic potential up to sub-leading order can be obtained according to:

$$T_d'=\frac{\alpha\rho_1 m}{\pi\eta^2}=\frac{\alpha\rho_1 m}{\pi\eta_x^2}\left(1-\frac{1.3346}{\sqrt{\lambda}}\right). \tag{22}$$

4 Results

Now we discuss our results. At first, we take $\lambda\to\infty$ in (12) and (22), which leads to the leading order case

$$T_{\mathrm{d}}=\frac{\alpha\rho_0 m}{\pi\eta_x^2}, \tag{23}$$

where T_{d} is the melting temperature of heavy quarkonium states with the leading order potential. The numerical results for T_{d} of J/Ψ and Υ are presented in Table 1, where we have chosen $m=1.65$ GeV,4.85 GeV for c and b quarks.

Table 1. T_{d} in MeV for J/Ψ and Υ under the holographic potential.

	$T_{\mathrm{d}}(\lambda=5.5)$	$T_{\mathrm{d}}(\lambda=6\pi)$
J/Ψ(1s)	143	265
J/Ψ(2s)	27	50
J/Ψ(1p)	31	58
Υ(1s)	421	780
Υ(2s)	80	148
Υ(1p)	92	171

Then we consider sub-leading order correction. For comparison, here we we show the curve about $T_{\mathrm{d}}'/T_{\mathrm{d}}$ vs λ in Fig. 1. Note that owing to the λ correction to the holographic potential, T_{d}' is smaller than T_d. With the typical interval $5.5<\lambda<6\pi$ [14], we find this correction gives rise to a 47% reduction of T_{d} when $\lambda=6\pi$, and it will increase as λ becomes smaller. One may doubt this result because with small values of λ the corrections term in (9) and (22) will be very large so that these corrections are meaningless. However, this consideration is unnecessary since the strong coupling expansion of the potential relies on the assumption that the λ is large. Indeed, it appears that this correction will vanish for $\lambda\to\infty$ as it must, since the sub-leading order to the potential vanishes in that limit.

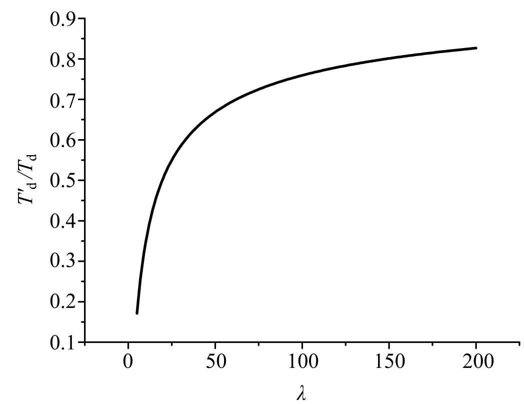

Fig. 1. $T_{\mathrm{d}}'/T_{\mathrm{d}}$ vs λ. Melting temperature with sub-leading order potential over its counterpart with potential versus λ.

5 Conclusion

In this paper, we have investigated the melting temperatures studied with the truncated holographic potential and we consider the holographic potential with its sub-leading order, which permits to take into account finite coupling corrections. It is found that this correction becomes smaller as λ increases. With this corrections, the dissociation temperatures of heavy quarkonium are lowered, leaving the corrected values further below the lattice result. This disagreement can be attributed to several reasons. Firstly, the short screening length $r_0\approx 0.25$ fm at $T=200$ MeV of the AdS/CFT potential and sharp cutoff nature of the screening. Secondly, one should take into account the different number of degrees of freedom in $N_c = 3$ SYM and 3 flavor QCD; a similar problem has been explained in the calculation of the jet quenching parameter beyond AdS/CFT correspondence in Ref. [15], where they have matched the corresponding entropy density to obtain $T^3 \approx 3T^3_{\rm SYM}$. Moreover, one should also bear in mind that the particles of $\mathcal{N} = 4$ SYM are quite different to that of QCD. In particular it does not include particles in the fundamental representation, but only in the adjoint representation.

To summarize, the melting temperature studied in this work relies on the holographic quark potential. However, in gauge-gravity duality, heavy quark potential at finite temperature is usually calculated with the pure AdS background, and the potential also does not contain any confining term in the deconfined phase. This has led some authors to consider a potential closer to QCD, for instance heavy quark potential in strongly-coupled $\mathcal{N} = 4$ SYM in a magnetic field [16] and with some deformed AdS$_5$ model [17]. Applying these rectified potentials, one can obtain the melting temperature as well.

However, we should admit that there are some shortcomings in our work. Firstly, our updated results do not seem to be close to "reality"; this may depend on the model. Secondly, recently many authors have suggested that the potential at non-zero temperature is complex [18, 19]: the real part is neither the free energy nor the internal energy, and the imaginary potential plays an important role in setting the dissociation temperature. This subject is very interesting, and we hope to do some future work in this regard.

We would like to thank Prof. Hai-cang Ren for useful discussions.

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