Rotational energy conversion and thermal evolution of neutron stars [*]

Cui Zhu(朱翠)[1,2,3] Xia Zhou(周霞)[1,2;1)] Na Wang(王娜)[1,2]

[1] Xinjiang Astronomical Observatory, Chinese Academy of Sciences, Urumqi 830011, China
[2] Key Laboratory of Radio Astronomy, Chinese Academy of Sciences, Urumqi 830011, China
[3] University of Chinese Academy of Sciences, 19A Yuquan Road, Beijing 100049, China

Abstract: Pulsars are rapidly spinning, strongly magnetized neutron stars. Their electromagnetic dipole radiation is usually assumed to be at the expense of the rotational energy. In this work, we consider a new channel through which rotational energy could be radiated away directly via neutrinos. With this new energy conversion channel, we can improve the chemical heating mechanism that originates in the deviation from β equilibrium due to spin-down compression. The improved chemical and thermal evolution equations with different magnetic field strengths are solved numerically. The results show that the new energy conversion channel could raise the surface temperature of neutron stars, especially for weak field stars at later stages of their evolution. Moreover, our results indicate that the new energy conversion channel induced by the non-equilibrium reaction processes should be taken into account in the study of thermal evolution.

Keywords: neutron star, rotation, thermal evolution, energy conversion

PACS: 26.60.-c, 97.10.Kc, 97.60.Jd **DOI:** 10.1088/1674-1137/41/12/125104

1 Introduction

A pulsar is a highly magnetized, rotating neutron star (NS). The period of a pulsar is extremely stable but it is not constant. The spin-down of a pulsar is usually considered to result from electromagnetic dipole radiation [1, 2]. By tracking the long term rotational and thermal evolution of NSs, we can study the nature of dense matter. For example, Zheng et al. [3] investigated the cooling of color-superconducting color-flavor locked (CFL) phase strange stars, with the effect of deconfinement heating and the existence of a Cooper pair breaking and formation (PBF) process. With the help of brightness constraints from young compact objects, Ref. [3] supplied a constraint on the existence of CFL phases in strange stars. Negreiros et al. [4] investigated the effects of rotation on the thermal evolution of NSs in detail, and their results show that the rotational evolution is linked to a reorganization of particle composition in the stellar interior, which leads to the switching on of exotic neutrino emission processes.

During the rotational and thermal evolution of NSs, several heating mechanisms may become important, especially in the later stages of the evolution. These include chemical heating [5–7], compositional transitions in the crust [8], release of strain energy stored by the solid crust due to spin down deformation [9], r-mode dissipation heating [10, 11] and deconfinement heating [3, 12, 13]. The energy resources of these heating mechanisms are closely related to the rotational evolution of the stars. Some of them are directly connected with the conversion of rotational energy. For example, the heating energy of the chemical heating mechanism comes from the rotational energy, which is converted into heating by storing rotational energy in terms of chemical energy [5], and the energy of r-mode dissipation heating comes from the dissipation of rotational energy due to viscous damping [11].

Sa'd and Schaffner-Bielich [14] investigated the dissipation of the density oscillations of condensed matter present in the interior of compact stars. They suggested that the energy of the r-mode dissipates not only through non-equilibrium effects but also via neutrino emission. The latter mechanism constitutes another source of the bulk viscosity, the so-called radiative viscosity. The results show that, in the case of non-strange quark matter, this effect is 1.5 times larger than that of the bulk viscosity [14]. Yang et al. [15, 16] studied the non-linear effects of radiative viscosity of *npe* matter for both the direct Urca process and modified Urca process in NSs. In Ref. [17], the authors extended radiative viscosity to superfluid matter. Moreover, Jaikumar and Sandalski [18]

Received 7 April 2017, Revised 11 September 2017

[*] Supported by National Natural Science Foundation of China (11373006) and the West Light Foundation of Chinese Academy of Sciences (ZD201302)

[1)] E-mail: zhouxia@xao.ac.cn

showed that non-equilibrium neutrino rates associated with the r-mode fluctuation could change the thermal and rotational evolution of isolated strange stars considerably. The above works show that energy dissipation is not only through non-equilibrium effects, but that neutrino emissions should be considered when deviation from β equilibrium is considered.

Several different processes could lead to a departure from β equilibrium, including a changing rotation rate [5, 7], radial pulsation [19], gravitational collapse [20, 21] and the time-variation of the gravitational constant [22]. As discussed above, the energy dissipation via neutrino emission induced by deviation from β equilibrium is caused by stellar pulsation. The purpose of this paper is to discuss the effect of this new channel induced by pulsar spin down, through which the rotational energy of the NS can be taken away directly by neutrinos.

In a rotating NS, chemical heating originates in the departure from β equilibrium due to spin down compression. When the departure takes place, there exists not only a net reaction rate of β decay and its inverse, but also an increase of total neutrino emission [23]. The net reaction rate and total neutrino emission increment have an equivalent effect on releasing the stored chemical energy, i.e., chemical energy can be taken away through either of the channels. This is different from the previous understanding of chemical heating, which suggested that the conversion of rotational energy into heat takes place first and then the neutrinos take away the internal energy of the stars. Therefore, the chemical heating mechanism scenario mentioned in Ref. [5] should be improved. Increased neutrino emissivity could escape directly from the star and change the amplitude of chemical deviation along with the change of net reaction rate and spin-down rate.

We will discuss the effect of the new channel through which rotational energy can be converted into heat and neutrinos. The thermal evolution of NSs is also discussed. The paper is organized as follows. Section 2 introduces our improvements to the modeling rotational energy conversion, and the corresponding evolutionary equations will also be introduced. Section 3 presents the numerical results, and comparisons of the old scenario with the improved one. Discussion and conclusions are given in Section 4.

2 Rotational energy conversion and improved model

The original framework of chemical heating was described by Reisenegger et al. [5]. Here we describe the fundamental equations and the improvements related to the rotational energy being taken away by neutrinos. We consider a simple model of NS, consisting of *npe* matter.

For a normal NS, the simplest weak interaction process that could proceed in the core is the direct Urca process [23],

$$\mathrm{n}\rightarrow\mathrm{p}+\mathrm{e}+\bar{\nu}_{\mathrm{e}},\quad \mathrm{p}+\mathrm{e}\rightarrow\mathrm{n}+\nu_{\mathrm{e}}. \tag{1}$$

This is the most powerful neutrino process that can potentially lead to the cooling of a NS. It contains two successive reactions: β decay and electron capture. These processes are only possible in NSs if the proton fraction exceeds a critical threshold. Otherwise energy and momentum cannot be conserved simultaneously for these reactions [23]. If this were the case, the modified Urca process would prevail:

$$\mathrm{n}+\mathrm{N}\rightarrow\mathrm{p}+\mathrm{N}+\mathrm{e}+\bar{\nu}_{\mathrm{e}},\quad \mathrm{p}+\mathrm{N}+\mathrm{e}\rightarrow\mathrm{n}+\mathrm{N}+\nu_{\mathrm{e}}, \tag{2}$$

where N is an additional spectator nucleon (either a neutron or a proton) that allows energy and momentum conservation.

The outcome of these reactions is a neutrino pair which carries energy away by escaping from the star. The composition remains unchanged if the rates of the direct and inverse reactions are equal, which happens in the β-equilibrium state (β equilibrium condition is $\mu_{\mathrm{n}}=\mu_{\mathrm{p}}+\mu_{\mathrm{e}}$) [23]. In the absence of this equilibrium, the rates of direct and inverse reactions are different, and the reactions affect the nuclear composition, driving matter towards the equilibrium. While the star spins down, the internal density of the star will increase. This density variation will change the chemical equilibrium state throughout the core. If a finite departure from the chemical equilibrium ($\delta\mu=\mu_{\mathrm{n}}-\mu_{\mathrm{p}}-\mu_{\mathrm{e}}$) is produced, the net effect of the reactions is to increase the chemical energy at the expense of the stored rotational energy. This is the chemical heating mechanism, a channel through which the rotational energy of NSs can be converted into heat [5, 24, 25].

In this work, we are going to discuss the non-equilibrium β reactions induced by a changing rotation rate. According to Ref. [5], the time evolution of chemical potential difference and temperature are given by

$$\delta\dot{\mu} = -E_{xx}\left(\alpha n\frac{E_{nx}}{E_{xx}}\frac{\Omega\dot{\Omega}}{G\rho_c}+\frac{\Gamma}{n}\right), \tag{3}$$

$$\dot{T} = \frac{1}{E_{ss}^{-1}T}\left(\frac{\Gamma\delta\mu}{n}-\frac{\epsilon}{n}-\dot{E}_{\gamma}\right), \tag{4}$$

where α is a positive and dimensionless number of order unity that depends slightly on radius within the core of a NS, n is the baryon number density, E_{nx}, E_{xx} and E_{ss} are partial derivatives of the total energy per baryon with respect to baryon number density, composition parameter $x=n_p/n$ and entropy s per baryon, respectively ($E_{nx}=\partial^2E/\partial n\partial x$, $E_{xx}=\partial^2E/\partial x\partial x$, $E_{ss}=\partial^2E/\partial s\partial s$), G is the gravitational constant, ρ_c is the central density of the star, Ω and $\dot{\Omega}$ are angular velocity and the derivatives angular velocity respectively, Γ is the reaction rate of the corresponding reactions, ϵ is the total emissivity, and $\dot{E}_\gamma$ represents the energy in photons radiated from the surface of a NS.

The chemical imbalance increases the phase space available to the products of the neutrino-emitting reactions, and the temperature also affects the chemical imbalance by determining the rate at which reactions proceed. This change of emissivity takes rotational energy away from the deviation from β equilibrium and increases the rate of energy loss via neutrinos. The increased of neutrino emissivity due to the deviation from equilibrium will not increase the temperature but radiate away directly in the form of neutrinos. This energy conversion process is coincident with the one mentioned in Ref. [14].

With the new energy conversion channel, the evolution functions should be improved at the same time. Two basic rates are relevant for following the coupled evolution of T and $\delta\mu$: the total emissivity ϵ (energy per unit volume per unit time) and the net reaction rate Γ (number of reactions or emitted lepton number per unit volume per unit time). The changed interaction rates always lead to an increase of the neutrino emissivity. This part of increasing neutrino emissivity induced by the deviation from chemical equilibrium runs directly out of the star, and it affects the deviation amplitude of chemical equilibrium without affecting temperature. Under this new scenario, the evolution equations can be written as

$$\delta\dot{\mu} = -E_{xx}\left(\alpha n\frac{E_{nx}}{E_{xx}}\frac{\Omega\dot{\Omega}}{G\rho_c}+\frac{\Gamma}{n}+\frac{\Delta\epsilon}{n\delta\mu}\right), \tag{5}$$

$$\dot{T} = \frac{1}{E_{ss}^{-1}T}\left(\frac{\Gamma\delta\mu}{n}-\frac{\epsilon_0}{n}-\dot{E}_\gamma\right), \tag{6}$$

where ϵ_0 is the equilibrium emissivity, and $\Delta\epsilon$ is neutrino emissivity induced by the deviation from chemical equilibrium. In Eq. 5, the first term in the parentheses accounts for the change in the chemical equilibrium state due to spin down, the second term for the change in the actual chemical state due to reactions, and the third term is equivalent to the stored chemical energy that is taken away directly by neutrinos. In Eq. 6, $\Gamma\delta\mu/n$ and ϵ_0/n represent the stored chemical energy released by the corresponding reactions and the energy radiated by neutrinos and anti-neutrinos during the reactions respectively. The improved evolution functions show that the chemical potential difference has changed, so the temperature evolution of the star is subsequently affected.

3 Numerical results

In this section, we are going to give the numerical results of the thermal evolution of a NS with chemical heating under the old scenario (mentioned in Ref. [5]), as well as the improved scenario. For comparison, we use the same parameters for the star mentioned in Ref. [5]. The corresponding parameters are set as follows: the equation of state used here is PAL [26] and for deriving the E_{nx}, E_{ss} and E_{xx}, and $n=2n_0$, $\alpha=0.73$, $\rho_c=3m_n n_0=8.0\times10^{14}$ g·cm^{-3}, and the initial rotation period P_i=1 ms and initial temperature $T_i=10^{11}$K. The surface temperature of the stars is related to internal temperature by a coefficient determined by the scattering processes occurring in the crust. We apply a formula $T_s=3.08\times10^6 g_{s,14}^{1/4}T_9^{0.5495}$, which is demonstrated in Ref. [27], where $g_{s,14}$ is the proper surface gravity of the star in units of 10^{14}cm s^{-2}. In principle, magnetic fields may change the relation of surface temperature and internal temperature. However, Potekhin et al. [28] have shown that the effect is negligible if the field strength is lower than $10^{13}G$. So this relation is a good approximation for our case. In order to reveal the effect of the new channel for energy conversion, we discuss the turning on of the direct Urca process and modified Urca process respectively.

Defining $u=\delta\mu/(\pi kT)$, for direct Urca processes [5]

$$\epsilon_d(T,\delta\mu)=\epsilon_d(T,0)\left(1+\frac{1071u^2+315u^4+21u^6}{457}\right), \tag{7}$$

$$\Gamma_d(T,\delta\mu)\delta\mu=\epsilon_d(T,0)\frac{714u^2+429u^4+42u^6}{457}, \tag{8}$$

and for modified Urca processes [5]

$$\epsilon_m(T,\delta\mu)=\epsilon_m(T,0)\left(1+\frac{22020u^2+5670u^4+420u^6+u^8}{11513}\right), \tag{9}$$

$$\Gamma_m(T,\delta\mu)\delta\mu=\epsilon_m(T,0)\frac{14680u^2+7560u^4+860u^6+24u^8}{11513}, \tag{10}$$

where $\epsilon(T,0)$ is the equilibrium neutrino emissivity rate.

In this paper, we use the equilibrium neutrino emissivity given by [29]

$$\epsilon_d(T,0)\approx4.3\times10^{21}\left(\frac{x_{eq}n}{n_0}\right)^{1/3}T_8^6\ \text{ergs}\cdot\text{cm}^{-3}\cdot\text{s}^{-1} \tag{11}$$

and

$$\epsilon_m(T,0)\approx3.5\times10^{13}\left(\frac{x_{eq}n}{n_0}\right)^{1/3}T_8^8\ \text{ergs}\cdot\text{cm}^{-3}\cdot\text{s}^{-1} \tag{12}$$

where $n_0=0.16$ fm^{-3} is the normal nuclear baryon density and the subscripts m and d denote the modified Urca process and the direct Urca process respectively.

Since the star is assumed to be spinning down due to magnetic dipole radiation ($\dot{\Omega}=-B_p^2R^6\Omega^3\sin^2\theta/(6Ic^3)$), we consider three different surface magnetic field strengths: $B=10^8$G,10^{10}G, and 10^{12}G. Figure 1 displays the evolution curves of effective surface temperature of

NSs, which are obtained by integrating the coupled evolution equations (5) and (6) (for the old scenario, solving the coupled equations (3) and (4)). Both the chemical heating under the old scenario and the improved one are shown in Fig. 1. We can see from Fig. 1 that the effect of chemical heating under the improved scenario with intermediate field (10^{10}G) and lowest field (10^8G, representative of a millisecond pulsar) is strengthened regardless of whether the direct Urca process is switched on or not, especially at later times. For the strongest magnetic field (10^{12}G, representative of a typical classical radio pulsar), the spin-down and heating are substantial only in the early stage of evolution, and its effect on the thermal evolution curves is not particularly obvious with chemical heating under either the old scenario or the improved one.

The chemical evolution curves of NSs with different magnetic field strengths are present in Fig. 2. The old scenario and the improved one are both considered. The absolute value of $\delta\mu$ under the old scenario is slightly smaller than with the improved scenario. When the magnetic field strength is low, the difference between these two scenarios decreases in the later evolution stage of NSs. When the magnetic field is strong, the difference is stable.

Figure 3 shows the ratio of $\varGamma\delta\mu$ to $\Delta\varepsilon$. It shows the capability of converting rotational energy to heat and neutrinos. Compared with Fig. 2, we find that the ratio of $\varGamma\delta\mu$ to $\Delta\varepsilon$ changes with temperature. As the star gets older, the ratio of $\varGamma\delta\mu$ to $\Delta\varepsilon$ increases dramatically, so the heating term plays a more important role than the photon-cooling stages.

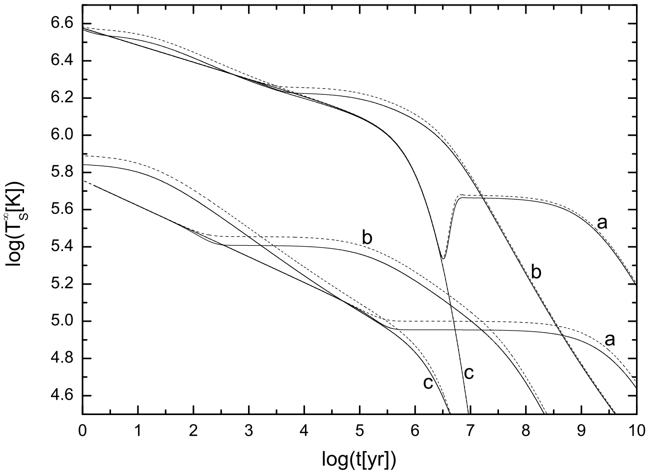

Fig. 1. Thermal evolution curves of NSs. The different effective surface temperature at infinity T_s^∞ is plotted logarithmically as a function of time for direct Urca reactions (bottom group of lines) and modified Urca reactions (top group of lines). The dotted curves represent solutions in the new scenario of chemical heating. The solid curves are solutions obtained by recovering the work of Ref. [5]. The curves correspond to magnetic dipole braking for different magnetic field strengths at (a) B=10^8G; (b) B=10^{10}G; (c) B=10^{12}G.

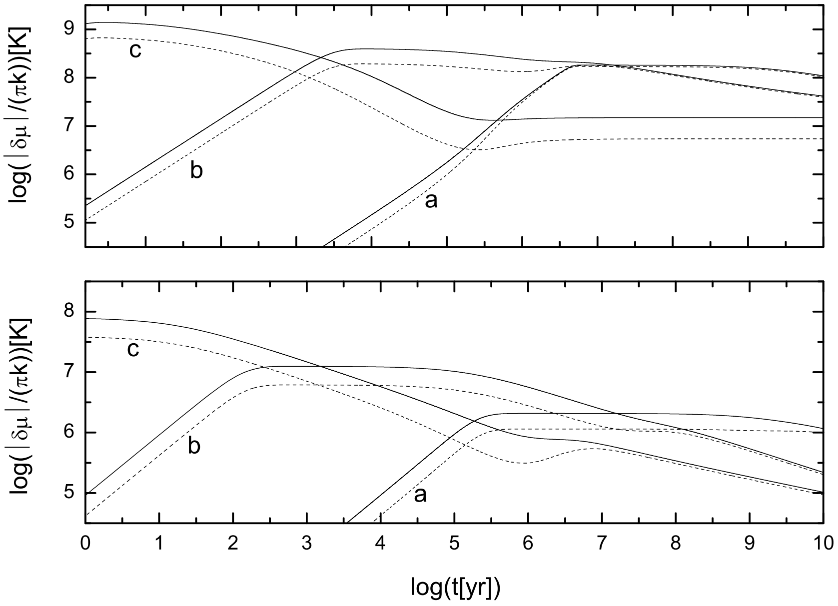

Fig. 2. Chemical evolution curves of NSs. $|\delta\mu|/(\pi k)$ is plotted logarithmically as a function of time with modified Urca reactions (top panel) and direct Urca reactions (bottom panel). The dotted curves are the solutions under the new scenario of chemical heating. The solid curves are the solutions obtained by recovering the work of Ref. [5]. The curves correspond to magnetic dipole braking with different magnetic field strengths: (a) B=10^8G; (b) B=10^{10}G; (c) B=10^{12}G.

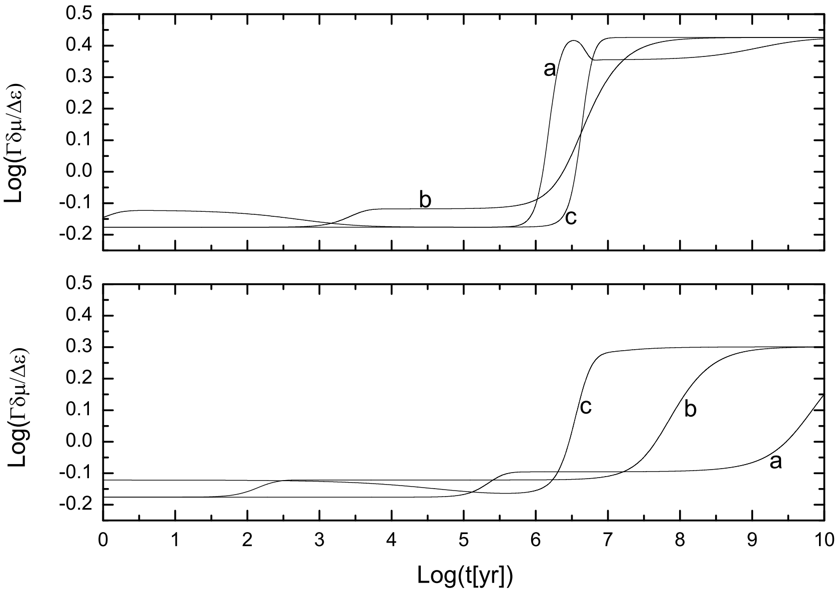

Fig. 3. The ratio of $\Gamma\delta\mu$ to $\Delta\varepsilon$ in the new scenario of chemical heating. The ratio is plotted logarithmically as a function of time with modified Urca reactions (top panel) and direct Urca reactions (bottom panel). The curves correspond to magnetic dipole braking with different magnetic field strengths: (a) $B=10^8$G; (b) $B=10^{10}$G; (c) $B=10^{12}$G.

If the relevant forcing ($\Omega\dot{\Omega}$) changes slowly with time, the star eventually arrives at a quasi-stationary state, where the rate at which the equilibrium concentrations are modified by this forcing is the same as that at which the reactions drive the system toward the new quasi-equilibrium configuration, with heating and cooling balancing each other [5]. Properties of this quasi-stationary state can be obtained by the simultaneous solutions for Eqs. (5) and (6) with $\dot{T}=\delta\dot{\mu}=0$. The existence of a quasi-stationary state makes it unnecessary to model the full evolution of the temperature of the star in order to get the final temperature, since the quasi-equilibrium state is independent of the initial conditions (see Fernández and Reisenegger [7] for a detailed analysis of the rotochemical heating). This implies that, for given values of $\Omega\dot{\Omega}$, it is possible to calculate the temperature for an old pulsar, that has reached the quasi-equilibrium state, without knowing its exact age.

Here we are going to give the surface temperature of the quasi-equilibrium state. Assuming direct Urca processes are switched on, the surface temperature of the quasi-equilibrium state can be written as:

$$T_s^D \simeq 1.50599\times10^5\left(\frac{\dot{P}_{-20}}{P_{ms}^3}\right)^{3/10},$$

$$T_s^{nD} \simeq 1.58582\times10^5\left(\frac{\dot{P}_{-20}}{P_{ms}^3}\right)^{3/10},$$

where T_s^D is the surface temperature with chemical heating under the old scenario, T_s^{nD} is the surface temperature with chemical heating under the new scenario, $\dot{P}_{-20}$ is the period derivative measured in units of 10^{-20} and P_{ms} is the period in milliseconds. The temperature increases by roughly 5.3% when rotation energy is taken away by neutrinos directly. When only modified Urca processes are switched on, the surface temperature of the quasi-equilibrium state can be written as:

$$T_s^M \simeq 3.35553\times10^5\left(\frac{\dot{P}_{-20}}{P_{ms}^3}\right)^{2/7},$$

$$T_s^{nM} \simeq 3.44569\times10^5\left(\frac{\dot{P}_{-20}}{P_{ms}^3}\right)^{2/7},$$

where T_s^M is the surface temperature with chemical heating under the old scenario, and T_s^{nM} is the surface temperature with chemical heating under the new scenario. We find that the temperature increase is 2.69%.

4 Discussion and conclusions

We have introduced a new channel through which rotational energy is radiated away via neutrinos directly. Considering this new channel of energy conversion, we studied the thermal evolution of NSs with the traditional chemical heating and the improved model. Its effects on a normal NS with direct Urca process and modified Urca process have been discussed. Investigation of the evolution of the surface temperature and chemical evolution of NSs with different magnetic field strengths indicates that the improved chemical heating mechanism can raise the surface temperature of the NSs to a certain extent. Moreover, when considering non-equilibrium cases, such as chemical deviations, this kind of energy dissipation through neutrinos should be considered.

In Ref. [7], the authors gave quasi-equilibrium effective temperatures obtained from different equations of state and stellar models. The highest predicted quasi-equilibrium effective temperature is lower than the blackbody fit to the UV emission of PSR J0437-4715 by about 20%. The quantitative results presented in our work are produced by a classical NS model in which some detailed features of real NSs are neglected. However, this simplified model can exhibit the main qualitative effects of the new channel of the conversion of rotational energy. Furthermore, consideration of the improved chemical heating mechanism in our work may help us to understand or constrain fundamental physics problems, or explain the observations of older millisecond pulsars with high surface temperatures [30–32]. Moreover, we can use it to discuss the thermal evolution of NSs in detail, including the presence of exotic particles, and the structure of the stars combined with spin evolution.

Detailed calculations, including the determination of the reaction rates of superfluid matter which is out of chemical equilibrium, are needed in order to make precise predictions for specific stellar models as mentioned in Ref. [24]. At the same time, particular equations of state and structure of the NS help us get further details about the energy conversion processes [15]. The inclusion of superfluidity is likely to raise the predicted temperatures, and this will also be the subject of our future work.

We grateful thank the anonymous reviewers for the thoughtful comments that helped to significantly improve the clarity of this work. We thank Dr. Rai Yuen and Dr. Yun-feng Chang for a careful reading of the manuscript.

References

1 W. Becker, *Neutron Stars and Pulsars*, Astrophys. Space Sci. Library, Vol 357 (New-York: Springer, 2008)
2 N. K. Glendenning, *Compact Stars: Nuclear Physics, Particle Physics, and General Relativity* (New-York: Springer, 2000)
3 X. P. Zheng, X. Zhou, Mon. Not. R. Astron. Soc., **371**, 1659 (2006)
4 R. Negreiros, S. Schramm, and F. Weber, Phys. Lett. B, **718**: 1176 (2013)
5 A. Reisenegger, Astrophys. J, **442**: 749 (1995)
6 K. S. Cheng and Z. G. Dai, Astrophys. J, **468**: 819 (1996)
7 R. Fernández and A. Reisenegger, Astrophys. J, **625**: 291 (2005)
8 K. Iida, K. Sato, Astrophys. J, **477**, 294 (1997)
9 K. S. Cheng, W. Y. Chau, J. L. Zhang, and H. F. Chau, Astrophys. J, **396**: 135 (1992)
10 Y. W. Yu and X. P. Zheng, Astron. Astrophys., **450**: 1071 (2006)
11 X. P. Zheng and Y. W. Yu, Astron. Astrophys., **445**: 627 (2006)
12 X. Zhou, L. Z. Wang, and A. Z. Zhou, Pub. Astron. Soc. Pac., **119**: 1367 (2007)
13 X. P. Zheng, X. Zhou, and S. H. Yang, Physics Letters B, **729**: 79 (2014)
14 B. A. Sa'd and J. Schaffner-Bielich, arXiv:0908.4190
15 S. H. Yang, X. P. Zheng, and C. M. Pi, Phys. Lett. B, **683**: 255 (2009)
16 S. H. Yang, X. P. Zheng, C. M. Pi, and Y. W. Yu, Mon. Not. R. Astron. Soc., **403**: 2007 (2010)
17 C. M. Pi, S. H. Yang, and X. P. Zheng, Chinese Physics Letters, **28**(10): 109701 (2011)
18 P. Jaikumar and S. Sandalski, Phys. Rev. D, **82**: 103013 (2010)
19 A. Finzi and R. A. Wolf, Astrophys. J, **153**: 835 (1968)
20 P. Haensel, Astron. Astrophys., **262**, 131 (1992)
21 E. Gourgoulhon and P. Haensel, Astron. Astrophys., **271**: 187 (1993)
22 P. Jofré, A. Reisenegger, and R. Fernández, Phys. Rev. Lett., **97**: 131102 (2006)
23 D. G. Yakovlev, A. D. Kaminker, O. Y. Gnedin, and P. Haensel, Phys. Rep., **354**: 1 (2001)
24 C. Petrovichand and A. Reisenegger, Astron. Astrophys., **521**: A77 (2009)
25 N. Gonzalez-Jimenez, C. Petrovich, and A. Reisenegger, Mon. Not. R. Astron. Soc., **447**: 2073 (2015)
26 M. Prakash, T. Ainsworth, and J. M. Lattimer, Phys. Rev. Lett., **61**: 2518 (1988)
27 E. H. Gudmundsson, C. J. Pethick, and R. I. Epstein, ApJ, **272**: 286 (1983)
28 A. Y. Potekhin, D. G. Yakovlev, and P. Prakash. A & A, **374**: 213 (2001)
29 P. Haensel, Astron. Astrophys., **262**: 131 (1992)
30 S. Mahmoodifar and T. Strohmayer, Astrophys. J., **840**: 94 (2017)
31 K. Schwenzer, T. Boztepe, T. Güver, and E. Vurgun, Mon. Not. R. Astron. Soc., **466**: 2560 (2017)
32 B. Rangelov et al, Astrophys. J., **835**: 264 (2017)