

Chiral corrections to the 1^{-+} exotic meson mass^{*}

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Abstract: We first construct the effective chiral Lagrangians for the 1^{-+} exotic mesons. With the infrared regularization scheme, we derive the one-loop infrared singular chiral corrections to the $\pi_1(1600)$ mass explicitly. We investigate the variation of the different chiral corrections with the pion mass under two schemes. Hopefully, the explicit non-analytical chiral structures will be helpful for the chiral extrapolation of lattice data from the dynamical lattice QCD simulation of either the exotic light hybrid meson or the tetraquark state.

Keywords: exotic mesons, hybrid state, lattice QCD

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1 Introduction

According to the naive non-relativistic quark model, mesons are composed of a quark and an anti-quark. The neutral mesons do not carry quantum numbers such as $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{-+}, \dots$. In contrast, non-conventional mesons such as the hybrid meson, tetraquark states and glueballs are allowed in quantum chromodynamics (QCD) and can have these quantum numbers. Sometimes these states are denoted as exotic states in order to emphasize the difference from mesons within the quark model. In fact, the exotic quantum numbers provide a powerful handle to probe the non-perturbative behavior of QCD [1–3]. In this work we focus on the exotic meson with $J^{PC} = 1^{-+}$, which is a good candidate for both hybrid meson and tetraquark state.

There are three candidates with $J^{PC} = 1^{-+}$: $\pi_1(1400)$, $\pi_1(1600)$ and $\pi_1(2000)$. Their masses and widths are $(1376 \pm 17, 300 \pm 40)$ MeV, $(1653_{-15}^{+18}, 225_{-28}^{+45})$ MeV and $(2014 \pm 20 \pm 16, 230 \pm 21 \pm 73)$ MeV [4] respectively. $\pi_1(1600)$ was first observed in the reaction

$\pi^- p \rightarrow \pi^- \pi^- \pi^+ p$ in 1998 [5, 6]. Later the $\pi_1(1600)$ was confirmed in the $\eta' \pi$ [7], $f_1(1285) \pi$ [8, 9] and $b_1(1235) \pi$ channels [10, 11]. Some experiments also indicated the possible existence of $\pi_1(1400)$ [12–14] and $\pi_1(2000)$ [8]. The existence of $\pi_1(2000)$ awaits further experimental confirmation. This state was not included in the PDG since 2010 [15].

The current status of the $\pi_1(1400)$ and $\pi_1(1600)$ is a little murky. There is speculation that the $\pi_1(1400)$ might be non-resonant or it may be a tetraquark candidate instead of a hybrid meson. Although there are other possible theoretical explanations such as a tetraquark candidate [16, 17] or a molecule/four-quark mixture [18], the $\pi_1(1600)$ remains a popular candidate for the light hybrid meson [19]. The present calculation is based on the following three facts: the 1^{-+} exotic quantum number, the SU(3) flavor structure and the current available decay modes. In other words, it is applicable to all possible interpretations of the π_1 mesons.

There are many investigations of the 1^{-+} light hybrid meson mass in the literature [20–33]. The 1^{-+} mass extracted from the quenched lattice QCD simulation ranges

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from 1.74 GeV [34] and 1.8 GeV [35] to 2 GeV [25], which is significantly larger than the experimental value. This apparent discrepancy is slightly disturbing. One possible reason may be due to the fact that all these lattice QCD simulations were performed with quenched configurations and a rather large pion mass on the lattice. One may wonder whether such a discrepancy may be removed with dynamical lattice QCD simulations using the physical pion mass. Then one may make chiral extrapolations to extract the physical mass of the hybrid meson.

In this work we shall derive the explicit expressions of the non-analytical chiral corrections to the $\pi_1(1600)$ mass up to one-loop order, which may be used to make the chiral extrapolations if the dynamical lattice QCD simulations are available. Throughout our analysis, we focus on the variation of the $\pi_1(1600)$ meson mass with $m_{u,d}$ or m_π . In the $SU_F(3)$ chiral limit $m_{u,d,s} \rightarrow 0$, $m_{\pi,\eta} \rightarrow 0$. The $SU_F(2)$ chiral limit is adopted where $m_{u,d} \rightarrow 0$ and m_s remains finite. Then, the eta meson mass does not vanish due to the large strange quark mass.

This paper is organized as follows. We construct the effective chiral Lagrangians in Section 2 and present the formalism in Section 3. In Section 4, we present the numerical results and conclude.

2 Lagrangians

In order to calculate the chiral corrections to the $\pi_1(1600)$ meson mass up to one-loop order, we first construct the effective chiral Lagrangian [36, 37], which can be expressed as follows

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\rho\pi} + \mathcal{L}_{b_1\pi} + \mathcal{L}_{f_1\pi} + \mathcal{L}_{\eta\pi} + \mathcal{L}_{\eta'\pi} + \mathcal{L}_{\pi_1\eta} + \mathcal{L}_{\pi_1\eta'} + \dots, \quad (1)$$

where \mathcal{L}_0 is the free part

$$\mathcal{L}_0 = \partial_\mu \vec{\pi}_{1\nu} \cdot \partial^\mu \vec{\pi}_1^\nu - m_0^2 \vec{\pi}_1^\mu \cdot \vec{\pi}_{1\mu}. \quad (2)$$

According to the decay modes of $\pi_1(1600)$, we can write down the interaction terms

$$\mathcal{L}_{\eta\pi} = g_{\eta\pi} \vec{\pi}_1^\mu \cdot \partial_\mu \vec{\pi}\eta, \quad (3)$$

$$\mathcal{L}_{\eta'\pi} = g_{\eta'\pi} \vec{\pi}_1^\mu \cdot \partial_\mu \vec{\pi}\eta', \quad (4)$$

$$\mathcal{L}_{\rho\pi} = g_{\rho\pi} \epsilon_{\mu\nu\alpha\beta} \vec{\pi}_1^\mu \times \partial^\alpha \vec{\rho}^\nu \cdot \partial^\beta \vec{\pi}, \quad (5)$$

$$\mathcal{L}_{b_1\pi} = g_{b_1\pi} \vec{\pi}_{1\mu} \times \vec{b}_1^\mu \cdot \vec{\pi}, \quad (6)$$

$$\mathcal{L}_{f_1\pi} = g_{f_1\pi} \vec{\pi}_{1\mu} \cdot \vec{f}_1^\mu. \quad (7)$$

Because of chiral symmetry and its spontaneous breaking, all the pionic coupling constants should vanish when either the pion momentum or its mass goes to zero. The S-wave coupling constants $g_{b_1\pi}$ and $g_{f_1\pi}$ arise

from the finite current quark mass correction. Therefore, these coupling constants are proportional to m_π^2 ,

$$g_{b_1\pi} = g_{b_1\pi}^* m_\pi^2, \quad g_{f_1\pi} = g_{f_1\pi}^* m_\pi^2. \quad (8)$$

The $\pi_1 \rightarrow \pi\pi\pi$ decay mode may lead to the two-loop self energy diagram $\pi_1(1600)$ in Fig. 1. We ignore the contribution from this diagram since we focus on the chiral corrections to the $\pi_1(1600)$ mass up to one-loop order in this work. Moreover, some contribution of this two-loop diagram may have been partly included in the one-loop diagram with the intermediate ρ and π meson because the ρ meson is the two-pion resonance.

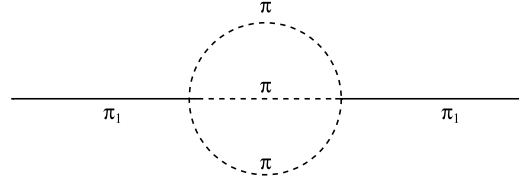


Fig. 1. The two-loop self energy diagram of the $\pi_1(1600)$ with three intermediate π mesons.

Furthermore, we need the chiral interaction between the $\pi_1(1600)$ and the pseudo scalar mesons, which is similar to the chiral Lagrangians of the vector mesons [38–42]. It should be stressed that the $\pi_1\pi_1\pi$ interaction is forbidden by G-parity conservation. We have

$$\mathcal{L}_{\pi_1\eta} = g_{\pi_1\eta} \epsilon_{\mu\nu\alpha\beta} \vec{\pi}_1^\mu \cdot \partial^\alpha \vec{\pi}_1^\nu \partial^\beta \eta, \quad (9)$$

$$\mathcal{L}_{\pi_1\eta'} = g_{\pi_1\eta'} \epsilon_{\mu\nu\alpha\beta} \vec{\pi}_1^\mu \cdot \partial^\alpha \vec{\pi}_1^\nu \partial^\beta \eta'. \quad (10)$$

For the $\pi_1\pi_1\pi\pi$ and $\pi_1\pi_1\eta\eta$ interaction, we have

$$\begin{aligned} \mathcal{L}_{\pi_1\pi_1\pi\pi} = & c_1 m_\pi^2 \vec{\pi} \cdot \vec{\pi} \vec{\pi}_1^\mu \cdot \vec{\pi}_{1\mu} + c_2 \partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} \vec{\pi}_1^\nu \cdot \vec{\pi}_{1\nu} \\ & + c_3 \partial_\mu \vec{\pi} \cdot \partial_\nu \vec{\pi} \vec{\pi}_1^\mu \cdot \vec{\pi}_1^\nu + \frac{c_4}{m_{\pi_1}} \partial_\mu \vec{\pi} \cdot \vec{\pi} \vec{\pi}_1^\nu \cdot \partial^\mu \vec{\pi}_{1\nu} \\ & + \frac{c_5}{m_{\pi_1}} \partial_\mu \vec{\pi} \cdot \vec{\pi} \vec{\pi}_1^\nu \cdot \partial_\nu \vec{\pi}_1^\mu \\ & + \frac{c_6 m_\pi^2}{m_{\pi_1}^2} \vec{\pi} \cdot \vec{\pi} \partial_\mu \vec{\pi}_1^\nu \cdot \partial^\mu \vec{\pi}_{1\nu}, \end{aligned} \quad (11)$$

$$\begin{aligned} \mathcal{L}_{\pi_1\pi_1\eta\eta} = & c_1^* m_\eta^2 \eta^2 \vec{\pi}_1^\mu \cdot \vec{\pi}_{1\mu} + c_2^* \partial_\mu \eta \partial^\mu \eta \vec{\pi}_1^\nu \cdot \vec{\pi}_{1\nu} \\ & + c_3^* \partial_\mu \eta \partial_\nu \eta \vec{\pi}_1^\mu \cdot \vec{\pi}_1^\nu + \frac{c_4^*}{m_{\pi_1}} \partial_\mu \eta \eta \vec{\pi}_1^\nu \cdot \partial^\mu \vec{\pi}_{1\nu} \\ & + \frac{c_5^*}{m_{\pi_1}} \partial_\mu \eta \eta \vec{\pi}_1^\nu \cdot \partial_\nu \vec{\pi}_1^\mu \\ & + \frac{c_6^* m_\eta^2}{m_{\pi_1}^2} \eta^2 \partial_\mu \vec{\pi}_1^\nu \cdot \partial^\mu \vec{\pi}_{1\nu}. \end{aligned} \quad (12)$$

In order to absorb the divergence in the one-loop chiral corrections, we need the following counter terms

$$\begin{aligned} \mathcal{L}_{\text{counter}} = & e_1 (m_\pi^2 + m_\eta^2) \vec{\pi}_{1\mu} \cdot \vec{\pi}_1^\mu \\ & + e_2 (m_\pi^2 + m_\eta^2)^2 \vec{\pi}_{1\mu} \cdot \vec{\pi}_1^\mu. \end{aligned} \quad (13)$$

$\mathcal{L}_{\text{counter}}$ is similar to the chiral Lagrangians of the vector mesons in the form of $\langle\chi_+\rangle\langle V_\mu V^\mu\rangle$ and $\langle\chi_+\rangle^2\langle V_\mu V^\mu\rangle$, where V_μ is the vector meson and the notation χ_+ is related to the current quark mass.

3 Chiral corrections to the $\pi_1(1600)$ mass

With the above preparation, we start to calculate the chiral corrections to the mass of $\pi_1(1600)$. The propagator of the $\pi_1(1600)$ is defined as

$$S_0^{\mu\nu} = i \int d^4x e^{ip \cdot x} \langle 0 | T \{ \pi_1^\mu(x) \pi_1^\nu(0) \} | 0 \rangle, \quad (14)$$

where p is the four momentum of π_1 . At the lowest order, the propagator simply reads

$$\begin{aligned} S_0^{\mu\nu} &= \frac{-i(g^{\mu\nu} - p^\mu p^\nu / m_0^2)}{p^2 - m_0^2 + i\epsilon} \\ &= \frac{-i \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right)}{p^2 - m_0^2} + \frac{i p_\mu p_\nu}{p^2 m_0^2} \end{aligned} \quad (15)$$

and its inverse is

$$(S_0^{-1})^{\mu\nu} = i((p^2 - m_0^2)g^{\mu\nu} - p^\mu p^\nu). \quad (16)$$

Here, m_0 denotes the bare mass of $\pi_1(1600)$.

We separate the self energy $\Sigma_{\mu\nu}(p^2)$ into the transversal and longitudinal parts

$$\Sigma_{\mu\nu}(p^2) = \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \Sigma_T(p^2) + \frac{p_\mu p_\nu}{p^2} \Sigma_L(p^2). \quad (17)$$

The full propagator reads

$$\begin{aligned} S^{\mu\nu} &= S_0^{\mu\nu} + S_0^{\mu\alpha} (i\Sigma)_{\alpha\beta}(p^2) S_0^{\beta\nu} + \dots \\ &= [(S_0^{-1} - i\Sigma)^{\mu\nu}]^{-1}. \end{aligned} \quad (18)$$

which can be expressed as

$$S^{\mu\nu} = \frac{-i(g_{\mu\nu} - p_\mu p_\nu / p^2)}{p^2 - m_0^2 - \Sigma_T(p^2)} + \frac{i p_\mu p_\nu}{p^2(m_0^2 + \Sigma_L(p^2))}. \quad (19)$$

Only the transverse part $\Sigma_T(p^2)$ will shift the pole position. Therefore we concentrate on the transverse part of the self energy [43] and consider all the Feynman diagrams shown in Fig. 2 and Fig. 3. The $\pi_1(1600)$ mass satisfies the relation

$$m_{\pi_1}^2 - m_0^2 - \Sigma_T(m_{\pi_1}^2) = 0. \quad (20)$$

In order to obtain the quark mass ($\sim m_\pi^2, m_\eta^2$) dependence of the self energy corrections, it is convenient to adopt the infrared regularization (IR) scheme [44–46] to calculate the loop integrals. Usually, the IR method is used in order not to break the power counting while dealing with the integral. Unfortunately, there is no proper

power counting rule for the issue we are dealing with. There are a few different mass scales such as the π_1 mass, the π, η meson masses, the masses of other meson resonances, and the chiral symmetry breaking scale. The mass of the π_1 is so high that the π, η and other light mesons can take large momenta, and thus the convergence of a chiral expansion is not ensured. However, for our purpose, the IR method can still be used to derive the non-analytic part of an integral. The non-analytical chiral corrections to the self-energy of the π_1 are inherent and intrinsic due to the presence of the chiral fields, and the non-analytical chiral structures are universal and model independent to a large extent. One may derive them using very different theoretical approaches such as the chiral quark model, effective chiral Lagrangians at the hadronic level or rigorous chiral perturbation theory (ChPT). With ChPT, one can include both analytical and non-analytical corrections order by order with consistent power counting. In contrast, with the effective chiral Lagrangians at the hadronic level as employed in this work, there is no consistent power counting. Fortunately, the non-analytical corrections from different approaches are similar if one considers the one-loop diagrams. The non-analytical structures may play an important role in the chiral extrapolation of the dynamical lattice QCD simulation of the 1^{-+} exotic meson mass, which is sensitive to the pion mass on the lattice. Within the IR scheme, the so-called ‘infrared singular part’ turns out to be the main contribution of the loop integral in the chiral limit. However, one can also find the full expressions of these loop integrals by performing the standard Lorentz invariant calculation in Refs. [47, 48].

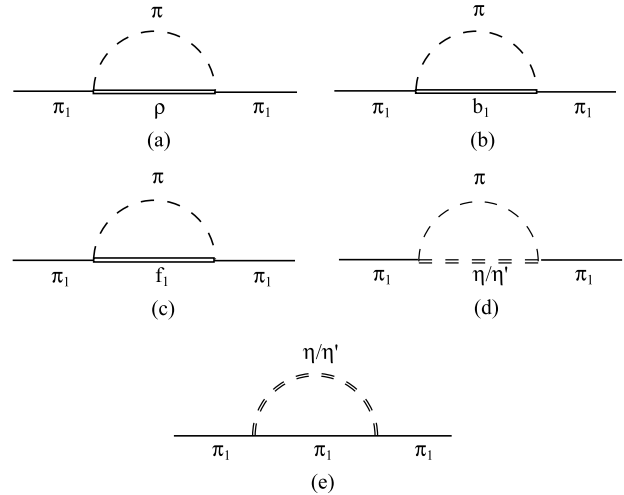


Fig. 2. The one-loop self energy diagrams of the $\pi_1(1600)$ with one light meson plus one π or η .

For a certain diagram, there are three mass scales, M_{π_1} and the masses of the two intermediate states m, M .

We assume $M > m$. The main contribution of a loop integral comes from the poles of the propagators, which are called the ‘soft poles’ and ‘hard poles’ in Refs. [49, 50].

When one expands the loop integral in terms of small parameters such as m/M or m/μ , where μ is the renormalization scale, the ‘soft part’ contribution contains all the terms which are non-analytic in the expansion parameter. In contrast, the ‘hard part’ is a local polynomial in these parameters which can be absorbed by the low energy constants of higher order Lagrangians [46].

Since we are interested in the small chiral fluctuations around the mass shell of $\pi_1(1600)$, we set the kinematical region $p^2 \sim M_{\pi_1}^2$. In particular, we set the regularization scale to be M_{π_1} . These self-energy diagrams can be divided into two categories. The first class of diagrams fulfills the condition $M_{\pi_1}^2 \gg (M+m)^2$ and $m^2 \ll M^2$, including those diagrams with the $\rho\pi, \eta\pi, b_1(1235)\pi, f_1(1285)\pi$ and $\eta'\pi$ as the intermediate states. The second class corresponds to the condition $M_{\pi_1}^2 \sim M^2$ and $m^2 \ll M^2$, where the intermediate states are the $\pi_1(1600)\eta$ and $\pi_1(1600)\eta'$.

3.1 The light meson pion loop

Now we deal with the light meson pion loop integration corresponding to diagrams (a)–(d) in Fig. 2. Consider the scalar loop integrals

$$I_{\pi X}(p^2) = \mu^{4-d} \int \frac{d^d l}{(2\pi)^d} \frac{1}{[l^2 - m_\pi^2 + i\epsilon][(p-l)^2 - M^2 + i\epsilon]}, \quad (21)$$

where X represents the ρ, b_1, f_1, η' mesons. l and p denote the loop momentum and external momentum respectively. After performing the l -integration, the above integral reads

$$I_{\pi X}(p^2) = \mu^{4-d} \Gamma\left(2 - \frac{d}{2}\right) \frac{iM^{d-4}}{(4\pi)^{\frac{d}{2}}} \int_0^1 dx (\Delta)^{\frac{d}{2}-2} \quad (22)$$

with

$$\begin{aligned} \Delta &= bx^2 - (a+b-1)x + a, \\ a &= \frac{m_\pi^2}{M^2}, \quad b = \frac{p^2}{M^2}. \end{aligned} \quad (23)$$

Since we choose the external momentum p near the mass shell of $\pi_1(1600)$, we always have $(p^2 - m_\pi^2 + M^2)^2 - 4p^2 M^2 > 0$. Δ can be re-expressed as $\Delta = b(x-x_1)(x-x_2)$, with

$$x_{1,2} = \frac{a+b-1}{2b} \left(1 \pm \sqrt{1 - \frac{4ab}{(a+b-1)^2}} \right). \quad (24)$$

Obviously we have $0 < x_2 < x_1 < 1$. We now divide the integral into three parts according to the integration

interval

$$I_{\pi X} = \mu^{4-d} \Gamma\left(2 - \frac{d}{2}\right) \frac{iM^{d-4}}{(4\pi)^{\frac{d}{2}}} \left(I_{\pi X}^{(1)} + I_{\pi X}^{(2)} + I_{\pi X}^{(3)} \right) \quad (25)$$

with

$$\begin{aligned} I_{\pi X}^{(1)}(p^2) &= \int_0^{x_2} dx [b(x-x_1)(x-x_2)]^{\frac{d}{2}-2}, \\ I_{\pi X}^{(2)}(p^2) &= \int_{x_2}^{x_1} dx [b(x-x_1)(x-x_2)]^{\frac{d}{2}-2}, \\ I_{\pi X}^{(3)}(p^2) &= \int_{x_1}^1 dx [b(x-x_1)(x-x_2)]^{\frac{d}{2}-2}. \end{aligned} \quad (26)$$

We first consider $I_{\pi X}^{(1)}$. The assumption $p^2 \gg (M+m_\pi)^2$ and $m_\pi^2 \ll M^2$ leads to

$$a \ll 1, \quad \frac{4ab}{(a+b-1)^2} \ll 1. \quad (27)$$

So we can expand $x_{1,2}$ in terms of the small parameter a ,

$$\begin{aligned} x_1 &= \frac{b-1}{b} - \frac{a}{b(b-1)} - \frac{a^2}{(b-1)^3} + \mathcal{O}(a^3), \\ x_2 &= \frac{a}{b-1} + \frac{a^2}{(b-1)^3} + \mathcal{O}(a^3). \end{aligned} \quad (28)$$

Then we have

$$I_{\pi X}^{(1)}(p^2) = (-bx_1)^{\frac{d}{2}-2} \int_0^{x_2} dx [(1-x/x_1)(x-x_2)]^{\frac{d}{2}-2}. \quad (29)$$

Recall that $x_1 \sim \mathcal{O}(1)$ and $x_2 \sim \mathcal{O}(a)$. When $x \in [0, x_2]$, we can expand the above integral in terms of the parameter x/x_1

$$\begin{aligned} I_{\pi X}^{(1)}(p^2) &= (-bx_1)^{\frac{d}{2}-2} \int_0^{x_2} dx (x-x_2)^{\frac{d}{2}-2} \\ &\cdot \sum_{m=0}^{\infty} \frac{\Gamma\left(\frac{d}{2}-1\right)}{\Gamma\left(\frac{d}{2}-1-m\right) m!} \left(-\frac{x}{x_1}\right)^m. \end{aligned} \quad (30)$$

After the interchange of summation and integration, we get

$$\begin{aligned} I_{\pi X}^{(1)}(p^2) &= (bx_1)^{\frac{d}{2}-1} x_2^{\frac{d}{2}-1} \sum_{m=0}^{\infty} \frac{\Gamma\left(\frac{d}{2}-1\right) \Gamma\left(\frac{d}{2}-1\right)}{\Gamma\left(\frac{d}{2}-1-m\right) \Gamma\left(\frac{d}{2}+m\right)} \\ &\cdot \left(-\frac{x_2}{x_1}\right)^m. \end{aligned} \quad (31)$$

Clearly $I_{\pi X}^{(1)}$ is non-analytic in a for non-integer dimension d .

We move on to the $I_{\pi X}^{(2)}$ part. After shifting the integration variable, we get

$$I_{\pi X}^{(2)}(p^2) = (-b)^{\frac{d}{2}-2} \int_0^{x_1-x_2} dx [x(x_1-x_2-x)]^{\frac{d}{2}-2}. \quad (32)$$

With the replacement $x = (x_1 - x_2)y$, one gets

$$\begin{aligned} I_{\pi X}^{(2)}(p^2) &= (-b)^{\frac{d}{2}-2} (x_1 - x_2)^{d-3} \int_0^1 dy [y(1-y)]^{\frac{d}{2}-2} \\ &= (-b)^{\frac{d}{2}-2} (x_1 - x_2)^{d-3} \frac{\left[\Gamma\left(\frac{d}{2}-1\right) \right]^2}{\Gamma(d-2)}. \end{aligned} \quad (33)$$

$I_{\pi X}^{(2)}$ is complex and proportional to $(x_1 - x_2)^{d-3}$ that can be expanded in powers of x_2 .

We expand the third integral $I_{\pi X}^{(3)}$ in terms of x_2/x , i.e.,

$$\begin{aligned} I_{\pi X}^{(3)}(p^2) &= \int_{x_1}^1 dx [b(x-x_1)]^{\frac{d}{2}-2} x^{\frac{d}{2}-2} \left(1 - \frac{x_2}{x}\right)^{\frac{d}{2}-2} \\ &= \int_{x_1}^1 dx [b(x-x_1)]^{\frac{d}{2}-2} x^{\frac{d}{2}-2} \\ &\quad \cdot \sum_{m=0}^{\infty} \frac{\Gamma\left(\frac{d}{2}-1\right)}{\Gamma\left(\frac{d}{2}-1-m\right) m!} \left(\frac{x_2}{x}\right)^m \\ &= \sum_{m=0}^{\infty} \frac{\Gamma\left(\frac{d}{2}-1\right)}{\Gamma\left(\frac{d}{2}-1-m\right) m!} x_2^m \\ &\quad \cdot \int_{x_1}^1 dx [b(x-x_1)]^{\frac{d}{2}-2} x^{\frac{d}{2}-2-m}. \end{aligned} \quad (34)$$

Obviously $I_{\pi X}^{(3)}$ only contains the integer powers of a .

It is clear that $I_{\pi X}^{(3)}$ and the real part of $I_{\pi X}^{(2)}$ are regular in a and will not produce any infrared singular terms for an arbitrary value of the dimension d . Thus these parts can be absorbed into the low energy constants of the effective Lagrangian. On the other hand, $I_{\pi X}^{(1)}$ develops an infrared singularity as $a \rightarrow 0$ for negative enough dimension d . This part is the so-called ‘infrared singular part’ of $I_{\pi X}$ in the IR method of Refs. [44–46]. The ‘infrared singular part’ contains all the terms which are non-analytic in a as the typical chiral log terms $\ln a$. Such terms cannot be absorbed into the low energy constants of the effective Lagrangian. Furthermore, the contribution of the ‘infrared singular part’ dominates the $I_{\pi X}$ as $a \rightarrow 0$.

Finally we obtain the ‘infrared singular part’ in $I_{\pi X}$ with the imaginary part,

$$\begin{aligned} I_{\pi X}^{\text{IR}}(p^2) &= \frac{i}{16\pi^2} x_2 \left[L + 1 - \ln\left(\frac{m_\pi^2}{\mu^2}\right) + \left(\frac{x_1 - x_2}{x_2}\right) \right. \\ &\quad \left. \cdot \ln\left(\frac{x_1 - x_2}{x_1}\right) \right] - \frac{1}{16\pi} (x_1 - x_2) \\ &= \frac{i}{16\pi^2} x_2 \left[L - \ln\left(\frac{m_\pi^2}{\mu^2}\right) \right] \\ &\quad + \frac{i}{16\pi^2} \left[x_2 - (x_1 - x_2) \left(\frac{x_2}{x_1} + \frac{1}{2} \frac{x_2^2}{x_1^2}\right) \right] \\ &\quad - \frac{1}{16\pi} (x_1 - x_2) \\ &= \frac{i}{16\pi^2} \left[L - \ln\left(\frac{m_\pi^2}{m_\pi^2}\right) \right] \left(\frac{a}{b-1} + \frac{a^2}{(b-1)^3} \right) \\ &\quad + \frac{i}{32\pi^2} \frac{a^2 b}{(b-1)^3} \\ &\quad - \frac{1}{16\pi} \left[\frac{b-1}{b} - \frac{(b+1)a}{b(b-1)} - \frac{2a^2}{(b-1)^3} \right] + \mathcal{O}(a^3), \end{aligned} \quad (35)$$

where $L = \frac{1}{\epsilon} - \gamma_E + \ln 4\pi + 1$ and we let $\mu = m_{\pi_1}$.

Up to $\mathcal{O}(m_\pi^4)$ and $\mathcal{O}(m_{\eta'}^4)$, we collect the one-loop chiral corrections to the self-energy of the $\pi_1(1600)$ below:

$$\begin{aligned} \Sigma_{\text{T,IR}}^{\rho\pi}(m_{\pi_1}^2) &= \frac{g_{\rho\pi}^2 m_{\pi_1}^2 m_\pi^4}{32\pi^2 (m_{\pi_1}^2 - m_\rho^2)} \left[1 - 2 \ln\left(\frac{m_\pi^2}{m_{\pi_1}^2}\right) \right] \\ &\quad - i g_{\rho\pi}^2 \left[\frac{(m_{\pi_1}^2 - m_\rho^2)^3}{48\pi m_{\pi_1}^2} - \frac{m_\pi^2 (m_{\pi_1}^4 - m_\rho^4)}{16\pi m_{\pi_1}^2} \right. \\ &\quad \left. + \frac{m_\pi^4 (m_{\pi_1}^4 + m_\rho^4)}{16\pi m_{\pi_1}^2 (m_{\pi_1}^2 - m_\rho^2)} \right], \end{aligned} \quad (36)$$

$$\begin{aligned} \Sigma_{\text{T,IR}}^{\pi\eta'}(m_{\pi_1}^2) &= \frac{g_{\eta'\pi}^2 m_\pi^4}{128\pi^2 (m_{\pi_1}^2 - m_{\eta'}^2)} \left[1 - 2 \ln\left(\frac{m_\pi^2}{m_{\pi_1}^2}\right) \right] \\ &\quad - i g_{\eta'\pi}^2 \left[\frac{(m_{\pi_1}^2 - m_{\eta'}^2)^3}{192\pi m_{\pi_1}^4} - \frac{m_\pi^2 (m_{\pi_1}^4 - m_{\eta'}^4)}{64\pi m_{\pi_1}^4} \right. \\ &\quad \left. + \frac{m_\pi^4 (m_{\pi_1}^4 + m_{\eta'}^4)}{64\pi m_{\pi_1}^4 (m_{\pi_1}^2 - m_{\eta'}^2)} \right], \end{aligned} \quad (37)$$

$$\begin{aligned} &\Sigma_{\text{T,IR}}^{b_1\pi}(m_{\pi_1}^2) \\ &= g_{b_1\pi}^2 \left\{ \frac{m_\pi^4 (m_{\pi_1}^4 - 6m_{\pi_1}^2 m_{b_1}^2 + m_{b_1}^4)}{64\pi^2 m_{b_1}^2 (m_{\pi_1}^2 - m_{b_1}^2)^3} \right. \\ &\quad + \left[\frac{m_\pi^2}{8\pi^2 (m_{\pi_1}^2 - m_{b_1}^2)} \right. \\ &\quad \left. - \frac{m_\pi^4 (m_{\pi_1}^4 - 2m_{\pi_1}^2 m_{b_1}^2 - 3m_{b_1}^4)}{32\pi^2 m_{b_1}^2 (m_{\pi_1}^2 - m_{b_1}^2)^3} \right] \ln\left(\frac{m_\pi^2}{m_{\pi_1}^2}\right) \left. \right\} \\ &\quad - i g_{b_1\pi}^2 \left[\frac{(m_{\pi_1}^2 - m_{b_1}^2)(m_{\pi_1}^4 + 10m_{\pi_1}^2 m_{b_1}^2 + m_{b_1}^4)}{96\pi m_{b_1}^2 m_{\pi_1}^4} \right] \end{aligned}$$

$$-\frac{m_\pi^2(m_{\pi_1}^2+m_{b_1}^2)^3}{32\pi m_{b_1}^2 m_{\pi_1}^4 (m_{\pi_1}^2-m_{b_1}^2)} + \frac{m_\pi^4(m_{\pi_1}^2+m_{b_1}^2)^2(m_{\pi_1}^4-4m_{\pi_1}^2 m_{b_1}^2+m_{b_1}^4)}{32\pi m_{b_1}^2 m_{\pi_1}^4 (m_{\pi_1}^2-m_{b_1}^2)^3} \Big], \quad (38)$$

$$\begin{aligned} & \Sigma_{\Gamma,IR}^{f_1\pi}(m_{\pi_1}^2) \\ &= g_{f_1\pi}^2 \left\{ \frac{m_\pi^4(m_{\pi_1}^4-6m_{\pi_1}^2 m_{f_1}^2+m_{f_1}^4)}{128\pi^2 m_{f_1}^2 (m_{\pi_1}^2-m_{f_1}^2)^3} \right. \\ & \quad + \left[\frac{m_\pi^2}{16\pi^2(m_{\pi_1}^2-m_{f_1}^2)} - \frac{m_\pi^4(m_{\pi_1}^4-2m_{\pi_1}^2 m_{f_1}^2-3m_{f_1}^4)}{64\pi^2 m_{f_1}^2 (m_{\pi_1}^2-m_{f_1}^2)^3} \right] \ln\left(\frac{m_\pi^2}{m_{\pi_1}^2}\right) \Big\} \\ & \quad - i g_{f_1\pi}^2 \left[\frac{(m_{\pi_1}^2-m_{f_1}^2)(m_{\pi_1}^4+10m_{\pi_1}^2 m_{f_1}^2+m_{f_1}^4)}{192\pi m_{f_1}^2 m_{\pi_1}^4} \right. \\ & \quad - \frac{m_\pi^2(m_{\pi_1}^2+m_{f_1}^2)^3}{64\pi m_{f_1}^2 m_{\pi_1}^4 (m_{\pi_1}^2-m_{f_1}^2)} \\ & \quad \left. + \frac{m_\pi^4(m_{\pi_1}^2+m_{f_1}^2)^2(m_{\pi_1}^4-4m_{\pi_1}^2 m_{f_1}^2+m_{f_1}^4)}{64\pi m_{f_1}^2 m_{\pi_1}^4 (m_{\pi_1}^2-m_{f_1}^2)^3} \right]. \quad (39) \end{aligned}$$

3.2 η-π loop

Consider the scalar loop integral for the η-π loop:

$$I_{\pi\eta}(p^2) = \mu^{4-d} \int \frac{d^d l}{(2\pi)^d} \frac{1}{[l^2-m_\pi^2+i\epsilon][(p-l)^2-m_\eta^2+i\epsilon]}. \quad (40)$$

After performing the *l*-integration, the above integral reads

$$I_{\pi\eta}(p^2) = \mu^{4-d} \Gamma\left(2-\frac{d}{2}\right) \frac{i p^{d-4}}{(4\pi)^{\frac{d}{2}}} \int_0^1 dx (\Delta)^{\frac{d}{2}-2} \quad (41)$$

with

$$\begin{aligned} \Delta &= x^2 - (a-b+1)x + a, \\ a &= \frac{m_\pi^2}{p^2}, \quad b = \frac{m_\eta^2}{p^2}. \end{aligned} \quad (42)$$

Similarly, Δ can be re-expressed as Δ = b(x-x₁)(x-x₂), with

$$x_{1,2} = \frac{a-b+1}{2} \left(1 \pm \sqrt{1 - \frac{4a}{(a-b+1)^2}} \right) \quad (43)$$

Obviously we have

$$a \ll 1, \quad b \ll 1, \quad \frac{4a}{(a-b+1)^2} \ll 1. \quad (44)$$

So we can expand x_{1,2} in terms of a and b

$$\begin{aligned} x_1 &= 1-b-ab+\dots, \\ x_2 &= a+ab+\dots \end{aligned} \quad (45)$$

With the same method, we divide the integral into three parts

$$I_{\pi\eta} = \mu^{4-d} \Gamma\left(2-\frac{d}{2}\right) \frac{i p^{d-4}}{(4\pi)^{\frac{d}{2}}} (I_{\pi\eta}^{(1)} + I_{\pi\eta}^{(2)} + I_{\pi\eta}^{(3)}) \quad (46)$$

with

$$\begin{aligned} I_{\pi\eta}^{(1)}(p^2) &= \int_0^{x_2} dx [(x-x_1)(x-x_2)]^{\frac{d}{2}-2} \\ &= x_1^{\frac{d}{2}-1} x_2^{\frac{d}{2}-1} \sum_{m=0}^{\infty} \frac{\Gamma\left(\frac{d}{2}-1\right)\Gamma\left(\frac{d}{2}-1\right)}{\Gamma\left(\frac{d}{2}-1-m\right)\Gamma\left(\frac{d}{2}+m\right)} \\ & \quad \cdot \left(-\frac{x_2}{x_1}\right)^m, \end{aligned} \quad (47)$$

$$\begin{aligned} I_{\pi\eta}^{(2)}(p^2) &= \int_{x_2}^{x_1} dx [(x-x_1)(x-x_2)]^{\frac{d}{2}-2} \\ &= (-1)^{\frac{d}{2}-2} (x_1-x_2)^{d-3} \frac{\left[\Gamma\left(\frac{d}{2}-1\right)\right]^2}{\Gamma(d-2)}, \end{aligned} \quad (48)$$

$$I_{\pi\eta}^{(3)}(p^2) = \int_{x_1}^1 dx [(x-x_1)(x-x_2)]^{\frac{d}{2}-2}. \quad (49)$$

The I_{πη}⁽¹⁾ and I_{πη}⁽²⁾ are similar for the case in the previous section, where I_{πη}⁽¹⁾ belongs to the ‘infrared singular part’ of I_{πη} and I_{πη}⁽²⁾ contains an imaginary part. However, the I_{πη}⁽³⁾ is quite different. To calculate the I_{πη}⁽³⁾, we first shift the integration variable

$$\begin{aligned} I_{\pi\eta}^{(3)}(p^2) &= \int_0^{1-x_1} dy [(1-x_1-y)(1-x_2-y)]^{\frac{d}{2}-2} \\ &= (1-x_2)^{\frac{d}{2}-2} \int_0^{1-x_1} dy \\ & \quad \cdot \left[(1-x_1-y) \left(1 - \frac{y}{1-x_2} \right) \right]^{\frac{d}{2}-2}. \end{aligned} \quad (50)$$

Since (1-x₁) ~ O(a) ~ O(b) and (1-x₂) ~ O(1), when y ∈ [0, 1-x₁], we can expand the above integral in terms of the parameter y/(1-x₂)

$$\begin{aligned} I_{\pi\eta}^{(3)}(p^2) &= (1-x_2)^{\frac{d}{2}-2} \int_0^{1-x_1} dy (1-x_1-y)^{\frac{d}{2}-2} \\ & \quad \cdot \sum_{m=0}^{\infty} \frac{\Gamma\left(\frac{d}{2}-1\right)}{\Gamma\left(\frac{d}{2}-1-m\right)m!} \left(-\frac{y}{1-x_2}\right)^m \\ &= (1-x_1)^{\frac{d}{2}-1} (1-x_2)^{\frac{d}{2}-1} \end{aligned}$$

$$\sum_{m=0}^{\infty} \frac{\Gamma\left(\frac{d}{2}-1\right)\Gamma\left(\frac{d}{2}-1\right)}{\Gamma\left(\frac{d}{2}-1-m\right)\Gamma\left(\frac{d}{2}+m\right)} \left(-\frac{1-x_1}{1-x_2}\right)^m. \quad (51)$$

Obviously $I_{\pi_X}^{(3)}$ is non-analytic in b for non-integer dimension d . In other words, $I_{\pi_X}^{(3)}$ also contributes to the ‘infrared singular part’. The ‘infrared singular part’ of $I_{\pi\eta}$ with the imaginary part is thus

$$\begin{aligned} I_{\pi\eta}^{\text{IR}}(p^2) &= \mu^{4-d} \Gamma\left(2-\frac{d}{2}\right) \frac{ip^{d-4}}{(4\pi)^{\frac{d}{2}}} (I_{\pi\eta}^{(1)} + \text{Im}(I_{\pi\eta}^{(2)}) + I_{\pi\eta}^{(3)}) \\ &= \frac{i}{16\pi^2} (1-x_1) \left[L + 1 - \ln\left(\frac{m_\eta^2}{\mu^2}\right) \right. \\ &\quad \left. + \frac{x_1-x_2}{1-x_1} \ln\left(\frac{x_1-x_2}{1-x_2}\right) \right] \\ &\quad + \frac{i}{16\pi^2} x_2 \left[L + 1 - \ln\left(\frac{m_\pi^2}{\mu^2}\right) \right. \\ &\quad \left. + \frac{x_1-x_2}{x_2} \ln\left(\frac{x_1-x_2}{x_1}\right) \right] - \frac{1}{16\pi} (x_1-x_2) \\ &= \frac{i}{16\pi^2} \left[L - \ln\left(\frac{m_\pi^2}{m_{\pi_1}^2}\right) \right] (a+ab) \\ &\quad + \frac{i}{16\pi^2} \left[L - \ln\left(\frac{m_\eta^2}{m_{\pi_1}^2}\right) \right] (b+ab) \\ &\quad + \frac{i}{32\pi^2} (a^2+b^2) - \frac{1}{16\pi} (1-a-b-2ab). \end{aligned} \quad (52)$$

The chiral correction from the $\eta\pi$ loop diagram reads

$$\begin{aligned} \Sigma_{\text{T,IR}}^{\pi\eta}(m_{\pi_1}^2) &= \frac{g_{\pi\eta}^2 m_\pi^4}{128\pi^2 m_{\pi_1}^2} \left[1 - 2 \ln\left(\frac{m_\pi^2}{m_{\pi_1}^2}\right) \right] \\ &\quad + \frac{g_{\pi\eta}^2 m_\eta^4}{128\pi^2 m_{\pi_1}^2} \left[1 - 2 \ln\left(\frac{m_\eta^2}{m_{\pi_1}^2}\right) \right] \\ &\quad - ig_{\pi\eta}^2 \left(\frac{m_{\pi_1}^2 - 3m_\pi^2 - 3m_\eta^2}{192\pi} + \frac{m_\pi^4 + m_\eta^4}{64\pi m_{\pi_1}^2} \right). \end{aligned} \quad (53)$$

3.3 $\eta(\eta')\text{-}\pi_1$ loop

The η' meson mass is dominated by the axial anomaly, which remains large in the chiral limit. The propagators in the $\eta'\text{-}\pi_1$ loop do not produce a ‘soft pole’ contribution. In other words, the loop integral does not contain the ‘infrared singular part’.

Now we consider the $\pi_1\eta$ loop diagram with $M_{\pi_1}^2 \sim M^2$ and $m^2 \ll M^2$, which is similar to the nucleon self energy diagram. We can use the standard IR method in Ref. [46] to obtain the ‘infrared singular part’. First we define the dimensionless variables

$$\Omega = \frac{p^2 - m_\eta^2 - m_{\pi_1}^2}{2m_\eta m_{\pi_1}}, \quad \alpha = \frac{m_\eta}{m_{\pi_1}}. \quad (54)$$

The corresponding scalar loop integral is

$$\begin{aligned} I_{\pi_1\eta}(p^2) &= \mu^{4-d} \int \frac{d^d l}{(2\pi)^d} \frac{1}{[l^2 - m_\eta^2 + i\epsilon][(p-l)^2 - m_{\pi_1}^2 + i\epsilon]} \\ &= \mu^{4-d} \Gamma\left(2-\frac{d}{2}\right) \frac{im_{\pi_1}^{d-4}}{(4\pi)^{\frac{d}{2}}} \int_0^1 dx (\Delta)^{\frac{d}{2}-2}, \end{aligned} \quad (55)$$

where

$$\Delta = x^2 - 2\alpha\Omega x(1-x) + \alpha^2(1-x)^2 - i\epsilon. \quad (56)$$

Within the IR scheme, the ‘infrared singular part’ of $I_{\pi_1\eta}$ reads

$$\begin{aligned} I_{\pi_1\eta}^{\text{IR}} &= \mu^{4-d} \Gamma\left(2-\frac{d}{2}\right) \frac{im_{\pi_1}^{d-4}}{(4\pi)^{\frac{d}{2}}} \int_0^\infty dx (\Delta)^{\frac{d}{2}-2} \\ &= \frac{i(p^2 - m_{\pi_1}^2 + m_\eta^2)}{32\pi^2 p^2} L + I'(p^2) \end{aligned} \quad (57)$$

with

$$\begin{aligned} I'(p^2) &= \frac{i}{16\pi^2} \frac{\alpha(\alpha+\Omega)}{1+2\alpha\Omega+\alpha^2} (1-2\ln\alpha) \\ &\quad - \frac{i}{8\pi^2} \frac{\alpha\sqrt{1-\Omega^2}}{1+2\alpha\Omega+\alpha^2} \arccos\left(-\frac{\alpha+\Omega}{\sqrt{1+2\alpha\Omega+\alpha^2}}\right), \end{aligned} \quad (58)$$

and the regularization scale $\mu = m_{\pi_1}$. The chiral correction from the $\pi_1\eta$ loop diagram reads

$$\begin{aligned} \Sigma_{\text{T,IR}}^{\pi_1\eta}(m_{\pi_1}^2) &= -g_{\pi_1\eta}^2 \left[\frac{m_{\pi_1} m_\eta^3}{24\pi} + \frac{m_\eta^4}{32\pi^2} \ln\left(\frac{m_\eta^2}{m_{\pi_1}^2}\right) \right] \\ &\quad + \mathcal{O}(m_\eta^5). \end{aligned} \quad (59)$$

3.4 Tadpole diagrams

The chiral corrections from the tadpole diagrams in Fig. 3 are

$$\begin{aligned} \Sigma_{\text{T,IR}}^{\pi,\text{tadpole}}(m_{\pi_1}^2) &= \left(d_1 + \frac{d_2}{4}\right) \frac{3m_\pi^4}{16\pi^2} \ln\left(\frac{m_\pi^2}{m_{\pi_1}^2}\right) \\ &\quad - \frac{3}{128\pi^2} d_2 m_\pi^4, \end{aligned} \quad (60)$$

$$\begin{aligned} \Sigma_{\text{T,IR}}^{\eta,\text{tadpole}}(m_{\pi_1}^2) &= \left(d_1^* + \frac{d_2^*}{4}\right) \frac{m_\eta^4}{16\pi^2} \ln\left(\frac{m_\eta^2}{m_{\pi_1}^2}\right) \\ &\quad - \frac{1}{128\pi^2} d_2^* m_\eta^4, \end{aligned} \quad (61)$$

where we have redefined the low energy constants

$$\begin{aligned} d_1 &= c_1 + c_2 + c_6, & d_2 &= c_3, \\ d_1^* &= c_1^* + c_2^* + c_6^*, & d_2^* &= c_3^*. \end{aligned} \quad (62)$$

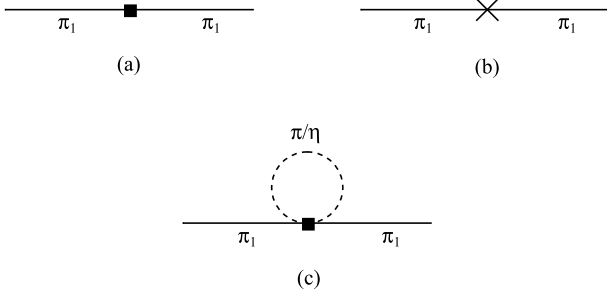


Fig. 3. The tadpole diagram of the $\pi_1(1600)$ self energy. The $\mathcal{O}(m_\pi^2)$ and $\mathcal{O}(m_\pi^4)$ LECs also contribute to the self energy, which are labeled by the squares and cross respectively.

All the divergence can be absorbed by the counter terms in Eq. (13), which also contribute to m_{π_1}

$$\Sigma_{\pi_1(1600)}^{\text{tree}} = e_1(m_\pi^2 + m_\eta^2) + e_2(m_\pi^2 + m_\eta^2)^2. \quad (63)$$

Finally we obtain the chiral corrections to the $\pi_1(1600)$ mass up to one-loop order, which is the main result of this work

$$\begin{aligned} \Delta M_{\pi_1(1600)}^{1\text{-loop}} &= \Sigma_{\text{T,IR}}^{\rho\pi}(m_{\pi_1}^2) + \Sigma_{\text{T,IR}}^{\pi_1\eta}(m_{\pi_1}^2) + \Sigma_{\text{T,IR}}^{\pi_1\eta'}(m_{\pi_1}^2) \\ &+ \Sigma_{\text{T,IR}}^{\pi\eta}(m_{\pi_1}^2) + \Sigma_{\text{T,IR}}^{\pi\eta'}(m_{\pi_1}^2) \\ &+ \Sigma_{\text{T,IR}}^{b_1\pi}(m_{\pi_1}^2) + \Sigma_{\text{T,IR}}^{f_1\pi}(m_{\pi_1}^2) + \Sigma_{\text{T,IR}}^{\pi,\text{tadpole}}(m_{\pi_1}^2) \\ &+ \Sigma_{\text{T,IR}}^{\eta,\text{tadpole}}(m_{\pi_1}^2) + \Sigma_{\pi_1(1600)}^{\text{tree}}. \end{aligned} \quad (64)$$

Note that we treat the intermediate states as stable particles in our above calculation. However, the widths of ρ , b_1 , f_1 are not small. The contributions from the widths of the intermediate states to the non-analytic chiral corrections to the $\pi_1(1600)$ mass are summarized in Appendix .

4 Results and discussion

We need to deal with the numerous effective coupling constants before the numerical analysis. Actually the experimental information on the $\pi_1(1600)$ decays is not rich. From the current experimental data of the $\pi_1(1600)$ decays, we can make a very rough estimate of the values of $g_{\rho\pi}$, $g_{\eta\pi}$, $g_{\eta'\pi}$, $g_{f_1\pi}$ and $g_{b_1\pi}$. The others still remain unknown.

A partial wave analysis in Ref. [51] gives the branching ratio

$$\text{Br}(\pi_1 \rightarrow b_1\pi) : \text{Br}(\pi_1 \rightarrow \rho\pi) : \text{Br}(\pi_1 \rightarrow \eta'\pi) = 1 : (1.5 \pm 0.5) : (1.0 \pm 0.3). \quad (65)$$

An analysis based on the VES experiment leads to [52]

$$\begin{aligned} &\text{Br}(\pi_1 \rightarrow b_1\pi) : \\ &\text{Br}(\pi_1 \rightarrow \rho\pi) : \text{Br}(\pi_1 \rightarrow \eta'\pi) : \text{Br}(\pi_1 \rightarrow f_1\pi) \\ &= (1.0 \pm 0.3) : < 0.3 : 1 : (1.1 \pm 0.3). \end{aligned} \quad (66)$$

The E852 collaboration reported [8]

$$\frac{\text{Br}(\pi_1 \rightarrow f_1\pi)}{\text{Br}(\pi_1 \rightarrow \eta'\pi)} = 3.80 \pm 0.78. \quad (67)$$

In order to make a very rough estimate of these coupling constants, we combine the above measurements and set the branching ratio to be

$$\begin{aligned} &\text{Br}(\pi_1 \rightarrow b_1\pi) : \text{Br}(\pi_1 \rightarrow \rho\pi) : \text{Br}(\pi_1 \rightarrow \eta'\pi) : \\ &\text{Br}(\pi_1 \rightarrow f_1\pi) : \text{Br}(\pi_1 \rightarrow \eta\pi) \\ &= 1 : 2 : 1 : 1 : 1. \end{aligned} \quad (68)$$

From Eqs. (3)–(7), the partial decay width of the $\pi_1(1600)$ reads

$$\Gamma(\pi_1 \rightarrow \rho\pi) = 2 \times \frac{g_{\rho\pi}^2}{12\pi} |\vec{p}_\pi|^3, \quad (69)$$

$$\Gamma(\pi_1 \rightarrow \eta\pi) = \frac{g_{\eta\pi}^2}{24\pi} \frac{|\vec{p}_\pi|^3}{m_{\pi_1}^2}, \quad (70)$$

$$\Gamma(\pi_1 \rightarrow \eta'\pi) = \frac{g_{\eta'\pi}^2}{24\pi} \frac{|\vec{p}_\pi|^3}{m_{\pi_1}^2}, \quad (71)$$

$$\Gamma(\pi_1 \rightarrow f_1\pi) = \frac{g_{f_1\pi}^2}{24\pi} \frac{|\vec{p}_\pi|}{m_{\pi_1}^2} \left(3 + \frac{|\vec{p}_\pi|^2}{m_{f_1}^2} \right), \quad (72)$$

$$\Gamma(\pi_1 \rightarrow b_1\pi) = 2 \times \frac{g_{b_1\pi}^2}{24\pi} \frac{|\vec{p}_\pi|}{m_{\pi_1}^2} \left(3 + \frac{|\vec{p}_\pi|^2}{m_{b_1}^2} \right), \quad (73)$$

where \vec{p}_π is the pion decay momentum.

With the total decay width of $\pi_1(1600)$ around 300 MeV as input [53], we get

$$\begin{aligned} |g_{\rho\pi}| &\simeq 2.7 \text{ GeV}^{-1}, \quad |g_{\eta\pi}| \simeq 5.1, \quad |g_{\eta'\pi}| \simeq 8.1, \\ |g_{f_1\pi}| &\simeq 3.3 \text{ GeV}, \quad |g_{b_1\pi}| \simeq 2.2 \text{ GeV}. \end{aligned} \quad (74)$$

For the $\pi_1\pi_1\eta$ coupling constant, we use $g_{\pi_1\eta} \sim \frac{1}{1.6F_\eta} \text{ GeV}^{-1} \sim 5.3 \text{ GeV}^{-1}$ where the $F_\eta \approx 0.1 \text{ GeV}$ is the decay constant of η . This ad hoc value was estimated with the very naive dimensional argument, which might be too large.

From the tree-level Lagrangian of chiral perturbation theory,

$$M_\pi^2 = 2B_0m, \quad M_\eta^2 = \frac{2}{3}B_0(m + 2m_s). \quad (75)$$

We consider two cases in the numerical analysis. Case 1 corresponds to the $SU_F(3)$ chiral limit where $M_\pi^2 = M_\eta^2 \rightarrow 0$ when $m_s = m$ approaches zero simultaneously.

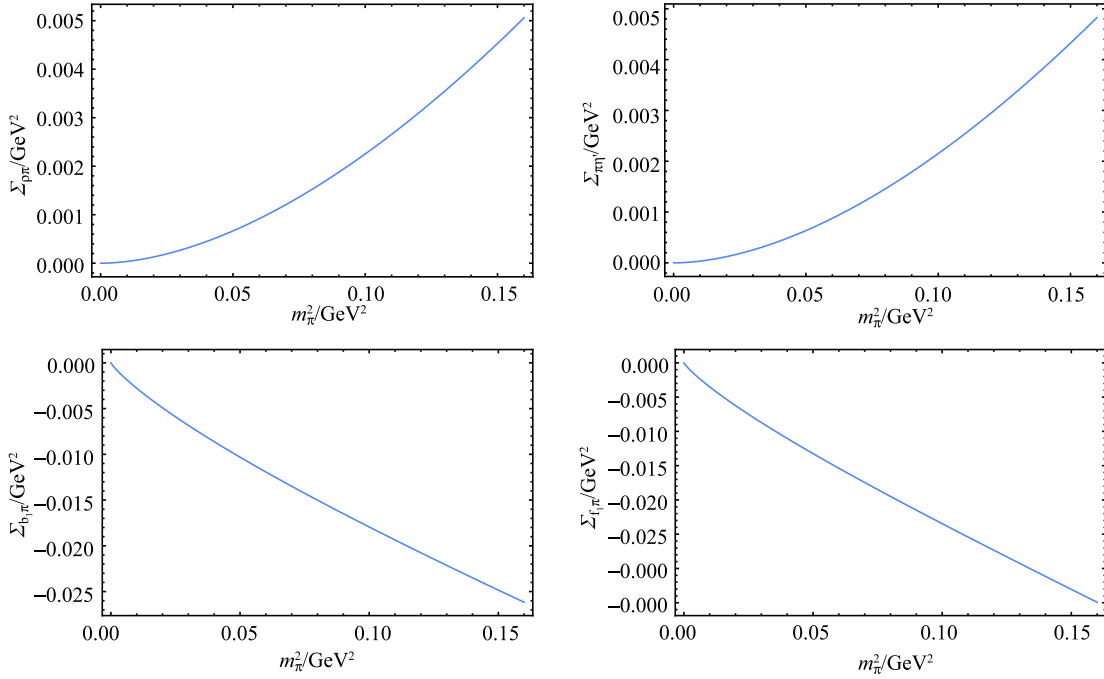


Fig. 4. (color online) The pion mass dependence of the chiral corrections to the $\pi_1(1600)$ mass from the $\rho\pi, \eta'\pi, b_1\pi, f_1\pi$ loops, where the top-left, top-right, bottom-left and bottom-right subfigures correspond to the $\rho\pi, \eta'\pi, b_1\pi, f_1\pi$ contributions respectively.

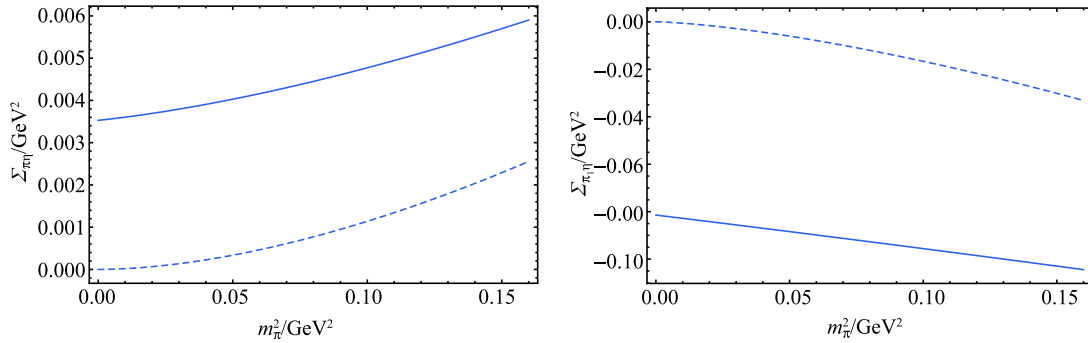


Fig. 5. (color online) The pion mass dependence of the chiral corrections to the $\pi_1(1600)$ mass from the $\eta\pi$ and $\pi_1\eta$ loops. The solid and dotted lines correspond to the $SU_F(2)$ and $SU_F(3)$ cases respectively.

Since the strange quark is sometimes treated as a heavy degree of freedom in the lattice QCD simulation, we also consider Case 2, which corresponds to the $SU_F(2)$ chiral limit. Now we fix the strange quark mass and let the up and down quark mass approach zero. In the $SU_F(2)$ chiral limit, the η meson mass remains finite. We have

$$M_\eta^2 = \frac{4}{3}B_0m_s + \frac{1}{3}M_\pi^2. \quad (76)$$

We collect the variation of the chiral corrections to the $\pi_1(1600)$ mass from different loop diagrams with the pion mass in Figs. (4)–(5). The most important chiral correction to the $\pi_1(1600)$ mass comes from the $\pi_1\eta$ loop. The chiral corrections from the $\pi\rho$, $\pi\eta$ and $\pi\eta'$ loops are

positive and increase with m_π while the corrections from the $\eta\pi_1$, πb_1 and πf_1 loops are negative. Furthermore, the chiral corrections from the $\eta\pi_1$, πb_1 and πf_1 loops are very sensitive to the pion mass.

The coupling constants d_i ($i = 1, 2$), d_j^* ($j = 1, 2$) contribute to the tadpole diagram while e_k ($k = 1, 2$) are low energy constants. They are unknown at present. Although this kind of contribution may be significant, we do not present their variations with the pion mass because there are too many unknown coupling constants.

According to PDG [4], the $\pi_1(1600)$ was observed in the $b_1\pi$, $\eta'\pi$ and $f_1\pi$ modes. The Compass collaboration reported the $\pi_1(1600)$ in the $\rho\pi$ mode [9]. The $\pi_1(1400)$ was observed in the $\eta\pi$ mode. Both the $\pi_1(1600)$ and $\pi_1(1400)$ signals are very broad with a decay width of

241 ± 40 MeV and 330 ± 35 MeV respectively [4]. These two signals overlap with each other. In this work, we have taken into account all the above possible decay modes and calculated the one-loop chiral corrections to the $\pi_1(1600)$ mass. We have employed two different methods to deal with the loop integrals and derived all the infrared singular chiral corrections explicitly.

From the available experimental measurement of the partial decay width of the $\pi_1(1600)$ meson, we extract

Appendix A

Contributions generated by the finite widths of the intermediate states

In this Appendix we deal with the scalar loop integrals when the intermediate states have a finite decay width Γ .

$$\begin{aligned}
 I_{\pi X}(p^2) &= \mu^{4-d} \int \frac{d^d l}{(2\pi)^d} \frac{1}{[l^2 - m_\pi^2 + i\epsilon][(p-l)^2 - M^2 + iM\Gamma]} \\
 &= \mu^{4-d} \Gamma \left(2 - \frac{d}{2}\right) \frac{iM^{d-4}}{(4\pi)^{\frac{d}{2}}} \int_0^1 dx (\Delta)^{\frac{d}{2}-2}, \quad (A1)
 \end{aligned}$$

with

$$\begin{aligned}
 \Delta &= bx^2 - \left(a + b - 1 + \frac{i\Gamma}{M}\right)x + a \\
 &= b(x - x_1)(x - x_2), \\
 a &= \frac{m_\pi^2}{M^2}, \quad b = \frac{p^2}{M^2}, \quad (A2)
 \end{aligned}$$

where the X represents ρ , b_1 , f_1 , M and Γ are the corresponding mass and width, and

$$x_{1,2} = \frac{a + b - 1 + \frac{i\Gamma}{M}}{2b} \left(1 \pm \sqrt{1 - \frac{4ab}{\left(a + b - 1 + \frac{i\Gamma}{M}\right)^2}} \right) \quad (A3)$$

We expand $x_{1,2}$ in terms of a

$$\begin{aligned}
 x_1 &= \frac{b-1 + \frac{i\Gamma}{M}}{b} - \frac{a\left(1 - \frac{i\Gamma}{M}\right)}{b\left(b-1 + \frac{i\Gamma}{M}\right)} - \frac{a^2\left(1 - \frac{i\Gamma}{M}\right)}{\left(b-1 + \frac{i\Gamma}{M}\right)^3} + \mathcal{O}(a^3), \\
 x_2 &= \frac{a}{b-1 + \frac{i\Gamma}{M}} + \frac{a^2\left(1 - \frac{i\Gamma}{M}\right)}{\left(b-1 + \frac{i\Gamma}{M}\right)^3} + \mathcal{O}(a^3). \quad (A4)
 \end{aligned}$$

In our case, $\Gamma_X \sim m_\pi$. We treat the $\left(\frac{\Gamma}{M}\right)^2$ as $\mathcal{O}(a)$ and get

the coupling constants. We investigate the variation of the different chiral corrections with the pion mass under two schemes. The present calculation is applicable to all possible interpretations of the π_1 mesons since our analysis does not rest on the inner structure of the π_1 mesons. Hopefully, the explicit non-analytical chiral structures will be helpful to the chiral extrapolation of lattice data from the dynamical lattice QCD simulation of either the exotic light hybrid meson or the tetraquark state.

$$\begin{aligned}
 x_1 &= \frac{b-1}{b} - \frac{a\left(b-1 - \frac{\Gamma^2}{M^2}\right)}{b\left[(b-1)^2 + \frac{\Gamma^2}{M^2}\right]} - \frac{a^2(b-1)^3}{\left[(b-1)^2 + \frac{\Gamma^2}{M^2}\right]^3} \\
 &\quad + i\frac{\Gamma}{M} \left[\frac{1}{b} + \frac{a}{(b-1)^2 + \frac{\Gamma^2}{M^2}} \right] + \dots, \\
 x_2 &= \frac{a(b-1)}{(b-1)^2 + \frac{\Gamma^2}{M^2}} + \frac{a^2(b-1)^3}{\left[(b-1)^2 + \frac{\Gamma^2}{M^2}\right]^3} \\
 &\quad - i\frac{\Gamma}{M} \frac{a}{(b-1)^2 + \frac{\Gamma^2}{M^2}} + \dots \quad (A5)
 \end{aligned}$$

The original integral can be re-expressed as

$$\begin{aligned}
 I_{\pi X}(p^2) &= \mu^{4-d} \Gamma \left(2 - \frac{d}{2}\right) \frac{iM^{d-4}}{(4\pi)^{\frac{d}{2}}} \\
 &\quad \int_0^1 dx [b(x - x_1)(x - x_2)]^{\frac{d}{2}-2}. \quad (A6)
 \end{aligned}$$

Now x_1, x_2 are complex while the integration variable x is real, which renders the evaluation of the integral straightforward. We have

$$\begin{aligned}
 I_{\pi X}(p^2) &= \frac{i}{16\pi^2} \left[L - \ln\left(\frac{M^2}{\mu^2}\right) - 1 - \int_0^1 dx \ln[b(x - x_1)(x - x_2)] \right] \\
 &= \frac{i}{16\pi^2} (1 - x_2) \left[L - \ln\left(\frac{M^2}{\mu^2}\right) \right] \\
 &\quad + \frac{i}{16\pi^2} x_2 \left[L - \ln\left(\frac{m_\pi^2}{\mu^2}\right) \right] \\
 &\quad + \frac{i}{16\pi^2} \left[1 - (1 - x_2) \ln\left(1 - \frac{i\Gamma}{M}\right) \right. \\
 &\quad \left. - (x_1 - x_2) \ln\left(\frac{-x_1}{1 - x_1}\right) \right]. \quad (A7)
 \end{aligned}$$

After extracting the non-analytic chiral corrections from the above expression, we get

$$I_{\pi X}^{\text{NA}}(p^2) = -\frac{i}{16\pi^2} x_2 \ln\left(\frac{m_\pi^2}{\mu^2}\right) = -\frac{i}{16\pi^2} \left[\frac{a(b-1)}{(b-1)^2 + \frac{\Gamma^2}{M^2}} + \frac{a^2(b-1)^3}{\left[(b-1)^2 + \frac{\Gamma^2}{M^2}\right]^3} - i\frac{\Gamma}{M} \frac{a}{(b-1)^2 + \frac{\Gamma^2}{M^2}} \right] \ln\left(\frac{m_\pi^2}{\mu^2}\right). \quad (\text{A8})$$

It is interesting to note that the above expression contains a non-analytic chiral correction to the imaginary part, which is proportional to $\frac{\Gamma}{M}$ and vanishes when $\Gamma \rightarrow 0$. In comparison, when we treat the intermediate states as stable particles, the imaginary parts of the chiral corrections to the self-energy of the $\pi_1(1600)$ are analytic in the pseudo-scalar meson mass. In the limit of $\Gamma = 0$, we recover the results in the previous sections in the text.

For the $\rho\pi$, $b_1\pi$, $f_1\pi$ loops, we collect the non-analytic chiral corrections to the mass of the $\pi_1(1600)$ up to $\mathcal{O}(m_\pi^4)$,

$$\Sigma_{\text{T,NA}}^{\rho\pi}(m_{\pi_1}^2) = -\frac{g_{\rho\pi}^2 m_\pi^2}{48\pi^2} \ln\left(\frac{m_\pi^2}{m_{\pi_1}^2}\right) \left\{ \frac{m_\rho^2 \Gamma_\rho^2 (m_{\pi_1}^2 - m_\rho^2)}{(m_{\pi_1}^2 - m_\rho^2)^2 + m_\rho^2 \Gamma_\rho^2} + \frac{m_\pi^2 (3m_{\pi_1}^4 - 2m_{\pi_1}^2 m_\rho^2 + m_\rho^2 \Gamma_\rho^2 - m_\rho^4)}{(m_{\pi_1}^2 - m_\rho^2)^2 + m_\rho^2 \Gamma_\rho^2} \right\}$$

$$\left. - \frac{m_\pi^2 m_\rho^2 (m_{\pi_1}^2 - m_\rho^2)^5}{[(m_{\pi_1}^2 - m_\rho^2)^2 + m_\rho^2 \Gamma_\rho^2]^3} \right\}, \quad (\text{A9})$$

$$\Sigma_{\text{T,NA}}^{b_1\pi}(m_{\pi_1}^2) = \frac{g_{b_1\pi}^2 m_\pi^2}{96\pi^2 m_{\pi_1}^2} \ln\left(\frac{m_\pi^2}{m_{\pi_1}^2}\right) \cdot \left\{ \frac{(m_{\pi_1}^2 - m_{b_1}^2)(12m_{\pi_1}^2 - \Gamma_{b_1}^2)}{(m_{\pi_1}^2 - m_{b_1}^2)^2 + m_{b_1}^2 \Gamma_{b_1}^2} - \frac{m_\pi^2 (3m_{\pi_1}^4 - 2m_{\pi_1}^2 m_{b_1}^2 + m_{b_1}^2 \Gamma_{b_1}^2 - m_{b_1}^4)}{m_{b_1}^2 [(m_{\pi_1}^2 - m_{b_1}^2)^2 + m_{b_1}^2 \Gamma_{b_1}^2]} + \frac{m_\pi^2 (m_{\pi_1}^2 - m_{b_1}^2)^3 (m_{\pi_1}^4 + 10m_{\pi_1}^2 m_{b_1}^2 + m_{b_1}^4)}{[(m_{\pi_1}^2 - m_{b_1}^2)^2 + m_{b_1}^2 \Gamma_{b_1}^2]^3} \right\}, \quad (\text{A10})$$

$$\Sigma_{\text{T,NA}}^{f_1\pi}(m_{\pi_1}^2) = \frac{g_{f_1\pi}^2 m_\pi^2}{192\pi^2 m_{\pi_1}^2} \ln\left(\frac{m_\pi^2}{m_{\pi_1}^2}\right) \cdot \left\{ \frac{(m_{\pi_1}^2 - m_{f_1}^2)(12m_{\pi_1}^2 - \Gamma_{f_1}^2)}{(m_{\pi_1}^2 - m_{f_1}^2)^2 + m_{f_1}^2 \Gamma_{f_1}^2} - \frac{m_\pi^2 (3m_{\pi_1}^4 - 2m_{\pi_1}^2 m_{f_1}^2 + m_{f_1}^2 \Gamma_{f_1}^2 - m_{f_1}^4)}{m_{f_1}^2 [(m_{\pi_1}^2 - m_{f_1}^2)^2 + m_{f_1}^2 \Gamma_{f_1}^2]} + \frac{m_\pi^2 (m_{\pi_1}^2 - m_{f_1}^2)^3 (m_{\pi_1}^4 + 10m_{\pi_1}^2 m_{f_1}^2 + m_{f_1}^4)}{[(m_{\pi_1}^2 - m_{f_1}^2)^2 + m_{f_1}^2 \Gamma_{f_1}^2]^3} \right\}. \quad (\text{A11})$$

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