Study of radially excited $D_s(2^1S_0)$ and $D_s(3P)$ *

Yu Tian(田玉)1) Ze Zhao(赵泽)2) Ai-Lin Zhang(张爱林)3)
Department of Physics, Shanghai University, Shanghai 200444, China

Abstract: The unobserved $J^P=0^-$ radial excitation $D_s(2^1S_0)$ is anticipated to have mass 2650 MeV (denoted as $D_s(2650)$). Study of hadronic production is an important way to identify highly excited states. We study hadronic production of $D_s(2650)$ from higher excited resonances in a $^3P_0$ model. Relevant hadronic partial decay widths are found to be very small, which implies it is difficult to observe $D_s(2650)$ in hadronic decays of higher excited resonances. Hadronic decay widths of radially excited $D_s(3P)$ have also been estimated. The total decay widths of four $D_s(3P)$ are large, but the branching ratios in the $D_s(2650)\eta$ channel are very small, which implies that it seems impossible to observe $D_s(2650)$ in hadronic decays of $D_s(3P)$. The dominant decay channels of the four $D_s(3P)$ have been pointed out, and $D_1(2420)$, $D_1(2430)$, $D_2^*(2460)$, D(2550), D(2600), (1^1D_2)D(2750) and $D_3^*(2760)$ are possible to observe in hadronic production from $D_s(3P)$.

Keywords: $^3P_0$ model, Hadronic decay, $D_s(2650)$, $D_s(3P)$

PACS: 13.25.Ft, 12.39.-x **DOI:** 10.1088/1674-1137/41/8/083107

1 Introduction

The lowest lying $1S$ and $1P$ excited D and D_s states are believed to be established [1, 2]. In recent years, more and more higher excited D and D_s resonances have been observed, though some of them have not been identified. Study of their spectroscopy, production and decay will be helpful for their identification and quark dynamics.

For D resonances, D(2550), $D^*(2600)$, D(2750) and $D^*(2760)$ were observed in inclusive e^+e^- and pp collisions by the BaBar [3] and LHCb [4] collaborations, respectively. $D^*(2760)$ was subsequently found to consist of ${D^*}_1(2760)$ and ${D^*}_3(2760)$ [5, 6].

For D_s resonances, ${D^*}_{s1}(2700)$ and ${D^*}_{sJ}(2860)$ were observed by Belle [7] and BaBar [8] collaborations. ${D^*}_{sJ}(2860)$ was also found to consist of ${D^*}_{s1}(2860)$ and ${D^*}_{s3}(2860)$ [9, 10]. According to analyses in Refs. [11–20], tentative assignments of these D and D_s resonances have been made as presented in Table 1. In the table, D(2550) is suggested to be the $J^P=0^-$ radial excitation $D(2^1S_0)$ [11, 12, 15, 16], and D(2750) is suggested to be the $J^P=2^-$ admixture of $D(1^1D_2)$ and $D(1^3D_2)$ [12, 14, 16].

The masses of the $2S$ and 1D D and D_s mesons have been predicted in $n^{2S+1}L_J$ and nj^P schemes in many models. In Table 1, theoretical results from two groups are presented [2, 21, 22]. Experimental resonances are also included in this table. However, two states with $c\bar{s}$ or $\bar{c}s$ quarks under the 2^1S_0 and $1^{1,3}D_2$ columns have not been observed. These are denoted with dots in the table.

Table 1. Predicted and observed masses of $2S$ and 1D excited D and D_s resonances (MeV).

$\backslash n^{2S+1}L_J$	2^1S_0	2^3S_1	$1^{1,3}D_2$	1^3D_1	1^3D_3
D([2])	2581	2643	2827/2834	2816	2833
D_s([2])	2673	2732	2911/2916	2899	2917
D([21])	2534	2593	2773/2779	2762	2779
D_s([22])	2646	2704	2861/2865	2848	2867
D(Exp)	D(2550)	$D^*(2600)$	D(2750)	${D^*}_1(2760)$	${D^*}_3(2760)$
D_s(Exp)	…	${D^*}_{s1}(2700)$	…	${D^*}_{s1}(2860)$	${D^*}_{s3}(2860)$

Received 11 April 2017
* Supported by National Natural Science Foundation of China (11475111)
1) E-mail: 1471632642@shu.edu.cn
2) E-mail: washingtonze@shu.edu.cn
3) E-mail: zhangal@staff.shu.edu.cn

When the experimental D and D_s resonances in the table are examined, it is interesting to note that the masses of $D^*{}_{s1}(2700)$, $D^*{}_{s1}(2860)$ and $D^*{}_{s3}(2860)$ with $c\bar{s}$ or $\bar{c}s$ quarks are about 100 MeV higher than those of $D^*(2600)$, $D^*{}_1(2760)$ and $D^*{}_3(2760)$ with $c\bar{q}$ or $\bar{c}q$ quarks, respectively, though theoretical predictions may not support this fact.

These two missing D_s states are naturally speculated to have masses 2650 MeV and 2850 MeV corresponding to the observed D(2550) and $D(2750)$, respectively. Similar to the assignments of D(2550) and D(2750), the two missing D_s are expected to have quantum numbers $J^P=0^-$ and $J^P=2^-$. In the following, the radially excited pseudoscalar $D_s(2^1S_0)$ with mass around 2650 MeV will be denoted as $D_s(2650)$.

In Ref. [16], two $D_{sJ}(2850)^{\pm}$ have been predicted to have mass 2850 MeV, which have $\Gamma_{\rm total}=125.1$ MeV and $\Gamma_{\rm total}=51.6$ MeV corresponding to $(J^P=2^-,j_q=\frac{3}{2})$ and $(J^P=2^-,j_q=\frac{5}{2})$, respectively, so these $J^P=2^-$ $D_s(1^{1,3}D_2)$ will not be addressed in our study.

In the 2600∼2700 MeV energy region, a surprisingly narrow ($\Gamma<17$ MeV) charmed strange state $D^+_{sJ}(2632)$ was once reported by SELEX [23] in the $D^+_s\eta$ and D^0K^+ decay channels with mass $M=2632.5\pm1.7$ MeV. However, subsequent searches for $D^+_{sJ}(2632)$ by BaBar [24], FOCUS and BELLE have not found any evidence. Now, it is popularly believed that $D^+_{sJ}(2632)$ is an experimental artefact.

In order to understand the nature of $D_s(2650)$, it is important to explore its hadronic decays and productions. Hadronic decays of $D_s(2^1S_0)$ with different predicted masses have been studied in some models, but its hadronic production has not been explored. In particular, it is not clear whether it is possible to observe $D_s(2650)$ in hadronic decays of higher excited resonances in experiment.

The $^3P_0$ model is a phenomenological method first put forth by Micu [26] and subsequently developed by Yaouanc et al. [27–29]. It has been employed successfully to explore Okubo-Zweig-Iizuka-allowed (OZI-allowed) hadronic decays of hadrons. Hadronic decays of the $J^P=0^-$ radial excitation D_s have been explored in the $^3P_0$ model [12, 22, 25], but the hadronic production of this missing $D_s(2650)$ from higher excited resonances has not been explored. As in Refs. [18, 19], hadronic productions of $D_s(2650)$ will be explored in this paper. $D_s(2650)$ may be produced from the near $D_s(3P)$. As an example, hadronic decays of $D_s(3P)$ will be explored in detail, paying attention to the partial decay widths and branching ratios related to the $D_s(2650)$ final state.

This paper is organized as follows. In the second section, the $^3P_0$ model and the parameters are briefly introduced. In Section 3, the numerical results relevant to hadronic decays of $D_s(2650)$ and $D_s(3P)$ are presented. A summary and discussions are presented in Section 4.

2 The $^3P_0$ model

Among models for hadronic decays, the $^3P_0$ model is popularly known as a quark-pair creation (QPC) model. Since the $^3P_0$ model was proposed [26–29], it has been extensively applied to mesons and baryons with success. To proceed with a practical computation, the formulae in Refs. [19, 30] are presented. In the $^3P_0$ model, the hadronic decay width of $A\to BC$ is

$$\Gamma=\pi^2\frac{|\vec{p}|}{M_A^2}\sum_{JL}|\mathcal{M}^{JL}|^2, \quad (1)$$

where $\vec{p}$ is the momentum of the final mesons B and C in the initial meson A's center-of-mass frame

$$|\vec{p}|=\frac{\sqrt{[m_A^2-(m_B-m_C)^2][m_A^2-(m_B+m_C)^2]}}{2m_A} \quad (2)$$

and M^{JL} is the partial wave amplitude of $A\to BC$. The partial wave amplitude M^{JL} is obtained from the helicity amplitude $\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}$ in terms of the Jacob-Wick formula as follows

$$\begin{aligned}\mathcal{M}^{JL}(A\to BC)=&\frac{\sqrt{2L+1}}{2J_A+1}\sum_{M_{J_B},M_{J_C}}\langle L0JM_{J_A}|J_AM_{J_A}\rangle\\ &\times\langle J_BM_{J_B}J_CM_{J_C}|J,JM_{J_A}\rangle\\ &\times\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}(\vec{p}),\end{aligned} \quad (3)$$

where $\vec{J}=\vec{J_B}+\vec{J_C}$, $\vec{J_A}=\vec{J_B}+\vec{J_C}+\vec{L}$ and $M_{J_A}=M_{J_B}+M_{J_C}$. The helicity amplitude reads

$$\begin{aligned}&\delta^3(\vec{p}_B+\vec{p}_C)\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}\\ &=\sqrt{8E_AE_BE_C}\gamma\sum_{\substack{M_{L_A},M_{S_A},\\ M_{L_B},M_{S_B},\\ M_{L_C},M_{S_C},m}}\langle L_AM_{L_A}S_AM_{S_A}|J_AM_{J_A}\rangle\\ &\times\langle L_BM_{L_B}S_BM_{S_B}|J_BM_{J_B}\rangle\langle L_CM_{L_C}S_CM_{S_C}|J_CM_{J_C}\rangle\\ &\times\langle 1m;1-m|00\rangle\langle\chi^{13}_{S_BM_{S_B}}\chi^{24}_{S_CM_{S_C}}|\chi^{12}_{S_AM_{S_A}}\chi^{34}_{1-m}\rangle\\ &\times\langle\varphi^{13}_B\varphi^{24}_C|\varphi^{12}_A\varphi^{34}_0\rangle I^{M_{L_A},m}_{M_{L_B},M_{L_C}}(\vec{p}),\end{aligned} \quad (4)$$

where $\vec{p}_B$ and $\vec{p}_C$ ($\vec{p}_B$=-$\vec{p}_C$=$\vec{p}$) are the momenta of the final mesons B and C in A's center-of-mass frame, respectively. γ is a phenomenological parameter, which indicates the strength of the quark-pair creation from the vacuum. $I^{M_{L_A},m}_{M_{L_B},M_{L_C}}(\vec{p})$ is a momentum integral

$$\begin{aligned}I^{M_{L_A},m}_{M_{L_B},M_{L_C}}(\vec{p})=&\int{\rm d}\vec{k}_1{\rm d}\vec{k}_2{\rm d}\vec{k}_3{\rm d}\vec{k}_4\\ &\times\delta^3(\vec{k}_1+\vec{k}_2-\vec{p}_A)\delta^3(\vec{k}_3+\vec{k}_4)\\ &\times\delta^3(\vec{p}_B-\vec{k}_1-\vec{k}_3)\delta^3(\vec{p}_C-\vec{k}_2-\vec{k}_4)\\ &\times\Psi^*_{n_BL_BM_{L_B}}(\vec{k}_{13})\Psi^*_{n_cL_CM_{L_c}}(\vec{k}_{24})\\ &\times\Psi_{n_AL_AM_{LA}}(\vec{k}_{12})Y_{1m}\left(\vec{k}_{34}\right)\end{aligned} \quad (5)$$

with a relative momentum of quark i and quark j: $\vec{k}_{ij}=\frac{m_j\vec{k}_i-m_i\vec{k}_j}{m_i+m_j}$ $(i,j=1,2,3,4)$. $\Psi_{nLM_L}(\vec{k})$ are simple harmonic oscillator (SHO) wave functions for the mesons, and $y_{1m}(\vec{k})=|\vec{k}|Y_{1m}(\Omega)$ is the solid harmonic polynomial of the created P-wave quark pair.

In terms of $9j$ symbols [29], the flavor matrix element reads

$$\langle\varphi_B^{13}\varphi_C^{24}|\varphi_A^{12}\varphi_0^{34}\rangle=\sum_{I,I^3}\langle I_C I_C^3;I_B I_B^3|I_A,I_A^3\rangle \times[(2I_B+1)(2I_C+1)(2I_A+1)]^{1/2} \times\begin{Bmatrix} I_1 & I_3 & I_B\\ I_2 & I_4 & I_C\\ I_A & 0 & I_A\end{Bmatrix} \quad (6)$$

where I_i $(i=1,2,3,4)$ is the isospin of the four u, d, s or c quark. I_A, I_B and I_C are the isospins of the mesons A, B and C, respectively. I_A^3, I_B^3 and I_C^3 are the third isospin components of the mesons, and $\langle I_C I_C^3;I_B I_B^3|I_A,I_A^3\rangle$ is the isospin matrix element.

The spin matrix element reads

$$\left\langle\chi^{13}_{S_B M_{S_B}}\chi^{24}_{S_C M_{S_C}}|\chi^{12}_{S_A M_{S_A}}\chi^{34}_{1-m}\right\rangle =(-1)^{S_C+1}[3(2S_B+1)(2S_C+1)(2S_A+1)]^{1/2} \times\sum_{S,M_S}\langle S_B M_{S_B} S_C M_{S_C}|S M_S\rangle\langle S M_S|S_A M_{S_A};1,-m\rangle \times\begin{Bmatrix} 1/2 & 1/2 & S_B\\ 1/2 & 1/2 & S_C\\ S_A & 1 & S\end{Bmatrix}. \quad (7)$$

With these formulae in hand, we proceed with our calculation. Constituent quark masses are model dependent and may be largely different in different constituent quark models. In many constituent quark models, u, d and s quarks approximately apply to $SU(3)$ flavor symmetry. In the Godfrey-Isgur quark model [2, 12], to solve the Hamiltonian to obtain the masses and wave functions of mesons, the masses of the u and d quarks were set small and the mass of the s quark was almost double the mass of the u and d quarks, which violates $SU(3)$ flavor symmetry. However, uncertainties of the constituent quark masses affect the partial decay widths little in the $^3P_0$ model [31]. In order to compare our results with those in Ref. [12], we have employed the same parameters. The masses of the constituent quarks are taken to be, $m_c=1628$ MeV, $m_u=m_d=220$ MeV, and $m_s=419$ MeV [12]. The masses and the effective scale parameters β of relevant mesons are presented in Table 2 [1, 12], where a common value $\beta=0.4$ GeV is employed for all light flavor mesons. For resonances included in PDG, their masses in PDG are employed (unlike those in Ref. [12]), while for resonances not included

Table 2. Masses and β values of mesons (MeV).

states	mass	β	states	mass	β
$(1^1S_0)\mathrm{K}^{\pm}$	493.677	400	$(1P_1)\mathrm{K}_1(1270)^{0(\pm)}$	1272	400
$(1^1S_0)\mathrm{K}^0$	497.614	400	$(1P_1')\mathrm{K}_1(1400)^{0(\pm)}$	1403	400
$(1^3S_1)\mathrm{K}^{*\pm}$	891.66	400	$(1^3P_2)\mathrm{K}_2^*(1430)^0$	1432.4	400
$(1^3S_1)\mathrm{K}^{*0}$	895.81	400	$(1^3P_2)\mathrm{K}_2^*(1430)^{\pm}$	1425.6	400
$(1^1S_0)\eta$	547.862	400	$(1^3P_0)\mathrm{K}_0^*(1430)^{0(\pm)}$	1425	400
$(1^3S_1)\phi$	1019.461	400	$(1P_1)\mathrm{D}_1(2430)^{0(\pm)}$	2427	475
$(1^1S_0)\mathrm{D}^{\pm}$	1869.61	601	$(1P_1')\mathrm{D}_1(2420)^0$	2421.4	475
$(1^1S_0)\mathrm{D}^0$	1864.84	601	$(1P_1')\mathrm{D}_1(2420)^{\pm}$	2423.2	475
$(1^3S_1)\mathrm{D}^{*\pm}$	2010.26	516	$(1^3P_0)\mathrm{D}_0(2400)^0$	2318	516
$(1^3S_1)\mathrm{D}^{*0}$	2006.96	516	$(1^3P_0)\mathrm{D}_0(2400)^{\pm}$	2403	516
$(1^1S_0)\mathrm{D_s}$	1968.30	651	$(1^3P_2)\mathrm{D}_2(2460)^{\pm}$	2464.3	437
$(1^3S_1)\mathrm{D_S^*}$	2112.1	562	$(1^3P_2)\mathrm{D}_2(2460)^0$	2462.6	437
$(2^1S_0)\mathrm{K}(1460)^{0(\pm)}$	1460	400	$(2^3S_1)\mathrm{D}^*(2600)^0$	2608.7	434
$(2^3S_1)\mathrm{K}^*(1410)^{0(\pm)}$	1414	400	$(2^3S_1)\mathrm{D}^*(2600)^+$	2621.3	434
$(2^1S_0)\mathrm{D}(2550)^{0(\pm)}$	2539.4	450	$(1^3\mathrm{D}_1)\mathrm{D}_1{}^*(2760)^0$	2763.3	456
$(1^3P_0)\mathrm{D_{s0}}(2317)$	2317.7	542	$(1^3\mathrm{D}_1)\mathrm{D}_1{}^*(2760)^+$	2769.7	456
$(1P_1)\mathrm{D_{s1}}(2460)$	2459.5	498	$(1^3D_3)\mathrm{D}_3^*(2760)^0$	2763.3	407
$(1P_1')\mathrm{D_{s1}}(2536)$	2535.10	498	$(1^3D_3)\mathrm{D}_3^*(2760)^+$	2769.7	407
$(1^3P_2)\mathrm{D_{s2}}(2573)$	2571.9	464	$(1^1\mathrm{D}_2)\mathrm{D}(2750)^0$	2752.4	428
$(2^3S_1)\mathrm{D^*{}_{s1}}(2700)$	2708.3	458	$(1^1\mathrm{D}_2)\mathrm{D}(2750)^+$	2752.4	428
$(1^3\mathrm{D}_1)\mathrm{D^*{}_{s1}}(2860)$	2859	469	$(1^3\mathrm{D}_2)\mathrm{D}(2750)^0$	2752.4	428
$(1^3\mathrm{D}_3)\mathrm{D^*{}_{s3}}(2860)$	2860.5	426	$(1^3\mathrm{D}_2)\mathrm{D}(2750)^+$	2752.4	428

in PDG or not observed by experiment, their masses and β are taken from Ref. [12]. $\gamma=6.95$ is employed in this paper, where γ is $\sqrt{96\pi}$ times as the $\gamma=0.4$ adopted in Ref. [12] for a different definition. The meson flavor follows the convention [12]: $D^0=c\bar{u}$, $D^+=-c\bar{d}$, ${D_s}^+=-c\bar{s}$, $K^+=-u\bar{s}$, $K^-=s\bar{u}$, $\phi=-s\bar{s}$, and $\eta=(u\bar{u}-d\bar{d})/2-s\bar{s}/\sqrt{2}$.

3 Hadronic decays

3.1 $D_s(2650)$

Hadronic decays of the $J^P=0^-$ radial excitation $D_s(2^1S_0)$ have been explored in Refs. [12, 25]. There are two possible hadronic decay channels, and the partial decay widths of the $J^P=0^-$ radial excitation D_s with $M=2650$ MeV ($D_s(2650)$) were calculated in the $^3P_0$ model [25]:

$$\Gamma(D_s(2^1S_0)\to D^*K)=78 \text{ MeV},$$
$$\Gamma(D_s(2^1S_0)\to {D^*}_s\eta)=0.$$

In fact, the D^*K decay channel is the only hadronic decay of $D_s(2650)$.

$D_s(2650)$ may be produced from hadronic decays of higher excited D mesons in the $D_s(2650)K$ channel or from hadronic decays of higher excited D_s mesons in the $D_s(2650)\eta$ channel. Possible hadronic decay channels from higher excited D and D_s mesons are presented in the first and the third columns in Table 3, and relevant hadronic partial decay widths have been estimated and presented in the second and the fourth columns.

Table 3. Hadronic partial decay widths of $D_s(2650)$ from highly excited resonances (MeV).

mode	width	mode	width
$D(4^3S_1)\to D_s(2650)K$	0.1	$D_s(4^3S_1)\to D_s(2650)\eta$	0.0
$D(5^3S_1)\to D_s(2650)K$	0.0	$D_s(5^3S_1)\to D_s(2650)\eta$	0.0
$D(3^3P_0)\to D_s(2650)K$	1.8	$D_s(3^3P_0)\to D_s(2650)\eta$	0.1
$D(3^3P_2)\to D_s(2650)K$	2.8	$D_s(3^3P_2)\to D_s(2650)\eta$	2.4
$D(4^3P_0)\to D_s(2650)K$	0.2	$D_s(4^3P_0)\to D_s(2650)\eta$	0.0
$D(4^3P_2)\to D_s(2650)K$	0.0	$D_s(4^3P_2)\to D_s(2650)\eta$	0.2
$D(2^3D_1)\to D_s(2650)K$	6.1	$D_s(2^3D_1)\to D_s(2650)\eta$	4.7
$D(2^3D_3)\to D_s(2650)K$	0.0	$D_s(2^3D_3)\to D_s(2650)\eta$	0.1
$D(3^3D_1)\to D_s(2650)K$	0.0	$D_s(3^3D_1)\to D_s(2650)\eta$	0.5
$D(3^3D_3)\to D_s(2650)K$	1.4	$D_s(3^3D_3)\to D_s(2650)\eta$	1.0
$D(1^3F_2)\to D_s(2650)K$	−−	$D_s(1^3F_2)\to D_s(2650)\eta$	0.0
$D(2^3F_2)\to D_s(2650)K$	4.5	$D_s(2^3F_2)\to D_s(2650)\eta$	3.0
$D(2^3F_4)\to D_s(2650)K$	0.6	$D_s(2^3F_4)\to D_s(2650)\eta$	0.8
$D(1^3G_3)\to D_s(2650)K$	0.6	$D_s(1^3G_3)\to D_s(2650)\eta$	0.6
$D(1^3G_5)\to D_s(2650)K$	0.0	$D_s(1^3G_5)\to D_s(2650)\eta$	0.0
$D(2^3G_3)\to D_s(2650)K$	1.6	$D_s(2^3G_3)\to D_s(2650)\eta$	1.0
$D(2^3G_5)\to D_s(2650)K$	0.7	$D_s(2^3G_5)\to D_s(2650)\eta$	1.0

Obviously, all the hadronic partial decay widths to $D_s(2650)K$ or $D_s(2650)\eta$ channels are very small. Therefore, these decay channels are difficult to observe even though those higher resonances may be observed in future experiments. D(2550) has been observed in inclusive e^+e^- and pp collisions, and ${D^*}_{1,3}(2760)$ has been observed in exclusive B decays. $D_s(2650)$ is naturally anticipated to be observed in these inclusive e^+e^- and pp collisions, or exclusive B decays in the D^*K channel.

3.2 Hadronic decay of $D_s(3P)$ resonances

From Table 3, $D_s(2650)$ can be produced from hadronic decays of $D_s(3^3P_{0,2})$ in the $D_s(2650)\eta$ channel. However, the branching fraction ratio is unknown, and it is not clear whether it is possible to observe $D_s(2650)$ in hadronic production from $D_s(3^3P_{0,2})$ in experiment. In order to have an overall picture on the hadronic decays of $D_s(3P)$, especially of their branching fractions, all the hadronic decay channels of $D_s(3P)$ have been studied in detail in this subsection. Many hadronic partial decay widths and branching ratios of $D_s(3P)$ have been calculated and presented in Tables XXX-XXXI in Ref. [12], but some decay channels were ignored.

In $D_s(3P)$ multiplets, $D_s(3^3P_1)$ and $D_s(3^1P_1)$ may mix with each other. Because of lack of information, the detail of the mixing of $D_s(3^3P_1)$ with $D_s(3^1P_1)$ is not clear. Therefore, the OZI-allowed hadronic decay channels of $D_s(3^3P_0)$, $D_s(3^1P_1)$, $D_s(3^3P_1)$ and $D_s(3^3P_2)$ were studied separately.

The masses of $D_s(3P)$ are employed as follows [12]: $M(D_s(3^3P_0))=3412$ MeV, $M(D_s(3^1P_1))=M(D_s(3^3P_1))=3425$ MeV, $M(D_s(3^3P_2))=3439$ MeV. The masses of $D_s(3^1P_1)$ and $D_s(3^3P_1)$ are assumed equal to the mean mass value of $D_s(3P_1)$ and $D_s(3P_1')$ in Ref. [12]. The final results of the hadronic decay widths are given in Tables 4-7. All the four $D_s(3P)$ resonances have large total decay widths, which are much larger than those in Ref. [12]. One reason is that the mixing has not been taken into account in our calculations, the other and the most important reason is that more decay channels have been included in our calculations. Our partial decay widths would be the same as those in Ref. [12] for the channels without mixing if $M=2673$ MeV was employed.

From Table 4, $\Gamma_{\text{total}}(D_s(3^3P_0))=175.5$ MeV (103.9 MeV in Ref. [12]) and the dominant decay channels of $D_s(3^3P_0)$ are: $(1^1D_2)D(2750)K$, DK, D(2550)K and $D_1(2420)K^*$.

From Table 5, $\Gamma_{\text{total}}(D_s(3^1P_1))=260.4$ MeV and the dominant decay channels of $D_s(3^1P_1)$ are: ${D^*}_2(2460)K$, $D_3^*(2760)K$, $DK^*(1410)$ and ${D^*}_2(2460)K^*$.

From Table 6, $\Gamma_{\text{total}}(D_s(3^3P_1))=211.6$ MeV and the dominant decay channels of $D_s(3^3P_1)$ are: $D_1(2430)K^*$, $D_3^*(2760)K$, D(2600)K and $D_2^*(2460)K$.

Table 4. Hadronic decay widths of $D_s(3^3P_0)$ (MeV).

channels	width	channels	width
D^0K^+	10.5	$D_{s1}(2536)\eta$	0.3
D^+K^0	10.3	$D_1(2430)^0K^+$	7.8
$D_s\eta$	0.2	$D_1(2430)^+K^0$	7.6
$D(2550)^0K^+$	9.8	$D_{s1}(2460)\eta$	0.2
$D(2550)^+K^0$	9.7	$D_1(2420)^0K^{*+}$	8.1
$D^0K(1460)^+$	5.5	$D_1(2420)^+K^{*0}$	8.2
$D^+K(1460)^0$	6.9	$D_1(2430)^0K^{*+}$	5.7
$D^{*0}K^{*+}$	3.2	$D_1(2430)^+K^{*0}$	5.7
$D^{*+}K^{*0}$	2.8	$D^{*0}K_1(1270)^+$	1.1
$D_s{}^*\phi$	4.0	$D^{*+}K_1(1270)^0$	1.1
$D_0^*(2400)^0K^{*+}$	0.6	$D^{*0}K_1(1400)^+$	0.3
$D_0^*(2400)^+K^{*0}$	2.2	$D_2^*(2460)^0K^{*+}$	3.0
$D_{s0}^*(2317)\phi$	2.0	$D_2^*(2460)^+K^{*0}$	2.5
$D^0K_1(1270)^+$	0.8	$(1^1D_2)D(2750)^0K^+$	19.3
$D^+K_1(1270)^0$	0.6	$(1^1D_2)D(2750)^+K^0$	18.8
$D^0K_1(1400)^+$	1.5	$(1^3D_2)D(2750)^0K^+$	6.7
$D^+K_1(1400)^0$	1.7	$(1^3D_2)D(2750)^+K^0$	6.5
$D_1(2420)^0K^+$	0.1	$(2^1S_0)D_s(2650)\eta$	0.1
$D_1(2420)^+K^0$	0.1	Γ_{total}	175.5

Table 5. Hadronic decay widths of $D_s(3^1P_1)$ (MeV).

channels	width	channels	width
D^0K^{*+}	3.9	$D_{s1}(2460)\eta$	0.0
D^+K^{*0}	3.7	$D_1(2420)^0K^{*+}$	7.9
$D_s\phi$	0.1	$D_1(2420)^+K^{*0}$	8.1
$D^0K^*(1410)^+$	13.0	$D_1(2430)^0K^{*+}$	9.3
$D^+K^*(1410)^0$	12.4	$D_1(2430)^+K^{*0}$	9.3
$D^{*0}K^+$	7.7	$D^{*0}K_1(1270)^+$	8.1
$D^{*+}K^0$	7.6	$D^{*+}K_1(1270)^0$	8.4
$D_s{}^*\eta$	0.2	$D^{*0}K_1(1400)^+$	2.6
$D^{*0}K^{*+}$	2.7	$D^{*+}K_1(1400)^0$	1.8
$D^{*+}K^{*0}$	2.4	$D^0K_2^*(1430)^+$	3.1
$D_s{}^*\phi$	2.0	$D^+K_2^*(1430)^0$	3.2
$D^{*0}K^*(1410)^+$	15.9	$D_2^*(2460)^0K^+$	14.6
$D^{*+}K^*(1410)^0$	4.5	$D_2^*(2460)^+K^0$	14.4
$D_0^*(2400)^0K^+$	2.5	$D_{s2}^*(2573)\eta$	3.9
$D_0^*(2400)^+K^0$	1.3	$D_2^*(2460)^0K^{*+}$	13.0
$D_{s0}^*(2317)\eta$	0.2	$D_2^*(2460)^+K^{*0}$	12.4
$D^0K_0^*(1430)^+$	1.4	$D^*(2600)^0K^+$	11.9
$D^+K_0^*(1430)^0$	1.6	$D^*(2600)^+K^0$	12.0
$D_0^*(2400)^0K^{*+}$	0.0	$(1^1D_2)D(2750)^0K^+$	0.0
$D_0^*(2400)^+K^{*0}$	0.0	$(1^1D_2)D(2750)^+K^0$	0.0
$D_{s0}^*(2317)\phi$	0.0	$(1^3D_2)D(2750)^0K^+$	0.0
$D^0K_1(1270)^+$	0.0	$(1^3D_2)D(2750)^+K^0$	0.0
$D^+K_1(1270)^0$	0.0	$D_1{}^*(2760)^0K^+$	0.7
$D^0K_1(1400)^+$	0.0	$D_1{}^*(2760)^+K^0$	0.7
$D^+K_1(1400)^0$	0.0	$D_3^*(2760)^0K^+$	14.4
$D_1(2420)^0K^+$	0.0	$D_3^*(2760)^+K^0$	13.1
$D_1(2420)^+K^0$	0.0	$D_{s1}^*(2860)\eta$	0.5
$D_{s1}(2536)\eta$	0.0	$D_{s3}^*(2860)\eta$	0.0
$D_1(2430)^0K^+$	0.0	$D_{s1}^*(2700)\eta$	3.5
$D_1(2430)^+K^0$	0.4	Γ_{total}	260.4

Table 6. Hadronic decay widths of $D_s(3^3P_1)$ (MeV).

channels	width	channels	width
D^0K^{*+}	4.1	$D_{s1}(2460)\eta$	0.0
D^+K^{*0}	3.8	$D_1(2420)^0K^{*+}$	4.5
$D_s\phi$	0.2	$D_1(2420)^+K^{*0}$	4.6
$D^0K^*(1410)^+$	7.2	$D_1(2430)^0K^{*+}$	11.9
$D^+K^*(1410)^0$	6.6	$D_1(2430)^+K^{*0}$	12.0
$D^{*0}K^+$	8.5	$D^{*0}K_1(1270)^+$	7.6
$D^{*+}K^0$	8.3	$D^{*+}K_1(1270)^0$	8.0
$D_s{}^*\eta$	0.1	$D^{*0}K_1(1400)^+$	2.3
$D^{*0}K^{*+}$	3.7	$D^{*+}K_1(1400)^0$	1.7
$D^{*+}K^{*0}$	3.3	$D^0K_2^*(1430)^+$	2.5
$D_s{}^*\phi$	2.0	$D^+K_2^*(1430)^0$	2.8
$D^{*0}K^*(1410)^+$	0.0	$D_2^*(2460)^0K^+$	9.1
$D^{*+}K^*(1410)^0$	0.0	$D_2^*(2460)^+K^0$	9.0
$D_0^*(2400)^0K^+$	0.0	$D_{s2}^*(2573)\eta$	2.6
$D_0^*(2400)^+K^0$	0.0	$D_2^*(2460)^0K^{*+}$	3.6
$D_{s0}^*(2317)\eta$	0.0	$D_2^*(2460)^+K^{*0}$	3.2
$D^0K_0^*(1430)^+$	0.0	$D^*(2600)^0K^+$	9.8
$D^+K_0^*(1430)^0$	0.0	$D^*(2600)^+K^0$	9.3
$D_0^*(2400)^0K^{*+}$	0.3	$(1^1D_2)D(2750)^0K^+$	3.4
$D_0^*(2400)^+K^{*0}$	1.7	$(1^1D_2)D(2750)^+K^0$	3.4
$D_{s0}^*(2317)\phi$	1.8	$(1^3D_2)D(2750)^0K^+$	6.7
$D^0K_1(1270)^+$	0.0	$(1^3D_2)D(2750)^+K^0$	6.5
$D^+K_1(1270)^0$	0.0	$D_1{}^*(2760)^0K^+$	0.1
$D^0K_1(1400)^+$	1.2	$D_1{}^*(2760)^+K^0$	0.1
$D^+K_1(1400)^0$	1.4	$D_3^*(2760)^0K^+$	12.0
$D_1(2420)^0K^+$	1.7	$D_3^*(2760)^+K^0$	11.0
$D_1(2420)^+K^0$	1.6	$D_{s1}^*(2860)\eta$	0.3
$D_{s1}(2536)\eta$	0.2	$D_{s3}^*(2860)\eta$	0.0
$D_1(2430)^0K^+$	1.8	$D_{s1}^*(2700)\eta$	2.3
$D_1(2430)^+K^0$	1.8	Γ_{total}	211.6

From Table 7, $\Gamma_{\text{total}}(D_s(3^3P_2)) = 307.9$ MeV (138.4 MeV in Ref. [12]) and the dominant decay channels of $D_s(3^3P_2)$ are: $D^*K^*(1410)$, $D^*{}_2(2460)K^*$, $DK^*(1410)$ and $D(2600)K$.

$D_s(2650)$ cannot be produced from hadronic decay of $D_s(3^{1,3}P_1)$, but it may be produced from hadronic decays of $D_s(3^3P_{0,2})$ in $D_s(2650)\eta$ channels. The partial decay width $\Gamma_{D_s(3^3P_0)\to D_s(2650)\eta}=0.1$ MeV and the branching fraction $\Gamma_{D_s(3^3P_0)\to D_s(2650)\eta}/\Gamma_{\text{total}}=0.06\%$. The partial decay width $\Gamma_{D_s(3^3P_2)\to D_s(2650)\eta}=2.4$ MeV, and the branching fraction $\Gamma_{D_s(3^3P_2)\to D_s(2650)\eta}/\Gamma_{\text{total}}=0.78\%$. Obviously, partial decay widths and their branching fraction ratios for hadronic production of $D_s(2650)$ from higher excited $D_s(3^3P_{0,2})$ are too small to be observed in experiment. In summary, it is really difficult to observe the $D_s(2650)\eta$ channel from hadronic decays of $D_s(3P)$.

4 Discussion and conclusions

The missing $J^P=0^-$ radial excitation $D_s(2650)$ is anticipated to have mass 2650 MeV. The D^*K channel

Table 7. Hadronic decay widths of $D_s(3^3P_2)$ (MeV).

channels	width	channels	width
D^0K^+	1.0	$D^{*0}K_0^*(1430)^+$	0.3
D^+K^0	1.1	$D^{*+}K_0^*(1430)^0$	0.2
$D_s\eta$	0.3	$D_1(2420)^0K^{*+}$	4.1
$D(2550)^0K^+$	2.6	$D_1(2420)^+K^{*0}$	4.3
$D(2550)^+K^0$	2.7	$D_1(2430)^0K^{*+}$	3.1
$D^0K(1460)^+$	5.3	$D_1(2430)^+K^{*0}$	3.0
$D^+K(1460)^0$	5.0	$D^{*0}K_1(1270)^+$	3.1
D^0K^{*+}	2.3	$D^{*+}K_1(1270)^0$	3.2
D^+K^{*0}	2.2	$D^{*0}K_1(1400)^+$	1.8
$D_s\phi$	0.0	$D^{*+}K_1(1400)^0$	1.6
$D^0K^*(1410)^+$	11.1	$D^0K_2^*(1430)^+$	1.1
$D^+K^*(1410)^0$	10.9	$D^+K_2^*(1430)^0$	1.0
$D(2550)^0K^{*+}$	0.0	$D_2^*(2460)^0K^+$	6.5
$D(2550)^+K^{*0}$	0.0	$D_2^*(2460)^+K^0$	6.5
$D^{*0}K^+$	3.8	$D_{s2}^*(2573)\eta$	1.8
$D^{*+}K^0$	3.8	$D^{*0}K_2^*(1430)^+$	1.6
$D_s{}^*\eta$	0.3	$D_2^*(2460)^0K^{*+}$	25.0
$D^{*0}K^{*+}$	4.8	$D_2^*(2460)^+K^{*0}$	24.9
$D^{*+}K^{*0}$	4.4	$D^*(2600)^0K^+$	7.1
$D_s{}^*\phi$	1.7	$D^*(2600)^+K^0$	7.6
$D^{*0}K^*(1410)^+$	45.2	$(1^1D_2)D(2750)^0K^+$	2.0
$D^{*+}K^*(1410)^0$	44.5	$(1^1D_2)D(2750)^+K^0$	2.0
$D_0^*(2400)^0K^{*+}$	0.1	$(1^3D_2)D(2750)^0K^+$	0.4
$D_0^*(2400)^+K^{*0}$	1.1	$(1^3D_2)D(2750)^+K^0$	0.4
$D_{s0}^*(2317)\phi$	1.3	$D_1{}^*(2760)^0K^+$	0.2
$D^0K_1(1270)^+$	2.9	$D_1{}^*(2760)^+K^0$	0.2
$D^+K_1(1270)^0$	2.7	$D_3^*(2760)^0K^+$	4.7
$D^0K_1(1400)^+$	1.6	$D_3^*(2760)^+K^0$	4.3
$D^+K_1(1400)^0$	1.5	$D_s(2650)\eta$	2.4
$D_1(2420)^0K^+$	1.9	$D_{s1}^*(2860)\eta$	0.0
$D_1(2420)^+K^0$	1.8	$D_{s3}^*(2860)\eta$	0.0
$D_{s1}(2536)\eta$	0.1	$D_{s1}^*(2700)\eta$	3.0
$D_1(2430)^0K^+$	4.9	$\Gamma_{\rm total}$	307.9
$D_1(2430)^+K^0$	4.9		
$D_{s1}(2460)\eta$	2.7		

is the only hadronic decay channel of $D_s(2650)$. $D_s(2650)$ may be produced from some higher excited D mesons in the hadronic $D_s(2650)K$ channel, and it may be produced from D_s mesons in the hadronic $D_s(2650)\eta$ channel. The relevant hadronic partial decay widths are very small, which implies that $D_s(2650)$ is difficult to observe in hadronic decay channels from those higher excited D or D_s mesons. $D_s(2650)$ is anticipated to be observed in inclusive e^+e^- and pp collisions, or exclusive B decays in the D^*K channel.

Four $D_s(3P)$ resonances have large total decay widths and many hadronic decay channels, but $D_s(2650)$ seems impossible to observe in hadronic decays of $D_s(3P)$. $D_s(2650)$ cannot be produced from hadronic decay of $D_s(3^{1,3}P_1)$, but it may be produced from hadronic decays of $D_s(3^3P_{0,2})$ in the $D_s(2650)\eta$ channels. The partial decay width $\Gamma_{D_s(3^3P_0)\to D_s(2650)\eta}=0.1$ MeV and the branching fraction $\Gamma_{D_s(3^3P_0)\to D_s(2650)\eta}/\Gamma_{\rm total}=0.06\%$. The partial decay width $\Gamma_{D_s(3^3P_2)\to D_s(2650)\eta}=2.4$ MeV, and the branching fraction $\Gamma_{D_s(3^3P_2)\to D_s(2650)\eta}/\Gamma_{\rm total}=0.78\%$. Both the partial decay widths and the branching fraction ratios are very small, which indicates that it is difficult to observe $D_s(2650)$ in the hadronic decays of $D_s(3^3P_{0,2})$.

The dominant decay channels of $D_s(3^3P_0)$ are: $(1^1D_2)D(2750)K$, DK, $D(2550)K$ and $D_1(2420)K^*$. The dominant decay channels of $D_s(3^1P_1)$ are: $D^*{}_2(2460)K$, $D_3^*(2760)K$, $DK^*(1410)$ and $D^*{}_2(2460)K^*$. The dominant decay channels of $D_s(3^3P_1)$ are: $D_1(2430)K^*$, $D_3^*(2760)K$, $D(2600)K$ and $D^*{}_2(2460)K$. The dominant decay channels of $D_s(3^3P_2)$ are: $D^*K^*(1410)$, $D^*{}_2(2460)K^*$, $DK^*(1410)$ and $D(2600)K$. Therefore, $D_1(2420)$, $D_1(2430)$, $D^*{}_2(2460)$, $D(2550)$, $D(2600)$, $(1^1D_2)D(2750)$ and $D_3^*(2760)$ are possible to observe in hadronic production from $D_s(3P)$, which requires more experiments.

References

1 C. Patrignani et al (Particle Data Group), Chin. Phys. C, **40**: 100001 (2016)
2 S. Godfrey and N. Isgur, Phys. Rev. D, **32**: 189 (1985)
3 P.del Amo Sanchez et al (BABAR Collaboration), Phys. Rev. D, **82**: 111101 (2010)
4 R. Aaij et al (LHCb collaboration), J. High Energy Phys., **09**: 145 (2013)
5 R. Aaij et al (LHCb collaboration), Phys. Rev. D, **91**: 092002 (2015)
6 R. Aaij et al (LHCb collaboration), Phys. Rev. D, **92**: 032002 (2015)
7 J. Brodzicka et al (Belle Collaboration), Phys. Rev. Lett., **100**: 092001 (2008)
8 B. Aubert et al (BaBar Collaboration), Phys. Rev. D, **80**: 092003 (2009)
9 R. Aaij et al (LHCb collaboration), Phys. Rev. D, **90**: 072003 (2014)
10 R. Aaij et al (LHCb Collaboration), Phys. Rev. Lett., **113**: 162001 (2014)
11 P. Colangelo, F. De Fazio, F. Giannuzzi, and S. Nicotri, Phys. Rev. D, **86**: 054024 (2012)
12 S. Godfrey and K. Moats, Phys. Rev. D, **93**: 034035 (2016)
13 Zhi-Feng Sun, Jie-Sheng Yu, Xiang Liu, and Takayuki Matsuki, Phys. Rev. D, **82**: 111501(R) (2010)
14 Xian-Hui Zhong, Phys. Rev. D, **82**: 114014 (2010)
15 Zhi-Gang Wang, Phys. Rev. D, **83**: 014009 (2011)
16 Bing Chen, Ling Yuan, and Ailin Zhang, Phys. Rev. D, **83**: 114025 (2011)
17 Bing Chen, Xiang Liu, and Ailin Zhang, Phys. Rev. D, **92**: 034005 (2015)
18 Jing Ge, Dan-Dan Ye, and Ailin Zhang, Eur. Phys. J. C, **75**: 178 (2015)
19 Ze Zhao, Yu Tian, and Ailin Zhang, Phys. Rev. D, **94**: 114035 (2016)
20 Hua-Xing Chen, Wei Chen, Xiang Liu, Yan-Rui Liu, and Shi-Lin Zhu, arXiv:1609.08928
21 Qin-Tao Song, Dian-Yong Chen, Xiang Liu, and Takayuki Matsuki, Phys. Rev. D, **92**: 074011 (2015)
22 Qin-Tao Song, Dian-Yong Chen, Xiang Liu, and Takayuki Matsuki, Phys. Rev. D, **91**: 054031 (2015)
23 A.V. Evdokimov et al (SELEX Collaboration), Phys. Rev. Lett., **93**: 242001 (2004)
24 B. Aubert et al (BABAR Collaboration), arXiv: hep-ex/0408087
25 S. Godfrey and K. Moats, Phys Rev. D, **90**: 117501 (2014)
26 L. Micu, Nucl. Phys. B, **10**: 521 (1969)
27 A. Le Yaouanc, L. Oliver, O. Pène, and J.C. Raynal, Phys. Rev. D, **8**: 2223 (1973); **9**, 1415 (1974); **11**, 1272 (1975)
28 A. Le Yaouanc, L. Oliver, O. Pène, and J.C. Raynal, Phys. Lett. B, **71**: 397 (1977); **72**, 57 (1977)
29 A. Le Yaouanc, L. Oliver, O. Pène, and J.C. Raynal, *Hadron Transitions in the Quark Model* (New York: Gordon and Breach Science Publishers, 1987)
30 Ze Zhao, Dan-Dan Ye, and Ailin Zhang, Phys. Rev. D, **94**: 114020 (2016)
31 Ling Yuan, Bing Chen, and Ailin Zhang, arXiv: 1203.0370