A formula for half-life of proton radioactivity *

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Abstract: We present a formula for proton radioactivity half-lives of spherical proton emitters with the inclusion of the spectroscopic factor. The coefficients in the formula are calibrated with the available experimental data. As an input to calculate the half-life, the spectroscopic factor that characterizes the important information on nuclear structure should be obtained with a nuclear many-body approach. This formula is found to work quite well, and in better agreement with experimental measurements than other theoretical models. Therefore, it can be used as a powerful tool in the investigation of proton emission, in particular for experimentalists.

Keywords: proton radioactivity, half-life, spectroscopic factor

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1 Introduction

The study of exotic nuclei far from the β-stable line has become one of the most important fields in current nuclear physics both experimentally and theoretically, because of the rapid development of radioactive beam facilities. Exotic nuclei may exhibit intriguing features that are different from the well-known stable nuclei, such as new shell closures [1]. Furthermore, the properties of these exotic nuclei play an important role in the nucleosynthesis of elements in explosive objects. Therefore, exotic nuclei with extreme numbers of protons or neutrons attract extensive interest in current nuclear physics. Nuclei with a large excess of protons could undergo spontaneous proton emission towards stability.

Proton emission was first observed in an isomeric state of ^{53}Co in 1970 [2]. As an important decay mode of unstable nuclei, it is useful to extract important information about nuclear structure, including the shell structure and the coupling between bound and unbound nuclear states [3]. Since the proton emission half-life is more sensitive to the decay energy Q and angular momentum transfer l than α-decay, one could extract nuclear structure information effectively by measured correlated quantities. In addition, proton emission from nuclear ground states or low-lying isomeric states can be treated as the inverse of the rapid proton capture process, which is of great importance in the understanding of the origin of the elements and the evolution of stars. At present, proton emission from ground states or low-lying isomeric states between $Z=51$ and $Z=83$ has been identified experimentally, and exploration of such radioactivity is continuing, since more new proton-rich nuclei will be synthesized in the future.

Theoretically, proton radioactivity is treated as a simple quantum tunneling phenomenon through a potential barrier, just as for the α-decay that was observed in early nuclear physics. Various theoretical methods have been applied to describe it and calculate the correlated quantities, such as the density-dependent M3Y (DDM3Y) effective interaction [4], the distorted-wave Born approximation [5], the Jeukenne, Lejeune and Mahaux (JLM) effective interaction [6], the generalized liquid drop model (GLDM) [7, 8], the single folded integral of the M3Y interaction [9, 10], the finite-range effective interaction of Yukawa form [11], the R-matrix approach [12], the Skyrme interactions [13], the relativistic density functionals [14], the Gamow-like model [15], and the phenomenological unified fission model [16, 17]. In particular, recently, Zhao et al. calculated the half-life of proton radioactivity for spherical proton emitters within the scheme of covariant density functional theory where the potential barrier that prevents the emitted proton is extracted with the similarity renormalization group method, and the spectroscopic factor is also extracted from the covariant density functional approach [18]. This

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method is more microscopic and self-consistent, which in turn promotes the development of nuclear many-body theory. In this study, we give an analytic formula that is more convenient for use in the investigation of proton radioactivity, in particular for experimentalists. The formula is the relation of decay energy $Q_{\rm p}$, half-life $T_{\rm p}$, angular momentum transfer l, and spectroscopic factor, where the parameters are fitted with the measured data of proton emission.

2 Method

In our previous work, we proposed a formula for proton emission half-life [7]

$$\log_{10}[T_{\rm p}({\rm s})] = a+bA^{1/6}\sqrt{Z}+cZQ^{-1/2} +d\frac{l(l+1)}{\sqrt{(A-1)(Z-1)A^{-2/3}}}, \tag{1}$$

where Z (A) is the charge (mass) number of the parent nucleus, and Q is the decay energy in MeV. The first two terms are similar to Royer's formulae [19] as a natural extension of the Geiger-Nuttall law for α-decay. The last term stems from the contribution of the centrifugal barrier due to the nonzero angular momentum transfer. However, in this equation, the spectroscopic factor is not included. The spectroscopic factor gives important information about nuclear structure, and is necessary for the accurate prediction of half-life. For spherical emitters, the spectroscopic factor can be calculated by $S_{\rm p}=u_j^2$ [5], where the u_j^2 is the probability that the spherical orbit of the emitted proton is empty in the corresponding daughter nucleus [5]. In Ref. [7], the detailed calculations were carried out in the framework of the nuclear many-body approach. Here, with the inclusion of the spectroscopic factor $S_{\rm p}$ and with the approximation $\sqrt{(A-1)(Z-1)A^{-2/3}}\simeq A^{1/6}Z^{1/2}$, we give the formula as follows:

$$\log_{10}[T_{\rm p}(s)]=a+bA^{1/6}Z^{1/2}+cZQ^{-1/2} +dl(l+1)A^{-1/6}Z^{-1/2}-\log_{10}S_{\rm p}. \tag{2}$$

3 Results and discussions

By performing a least-squares fit to the half-lives of the 26 spherical proton emitters that are presented in Table 1, with the spectroscopic factor $S_{\rm p}$ calculated by the relativistic mean field theory combined with the BCS

Table 1. Comparison between experimental and calculated proton radioactivity logarithmic half-lives of spherical proton emitters. Asterisks (*) by parent nuclei denote isomeric states. The experimental data are from Ref. [19, 20].

parent	l [7]	Q/MeV [7] expt.	$S_{\rm p}$ [7]	$\log_{10}T_{\rm p}(s)$ expt.	$\log_{10}T_{\rm p}(s)$ GLDM [7]	$\log_{10}T_{\rm p}(s)$ SRG [18]	$\log_{10}T_{\rm p}(s)$ DDM3Y [6]	$\log_{10}T_{\rm p}(s)$ JLM [6]	$\log_{10}T_{\rm p}(s)$ cal
^{105}Sb	2	0.491	0.999	2.049	1.831	–	2.27	1.69	2.171
^{145}Tm	5	1.753	0.580	−5.409	−5.656	–	−5.20	−5.10	−5.417
^{147}Tm	5	1.071	0.581	0.591	0.572	0.645	0.98	1.07	0.659
^{147}Tm*	2	1.139	0.953	−3.444	−3.350	−3.39	−3.26	−3.27	−3.429
^{150}Lu	5	1.283	0.497	−1.180	−1.309	−1.169	−0.59	−0.49	−1.208
^{150}Lu*	2	1.317	0.859	−4.523	−4.755	−4.553	−4.24	−4.24	−4.759
^{151}Lu	5	1.255	0.490	−0.896	−1.017	−0.869	−0.65	−0.55	−0.926
^{151}Lu*	2	1.332	0.858	−4.796	−4.913	−4.824	−4.72	−4.73	−4.919
^{155}Ta	5	1.453	0.422	−2.538	−2.410	−2.367	−4.67	−4.57	−2.314
^{156}Ta	2	1.028	0.761	−0.620	−0.642	−0.607	−0.22	−0.23	−0.698
^{156}Ta*	5	1.130	0.493	0.949	0.991	1.269	1.66	1.76	1.007
^{157}Ta	0	0.946	0.797	−0.523	−0.170	−0.420	−0.21	−0.23	−0.192
^{159}Re	5	1.816	0.308	−4.678	−4.636	–	–	–	−4.503
^{160}Re	2	1.284	0.507	−3.046	−3.111	−3.128	−2.86	−2.87	−3.175
^{161}Re	0	1.214	0.892	−3.432	−3.319	−3.481	−3.28	−3.29	−3.358
^{161}Re*	5	1.338	0.290	−0.488	−0.677	−0.539	−0.57	−0.49	−0.654
^{164}Ir	5	1.844	0.188	−3.959	−4.214	−4.066	−3.95	−3.86	−4.126
^{165}Ir*	5	1.733	0.187	−3.468	−3.460	−3.310	−3.52	−3.44	−3.388
^{166}Ir	2	1.168	0.415	−0.824	−1.099	−1.102	−0.96	−0.96	−1.202
^{166}Ir*	5	1.340	0.188	−0.076	−0.025	0.143	0.22	0.30	−0.037
^{167}Ir	0	1.086	0.912	−0.959	−1.074	−1.367	−1.05	−1.07	−1.149
^{167}Ir*	5	1.261	0.183	0.875	0.858	1.009	0.74	0.81	0.820
^{171}Au	0	1.469	0.848	−4.770	−4.872	−4.987	−4.84	−4.86	−4.971
^{171}Au*	5	1.718	0.087	−2.654	−2.613	−2.472	−3.03	−2.96	−2.596
^{177}Tl	0	1.180	0.733	−1.174	−1.049	−1.252	−1.17	1.20	−1.188
^{177}Tl*	5	1.986	0.022	−3.347	−3.471	−3.304	−4.52	−4.46	−3.454

method [7], we obtain a set of parameters listed in Table 2. The average deviation and root mean square deviation between the experimental and the calculated half-life are given by

$$\overline{\sigma}=\frac{1}{26}\sum_{i=1}^{26}\left|\log_{10}T_{1/2}^{\rm expt}(i)-\log_{10}T_{1/2}^{\rm cal}(i)\right|=0.105, \quad (3)$$

$$\sqrt{\overline{\sigma^2}}=\sqrt{\frac{1}{26}\sum_{i=1}^{26}\left[\log_{10}T_{1/2}^{\rm expt}(i)-\log_{10}T_{1/2}^{\rm cal}(i)\right]^2}=0.139. \quad (4)$$

The average deviations and root mean square deviations of other calculations, i.e., GLDM, DDM3Y, JLM, SRG, are listed in Table 3 for comparison. Because the spectroscopic factor $S_{\rm p}$ is introduced, the results of the GLDM, SRG and Eq. (2) calculations are much better than those of DDM3Y and JLM. Thus, consideration of $S_{\rm p}$ substantially improves the agreement between experimental and theoretical values, indicating the particular importance of the spectroscopic factor. The half-lives obtained by Eq. (2), with an average discrepancy of ~30%, give better agreement with the experimental data than those obtained by the GLDM and SRG from Table 3, although it is quite simple in formulism. Therefore, it can be employed as a useful tool to study proton emission and provide a reference for future experiments.

Table 2. The fitted coefficients of Eq. (2).

a	b	c	d
-20.8221	-0.5316	0.4150	2.3234

Table 3. The average deviations $\overline{\sigma}$ and root mean square deviations $\sqrt{\overline{\sigma^2}}$ of the calculations from GLDM, SRG, DDM3Y, JLM and Eq. (2) with respect to the experimental data.

model	$\overline{\sigma}$	$\sqrt{\overline{\sigma^2}}$
GLDM	0.121	0.153
SRG	0.124	0.162
DDM3Y	0.341	0.559
JLM	0.450	0.732
Eq. (2)	0.105	0.139

We would like to give a short discussion about the contribution of the centrifugal barrier in Eq. (2). According to an approximate derivation, the d coefficient is given as

$$d\approx\frac{2}{\sqrt{\frac{2\mu}{\hbar}e^2r_0}}/\ln 10,$$

where $\mu=(A-1)/A$ is the reduced mass of the proton emitter, and r_0 is the radius constant. With the typical value of $2\mu/\hbar=0.048$ and $r_0\approx1.3$, the d coefficient is $d\approx2.9$, which is not so different from the fitted value of 2.3234. This result indicates the validity of the contribution of the centrifugal barrier.

In order to further test the validity of Eq. (2), we define a quantity $\Delta=\log_{10}[T_{\rm p}^{\rm exp}(s)]-bA^{1/6}Z^{1/2}-dl(l+1)A^{-1/6}Z^{-1/2}+\log_{10}S_{\rm p}$. It is shown as a function of $ZQ^{-1/2}$ in Fig. 1. This satisfies a linear relationship quite well, which is supported by the fact that the correlation coefficient is as high as r=0.9985. Equation (2) reproduces the available experimental half-lives within a factor of about 30%. We plot the deviations between Eq. (2) and the experimental values in Fig. 2. For most cases, the deviation is located in the range of -0.2 to 0.2, which means that Eq. (2) works quite well. However, for ^{185}Bi, if the $S_{\rm p}$ calculated by the relativistic mean field theory combined with the BCS method is used, the deviation of the calculation with Eq. (2) is large, and also for other models. The underlying reason perhaps lies in the fact that its very small spectroscopic factor, at the end of a shell, is not calculated with a sufficiently high accuracy. Since Eq. (2) achieves a high accuracy for half-life calculation, it can be used to extract the spectroscopic factor combined with experimental measurements. As a consequence, the $S_{\rm p}$ of ^{185}Bi is estimated to be 0.051.

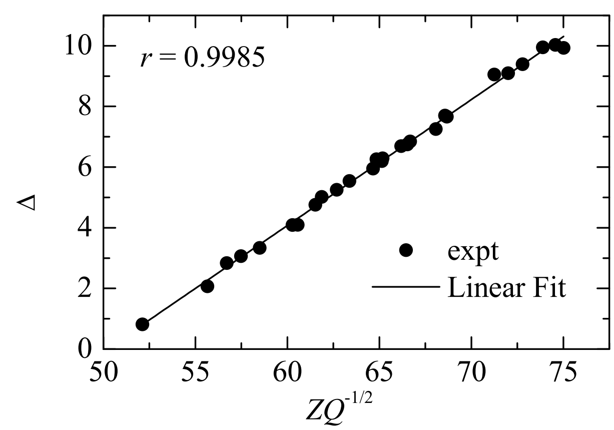

Fig. 1. Δ as a function of $ZQ^{-1/2}$, showing the linear relation.

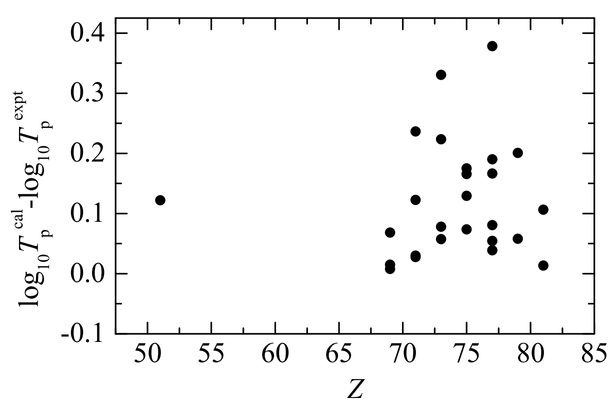

Fig. 2. Dimensionless deviation between Eq. (2) and experimental logarithm of half-lives for proton emission.

Very recently, proton emission from isomeric states in ^{151m}Lu has been reinvestigated experimentally [21], and the decay energy as well as the half-life were measured to be 1295(5) keV and 15.4(8) μs, respectively. With the decay energy and the relevant information in Table 1 as inputs, we obtain a half-life of $32.9^{+4.0}_{-3.6}$ μs, agreeing with the experimental measurement with a deviation of a factor of 2.

It should be stressed that, if the parent and daughter nuclei have distinctly different deformations, the spectroscopic factor S_p cannot be calculated by u_j^2 as for spherical nuclei, because the BCS ground state for a parent nucleus and the corresponding daughter nucleus cannot be regarded as the same. Accordingly, the overlap between their BCS ground states should be taken into account. To calculate such an overlap with sufficient accuracy is difficult.

4 Summary

With the inclusion of spectroscopic factor, a formula of half-lives for spherical proton radioactivity has been presented, and we have fitted the corresponding coefficients in the formula combined with the available experimental data. Here, the spectroscopic factor is an important input that should be obtained with nuclear many-body models. This formula is found to work quite well with the average discrepancy of 30%, and is in better agreement with experimental measurements than other existing theoretical models. Importantly, the formula is very convenient to study proton emission, and could be useful for future experiments.

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