Spectroscopy of the Ω_{ccb} baryon in the hypercentral constituent quark model

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Abstract: We extract the mass spectrum of the triply heavy baryon Ω_{ccb} using the hypercentral constituent quark model. The first order correction is also added to the potential term of the Hamiltonian. The radial and orbital excited state masses are determined, and the Regge trajectories and magnetic moments for this baryon are also given.

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1 Introduction

Last year, a number of particles were found in the heavy hadron sector. LHCb has determined the five excited states of the Ω_c baryon and the ground state of the Ξ_{cc}^{++} baryon [1, 2]. Latterly, Belle-II also confirmed the excited states of the Ω_c baryon [3]. Reviewing the heavy baryon sector, the ground states of singly heavy baryons and two doubly heavy baryons have been found experimentally, to date [4]. The excited state masses are well-known only for the singly heavy baryons in the charm sector [5, 6]. Now, we can expect such discovery of triply heavy baryons as well. The triply heavy baryon is a combination of three heavy quarks (c and/or b). These baryons are at the topmost layer of SU(3) flavor symmetry. The spectroscopic properties of triply charm $\Omega_{\rm ccc}$ and triply bottom $\Omega_{\rm bbb}$ baryons have already been presented in our recent work [7]. In this article, we discuss one of the triply heavy baryons, $\Omega_{\rm ccb}$. Many theoretical approaches have been used to determine the masses of this baryon, including the non-relativistic quark model [8], Faddeev approach [9], sum rules[10–12], bag model [13], di-quark model [14], lattice QCD [15], relativistic quark model [16], and variational Cornell model [17].

The hypercentral constituent quark model (hCQM) with color Coulomb plus linear potential has been used for systematic calculations of this baryon. Here, we have also incorporated the first order correction to the potential energy term of the Hamiltonian as it was in case of quarkonia [18]. The excited state mass spectra of heavy flavor baryons (singly, doubly and also some triply) were determined in our previous work using the same model [5, 7, 19–22]. Likewise, here, we have determined the mass spectrum of the $\Omega_{\rm ccb}$ baryon, which is a combination of two charm and one bottom quark. We also construct the Regge trajectories for these baryons in the (n, M^2) and (J, M^2) planes, where one can test several properties from the graph such as linearity, divergence, and parallelism. We also calculate the magnetic moment for the ground state mass. We calculate it for both $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ states.

This paper is organized as follows. A description of the hCQM is given in Section 2. A systematic mass spectroscopy calculation has been performed using this model and we analyze and discuss our results in Section 3. We also plot the Regge trajectories, and the magnetic moments are also determined. Finally, we draw conclusions in Section 4.

2 The model

The hCQM has successfully given the mass spectra of the baryons in the heavy sector [19–23]. We use the same methodology in this paper. A brief description of the hCQM model follows.

The relevant degrees of freedom for the motion of heavy quarks are related by the Jacobi coordinates ($\vec{\rho}$ and $\vec{\lambda}$), which are [24–26]

$$\vec{\rho} = \frac{1}{\sqrt{2}} (\vec{r_1} - \vec{r_2}),$$
 (1a)

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$$\vec{\lambda} = \frac{m_1 \vec{r_1} + m_2 \vec{r_2} - (m_1 + m_2) \vec{r_3}}{\sqrt{m_1^2 + m_2^2 + (m_1 + m_2)^2}},$$
 (1b)

Here m_i and $\vec{r_i}$ (i = 1, 2, 3) denote the mass and coordinates of the *i*-th constituent quark. Here, we only discuss the $\Omega_{\rm ccb}$ system, so (i=1, 2) for the c quark and (i=3) for the b quark.

The Hamiltonian of the system is defined as

$$H = \frac{P_x^2}{2m} + V(x), \qquad (2)$$

where m is the reduced mass and x is the six-dimensional radial hypercentral coordinate of the three-body system. The respective reduced masses are given by

$$m_{\rho} = \frac{2m_1m_2}{m_1 + m_2}, \tag{3a}$$

$$m_{\lambda} = \frac{2m_3(m_1^2 + m_2^2 + m_1 m_2)}{(m_1 + m_2)(m_1 + m_2 + m_3)},$$
 (3b)

In the center-of-mass frame $(R_{\text{c.m.}}=0)$, the kinetic energy operator can be written as

$$-\frac{\hbar^2}{2m}(\Delta_{\rho} + \Delta_{\lambda}) = -\frac{\hbar^2}{2m} \left(\frac{\partial^2}{\partial x^2} + \frac{5}{x}\frac{\partial}{\partial x} + \frac{L^2(\Omega)}{x^2}\right)$$
(4)

where $L^2(\Omega) = L^2(\Omega_{\rho}, \Omega_{\lambda}, \xi)$ is the quadratic Casimir operator of the six-dimensional rotational group O(6) and its eigenfunctions are the hyperspherical harmonics $Y_{[\gamma]l_{\rho}l_{\lambda}}(\Omega_{\rho}, \Omega_{\lambda}, \xi)$ satisfying the eigenvalue relation $L^2 Y_{[\gamma]l_{\rho}l_{\lambda}}(\Omega_{\rho}, \Omega_{\lambda}, \xi) = -\gamma(\gamma+4)Y_{[\gamma]l_{\rho}l_{\lambda}}(\Omega_{\rho}, \Omega_{\lambda}, \xi)$. Here, γ is the grand angular momentum quantum number.

The reduced six-dimensional hyperradial Schrödinger equation corresponding to Eq. (2) of the Hamiltonian can be written as

$$\left[\frac{-1}{2m}\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \frac{\frac{15}{4} + \gamma(\gamma + 4)}{2mx^2} + V(x)\right]\phi_\gamma(x) = E\phi_\gamma(x).$$
(5)

The hypercentral potential V(x) as the color Coulomb plus linear potential with first order correction as well as spin interaction is defined by [18, 27–29]

$$V(x) = V^{0}(x) + \left(\frac{1}{m_{\rho}} + \frac{1}{m_{\lambda}}\right) V^{(1)}(x) + V_{\rm SD}(x), \quad (6)$$

$$V^{(0)}(x) = \frac{\tau}{x} + \beta x \quad \& \quad V^{(1)}(x) = -C_{\rm F} C_{\rm A} \frac{\alpha_{\rm s}^2}{4x^2}, \quad (7)$$

$$V_{\rm SD}(x) = V_{\rm SS}(x)(\vec{S_{\rho}}.\vec{S_{\lambda}}) + V_{\gamma S}(x)(\vec{\gamma}.\vec{S}) + V_T(x) \left[S^2 - \frac{3(\vec{S}.\vec{x})(\vec{S}.\vec{x})}{x^2} \right].$$
(8)

Here, the hyper-Coulomb strength $\tau = -\frac{2}{3}\alpha_{\rm s}$ where $\alpha_{\rm s}$ corresponds to the strong running coupling constant; $\frac{2}{3}$ is the color factor for baryons, and β corresponds to the string tension for baryons. $C_{\rm F}$ and $C_{\rm A}$ are the Casimir charges of the fundamental and adjoint representation with values $\frac{2}{3}$ and 3. The spin-dependent part of Eq. (8),

 $V_{\rm SD}(x)$, contains three types of interaction terms, including the spin-spin term $V_{\rm SS}(x)$, the spin-orbit term $V_{\gamma \rm S}(x)$ and a tensor term $V_{\rm T}(x)$ [19]. We numerically solve the six-dimensional Schrödinger equation using a Mathematica notebook [30]. The values of quark masses are $m_{\rm c}{=}1.275$ GeV and $m_{\rm b}{=}4.67$ GeV in the calculation. We fix the ground state mass and after that we calculate radial and orbital excited state masses.

Table 1. Ground state masses of $\Omega_{\rm ccb}$ baryon (in GeV), without first-order correction (A) and with correction (B).

1P	our work		Refs.				
5	А	В	[14]	[15]	[16]	[17]	
$\frac{1}{2}^{+}$	8.005	8.005	8.005	8.007	8.018	8.018	
$\frac{3}{2}^{+}$	8.049	8.049	8.027	8.037	8.025	8.046	

3 Mass spectra, Regge trajectories and magnetic moments

The ground state (1S), radial excited states (2S-4S) and orbital excited states (1P-5P, 1D-4D, 1F-2F) are calculated for $\Omega_{\rm ccb}$ in this paper. We consider the total spin $S = \frac{3}{2}^+$ and find the possible number of states for P, D and F are 3, 4, and 4, respectively. The possible J^P values for S states are $J^P = \frac{1}{2}^+, \frac{3}{2}^+$; for P states are $J^P = \frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-$; for D states are $J^P = \frac{1}{2}^+, \frac{3}{2}^+, \frac{5}{2}^+, \frac{7}{2}^+$; and for F states are $J^P = \frac{3}{2}^-, \frac{5}{2}^-, \frac{7}{2}^-, \frac{9}{2}^-$. The masses are tabulated in Tables (1–3). The radial and orbital excited states are determined by adding a first order correction to the potential. A are the masses without adding the correction. Both results are given in Tables (2–3). We do not have many theoretical predictions of masses to compare with our outcomes, though available predictions are mentioned.

Table 2. Radial excited state masses of $\Omega_{\rm ccb}$ baryon (in GeV), without first-order correction (A) and with correction (B).

state	А	В	[8]
$2S(\frac{1}{2}^+)$	8.611	8.621	8.537
$3S(\frac{1}{2}^{+})$	9.203	9.224	
$4S(\frac{1}{2}^{+})$	9.792	9.823	
$5S(\frac{1}{2}^+)$	10.379	10.424	
$2S(\frac{3}{2}^+)$	8.627	8.637	8.553
$3S(\frac{3}{2}^{+})$	9.211	9.232	
$4S(\frac{3}{2}^{+})$	9.797	9.828	
$5S(\frac{3}{2}^+)$	10.421	10.424	

The 1S state is computed for both $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ in Table (1).

The obtained results are closer to the other theoretical predictions [14–17]. References [10] and [31] have also calculated the ground states with values 8.5 GeV and 8.23 GeV, respectively. These values are much higher than the results in Table 1.

Table 3. Orbital excited state masses of $\Omega_{\rm ccb}$ (in GeV), without first-order correction (A) and with correction (B).

state	А	В	[8]	[10]	[31]
$(1^4 P_{1/2})$	8.388	8.400	8.418		8.36
$(1^4 P_{3/2})$	8.372	8.383	8.420	8.35	8.36
$(1^4 P_{5/2})$	8.354	8.365	8.432		
$(2^4 P_{1/2})$	8.970	8.992			
$(2^4 P_{3/2})$	8.954	8.976			
$(2^4 P_{5/2})$	8.934	8.955			
$(3^4 P_{1/2})$	9.554	9.585			
$(3^4 P_{3/2})$	9.538	9.569			
$(3^4 P_{5/2})$	9.516	9.547			
$(4^4 P_{1/2})$	10.139	10.181			
$(4^4 P_{3/2})$	10.122	10.164			
$(4^4 P_{5/2})$	10.099	10.140			
$(5^4 P_{1/2})$	10.723	10.775			
$(5^4 P_{3/2})$	10.706	10.758			
$(5^4 P_{5/2})$	10.684	10.735			
$(1^4 D_{1/2})$	8.828	8.848			
$(1^4 D_{3/2})$	8.810	8.831			
$(1^4 D_{5/2})$	8.788	8.808	8.568		
$(1^4 D_{7/2})$	8.760	8.780			
$(2^4 D_{1/2})$	9.405	9.437			
$(2^4 D_{3/2})$	9.388	9.420			
$(2^4 D_{5/2})$	9.366	9.396			
$(2^4D_{7/2})$	9.338	9.368			
$(3^4 D_{1/2})$	9.988	10.031			
$(3^4 D_{3/2})$	9.970	10.012			
$(3^4 D_{5/2})$	9.947	9.988			
$(3^4 D_{7/2})$	9.918	9.957			
$(4^4 D_{1/2})$	10.571	10.622			
$(4^4 D_{3/2})$	10.553	10.605			
$(4^4 D_{5/2})$	10.529	10.582			
$(4^4 D_{7/2})$	10.499	10.553			
$(1^4 F_{3/2})$	9.250	9.280			
$(1^4 F_{5/2})$	9.225	9.254			
$(1^4 F_{7/2})$	9.193	9.222			
$(1^4 F_{9/2})$	9.156	9.184			
$(2^4F_{3/2})$	9.827	9.864			
$(2^4 F_{5/2})$	9.802	9.840			
$(2^4 F_{7/2})$	9.771	9.809			
$(2^4 F_{9/2})$	9.734	9.773			

38 Mev less, respectively, than our determined masses. The $1D(\frac{5}{2}^+)$ state value is 200 MeV higher than that of

Ref. [8]. As far as we know, we are first to determine the F states of this triply heavy $\Omega_{\rm ccb}$ baryon.

Regge trajectories

The calculated masses are used to plot the Regge trajectories for the triply heavy $\Omega_{\rm ccb}$ baryon in the $M^2 \rightarrow n$ and $M^2 \rightarrow J$ planes. We use

$$n = \beta M^2 + \beta_0, \tag{9}$$

$$J = \alpha M^2 + \alpha_0, \tag{10}$$

where β, α , and β_0, α_0 are the slope and intercept, respectively, and n = n - 1, where n is the principal quantum number. The values of β and β_0 are shown in Table 4 and the values of α and α_0 are shown in Table 5. As described in the previous section, we have calculated the masses of the S, P and D states which are used to construct Regge trajectories. The ground and radial excited states S with $J^P = \frac{1}{2}^+$ and the orbital excited state P with $J^P = \frac{1}{2}^-$, D with $J^P = \frac{5}{2}^+$ are plotted in Fig. 1 in the (n, M^2) plane. The Regge trajectories for natural and unnatural parities are drawn in Fig. 2 [32]. Straight lines were obtained by linear fitting in both figures. We observe that the square of the calculated masses fits very well to a linear trajectory and is almost parallel and equidistant in the S, P and D states. We can determine the possible quantum numbers and prescribe them to a particular Regge trajectory with the help of our obtained results.

Table 4. Fitted slope (β) and intercept (β_0) of the Regge trajectories.

baryon	J^P	state	β	β_0
$\Omega_{\rm ccb}$	$\frac{1}{2}^{+}$	S	$0.092{\pm}0.0017$	-5.816 ± 0.151
	$\frac{1}{2}^{-}$	P	$0.0894{\pm}0.0019$	-6.231 ± 0.177
	$\frac{5}{2}^{+}$	D	$0.089 {\pm} 0.0017$	-6.848 ± 0.169

Table 5. Fitted slope (α) and intercept (α_0) of the Regge trajectories.

baryon	J^P	state	α	$lpha_0$
$\Omega_{\rm ccb}$	$\frac{3}{2}^{+}$	S	$0.090 {\pm} 0.002$	-4.725 ± 0.191
	$\frac{5}{2}^{-}$	P	$0.090 {\pm} 0.002$	-5.218 ± 0.181
	$\frac{7}{2}^{+}$	D	$0.089 {\pm} 0.0017$	-5.835 ± 0.166

Magnetic moments

The orbital excited P, D and F states are given in Table 3. In the 1P state, Refs. [10, 31] are 28 MeV and The magnetic moments of baryons are obtained in terms of the spin, charge and effective mass of the bound quarks as [5, 23]

$$\mu_B \!=\! \sum_i \langle \phi_{sf} | \mu_{iz} | \phi_{sf} \rangle)$$

where

$$\mu_i = \frac{e_i \sigma_i}{2m_i^{\text{eff}}},\tag{11}$$

where e_i is a charge and σ_i is the spin of the respective constituent quark corresponding to the spin flavor wavefunction of the baryonic state. The effective mass for each of the constituent quarks m_i^{eff} can be defined as

$$m_i^{\text{eff}} = m_i \left(1 + \frac{\langle H \rangle}{\sum_i m_i} \right), \tag{12}$$

where $\langle H \rangle = E + \langle V_{\rm spin} \rangle$. The spin flavor wave function and magnetic moments of the baryon is given in Table 6.

Table 6. Magnetic moments (in nuclear magnetons).

baryons	function [33]	our result	[35]	[34]	[23]	[14]
$\Omega_{\rm ccb}^+$	$\frac{4}{3}\mu_{\rm c} - \frac{1}{3}\mu_{\rm b}$	0.606	0.505	0.522	0.502	
$\Omega_{ m ccb}^{*+}$	$\mu_{\rm b} + 2\mu_{\rm c}$	0.819	0.659	0.703		0.807



Fig. 1. (color online) Regge trajectory in the (n, M^2) plane.



(color online) Regge trajectories in the (J, M^2) plane for natural and unnatural parities. Fig. 2.

$\mathbf{4}$ Conclusion

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 $M^{2}(GeV^{2})$ 95

We have discussed the baryon with the combination of two charm quarks and one beauty quark. The $\Omega_{\rm ccb}$ baryon is unknown from the experimental side. The mass spectrum of the $\Omega_{\rm ccb}$ baryon has been listed using the hypercentral constituent quark model starting from S state to F state. We have also added the first order correction to the potential (see Eq. (6)). The ground states have already been studied by various models and the results for some of them are in same range as our predictions, although the excited states have been calculated by very few theorists. We also notice that first radial (2S) and orbital (1P) excited states masses are in the same range as other predictions, while for the higher excited state 1D, the difference is larger. The 2S state has only been determined by Roberts et al. [8] as far as we know. Their

3/2

values are 74 (84) MeV less for the 2S state, although, the difference between the isospin doublet states is 16 MeV in Ref. [8] and in a similar manner our 2S state masses $(\frac{3}{2}^+ - \frac{1}{2}^+)$ also have a 16 MeV difference. Thus, the obtained masses are very useful to obtain the resonances of higher excited states.

The first order correction for the excited state varies from 0.1% - 0.4% for 1S-4S and 1P-4P states; from 0.2%- 0.5% for 1D-4D states; and from 0.3% for F states. It is clear from the results that as we move towards the higher excited states, the contribution of the correction increases. It will be interesting to see the effect of this correction in the light sector baryons [36].

The paper also features the values of ground state magnetic moments of the $\Omega_{\rm ccb}$ and $\Omega^*_{\rm ccb}$ baryons. Our obtained results are reasonably close to the other predictions. The acquired Regge trajectories will also be

helpful to define unknown states of the $\Omega_{\rm ccb}$ baryon, and

 J^P values can be assigned.

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