Two-neutron halo state of ${}^{15}B$ around 3.48 MeV by a three-body model^{*}

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Abstract: We investigate low-lying bound states of the neutron-rich nucleus ¹⁵B by assuming it is a three-body system made of an inert core ¹³B and two valence neutrons. The three-body wave functions are obtained using the Faddeev formalism. Special attention is paid to the excited state at 3.48(6) MeV observed in the ¹³C(¹⁴C, ¹²N)¹⁵B reaction, whose properties are less clear theoretically. In our three-body model, besides the ground state $3/2_1^-$, a second $3/2_2^-$ state is discovered at around 3.61 MeV, which might be identified with the excited state observed at 3.48(6) MeV. We study this $3/2_2^-$ state in detail. It turns out to be a two-neutron halo state with a large matter radius $r_m \approx 4.770$ fm.

Keywords: two-neutron halo, boron isotopes, three-body system, Faddeev equation

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1 Introduction

Neutron-rich nuclei have attracted much attention in the last few decades, and many novel structures are expected in these nuclei, such as neutron halos [1–4], Borromean structures, Efimov-like systems [1], two-neutron emissions [5], etc. In this article, we would like to study the neutron-rich nucleus ¹⁵B theoretically. Although discovered more than fifty years ago [6], it is fair to say that the spectrum of ¹⁵B has not been thoroughly understood yet. Experimentally, ¹⁵B has been studied using the in-beam γ -spectroscopy technique [7] and multinucleon transfer reactions [8]. Two bound excited states at $E_x = 1.327$, 2.734 MeV are observed in the γ -ray spectrum, and five excited states at $E_x = 3.48(6), 4.90(6),$ 5.95(8), 7.63(8), 10.25(8) MeV in the ${}^{13}C({}^{14}C, {}^{12}N){}^{15}B$ reaction, but the spins/parities of these states have not been determined experimentally yet. Recently, there have also been many effects made to measure reaction cross sections for ¹³B, ¹⁴B, and ¹⁵B on different targets [9, 10]. On the theoretical side, at least four influential predictions can be found in the literature, including three shell-model calculations [7, 11, 12] and one antisymmetrized-molecular-dynamics (AMD) calculation [13]. In these theoretical studies, properties of the excited state at $E_x=3.48(6)$ MeV are less clear, and different models predict different energies and spin/parity assignments.

In this article, we would like to help clarify the properties of the excited state at 3.48(6) MeV, which is just beneath the two-neutron disintegration threshold and is probably a three-body bound state. Thanks to the large hierarchy between the neutron separation energies of ¹³B, ¹⁴B, and ¹⁵B, it is plausible that ¹⁵B could also be studied using a three-body model made of an inert ¹³B core and two valence neutrons. These neutron separation energies can be found in Table 1, and indeed, we have

$$S_{\rm n}(^{15}{\rm B}), S_{\rm n}(^{14}{\rm B}) \ll S_{\rm n}(^{13}{\rm B}).$$
 (1)

Similar hierarchical structures of nucleon separation energies can also be found in other three-body nuclei, such as the two-neutron nuclei ⁶He, ¹¹Li [14, 15], ¹²Be [16, 17], ¹⁴Be [18], ¹⁷B [19], ²²C [20], ²³N [21], and the two-proton nuclei ¹⁷Ne [22, 23], ¹⁸Ne, ²⁸S [24], etc, and are viewed as clues of internal three-body structures. For other interesting discussions on nuclear three-body systems, see, for example, Refs. [25–28].

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The rest of this article is organized as follows. In Section 2, we briefly review the Faddeev formalism that is used to solve the three-body Schrödinger equation, and introduce the corresponding interaction models. In Section 3, we present the numerical results concerning the two-body and three-body calculations. We end this article with conclusions in Section 4.

Table 1. Physical properties of ¹³B, ¹⁴B and ¹⁵B. All energy scales are in units of MeV, and all lengths are in units of fm. The experimental data are taken from Ref. [29] unless otherwise noted. Experimental errors are shown in brackets. Data in square brackets are based on theoretical expectations rather than experimental measurements.

AX	J^{π}	T	$S_{\mathbf{n}}$	S_{2n}
$^{13}\mathrm{B}$	$3/2^{-}$	[3/2]	4.879(17)	8.248(10)
^{14}B	2^{-}	[2]	0.970(21)	5.848(21)
$^{15}\mathrm{B}$	$3/2^{-}$ [30]	[5/2]	2.780(3)	3.747(21)

2 Faddeev formalism and interaction models

In the present work, we solve the three-body wave functions using the Faddeev formalism proposed in Ref. [31],

$$\begin{split} \Psi^{JM} &= \Psi_1^{JM}(\boldsymbol{x}_1, \boldsymbol{y}_1) + \Psi_2^{JM}(\boldsymbol{x}_2, \boldsymbol{y}_2) + \Psi_3^{JM}(\boldsymbol{x}_3, \boldsymbol{y}_3), \\ (T_1 + V_{3b} - E)\Psi_1^{JM} &= -V_{23}(\Psi_1^{JM} + \Psi_2^{JM} + \Psi_3^{JM}), \\ (T_2 + V_{3b} - E)\Psi_2^{JM} &= -V_{13}(\Psi_1^{JM} + \Psi_2^{JM} + \Psi_3^{JM}), \\ (T_3 + V_{3b} - E)\Psi_3^{JM} &= -V_{12}(\Psi_1^{JM} + \Psi_2^{JM} + \Psi_3^{JM}), \end{split}$$
(2)

with $(\boldsymbol{x}_i, \boldsymbol{y}_i)$ one of the three Jacobi coordinate systems,

$$\begin{split} & \boldsymbol{x}_{i} = \sqrt{A_{jk}} \boldsymbol{r}_{jk}, \quad \boldsymbol{y}_{i} = \sqrt{A_{i,jk}} \boldsymbol{r}_{i,jk}, \\ & \boldsymbol{r}_{jk} = \boldsymbol{r}_{j} - \boldsymbol{r}_{k}, \quad \boldsymbol{r}_{i,jk} = \boldsymbol{r}_{i} - (A_{j}\boldsymbol{r}_{j} + A_{k}\boldsymbol{r}_{k})/(A_{j} + A_{k}), \\ & A_{jk} = A_{j}A_{k}/(A_{j} + A_{k}), \\ & A_{i,jk} = (A_{j} + A_{k})A_{i}/(A_{i} + A_{j} + A_{k}). \end{split}$$

 $T_i = T_{xi} + T_{yi}$ is the corresponding relative kinetic energy, V_{jk} is the two-body interaction between clusters j and k, and V_{3b} is the three-body force introduced to take into account all those effects that go beyond the two-body interactions. The indices (i, j, k) take values in (1, 2, 3)cyclically. The Faddeev equations Eq. (2) are solved by first transforming from the Jacobi coordinate system to the hyperspherical coordinate system

$$(\boldsymbol{x}_i, \boldsymbol{y}_i, s_j, s_k, I) \equiv (x_i, y_i, \Omega_{x_i}, \Omega_{y_i}, s_j, s_k, s_i)$$
$$\Longrightarrow (\rho, \theta_i, \Omega_{x_i}, \Omega_{y_i}, s_j, s_k, s_i) \equiv (\rho, \Omega_{5i}, s_j, s_k, s_i),$$

with $\rho^2 = x_i^2 + y_i^2$ the hyperradius, and $\theta_i = \arctan(x_i/y_i)$ the hyperangle. (s_j, s_k, s_i) are the spins. We then introduce the hyperharmonic functions of $(\Omega_{5i}, s_j, s_k, s_i)$,

excluding only the dependence on ρ ,

$$\begin{aligned} \mathcal{Y}_{K_{i}l_{x_{i}}l_{y_{i}}}^{L_{i}S_{i}J_{i}s_{i},JM}(\Omega_{5i},s_{j},s_{k},s_{i}) &= \varphi_{K_{i}}^{l_{x_{i}}l_{y_{i}}}(\theta_{i}) \Big\{ \Big([Y_{l_{x_{i}}}(\Omega_{x_{i}}) \\ \otimes Y_{l_{y_{i}}}(\Omega_{y_{i}})]_{L_{i}} &\otimes [X_{s_{j}} \otimes X_{s_{k}}]_{S_{i}} \Big)_{J_{i}} \otimes X_{s_{i}} \Big\}_{JM}, \end{aligned}$$

with

$$\varphi_{K_{i}}^{l_{x_{i}}l_{y_{i}}}(\theta_{i}) = \mathcal{N}_{K_{i}}^{l_{x_{i}}l_{y_{i}}}(\sin\theta_{i})^{l_{x_{i}}}(\cos\theta_{i})^{l_{y_{i}}} \\ \times P_{n_{i}}^{l_{x_{i}}+1/2,l_{y_{i}}+1/2}(\cos2\theta_{i})$$

Here, (l_{x_i}, l_{y_i}) are the orbital angular momenta. $Y_{l_{x_i}m_{x_i}}(\Omega_{x_i})$ and X_{s_i} are the spherical and spin harmonics, respectively, with square brackets being the standard Clebsch-Gordan combination of two angular momenta. $P_{n_i}^{l_{x_i}+1/2, l_{y_i}+1/2}(\cos 2\theta_i)$ are the Jacobi polynomials. $K_i = l_{x_i} + l_{y_i} + 2n_i \ (n_i = 0, 1, 2, \cdots)$ is the hyper-angularmomentum. The normalization constant $\mathcal{N}_{K_i}^{l_{x_i} l_{y_i}}$ is given by

$$\mathcal{N}_{K_{i}}^{l_{x_{i}}l_{y_{i}}} \!=\! \left[\frac{2(K_{i}\!+\!2)\Gamma(K_{i}\!+\!2\!-\!n_{i})\Gamma(n_{i}\!+\!1)}{\Gamma(n_{i}\!+\!l_{x_{i}}\!+\!3/2)\Gamma(n_{i}\!+\!l_{y_{i}}\!+\!3/2)} \right]^{1/2}$$

With the help of hyperharmonic functions, the Faddeev components Ψ_i^{JM} could be rewritten as

$$\Psi_{i}^{JM}(\boldsymbol{x}_{i},\boldsymbol{y}_{i}) = \rho^{-5/2} \sum_{\substack{l_{x_{i}}l_{y_{i}}L_{i}S_{i}J_{i}s_{i},K_{i}\\ k_{i}K_{i}l_{x_{i}}l_{y_{i}}}} \mathcal{X}_{i,K_{i}l_{x_{i}}l_{y_{i}}}^{L_{i}S_{i}J_{i}s_{i},J}(\rho)$$

$$\times \mathcal{Y}_{K_{i}l_{x_{i}}l_{y_{i}}}^{L_{i}S_{i}J_{i}s_{i},JM}(\Omega_{5i},s_{j},s_{k},s_{i}).$$
(3)

After inserting Eq. (3) into the Faddeev equations, one obtains a set of coupled ordinary differential equations for $\mathcal{X}_{i,K_i l_{x_i} l_{y_i}}^{L_i S_i J_i s_i J}(\rho)$, which can be solved using the modern numerical algorithms for differential equations. We recommend Ref. [31] for a detailed discussion on the implementation of the Faddeev formalism including the threebody forces.

To predict physical properties of ¹⁵B quantitatively, we need to determine first the neutron-neutron and neutron-core interaction models by fitting experimental data. Here, we assume that the ¹³B ground state has the neutron configuration $(0s_{1/2})^2(0p_{3/2})^4(0p_{1/2})^2$. The ground state, first and second excited states of ¹⁴B are assumed to have the neutron configurations $(0s_{1/2})^2 (0p_{3/2})^4 (0p_{1/2})^2 (1s_{1/2})^1$, $(0s_{1/2})^2 (0p_{3/2})^4 (0p_{1/2})^2 (1s_{1/2})^1$, and $(0s_{1/2})^2 (0p_{3/2})^4 (0p_{1/2})^2$ $(0d_{5/2})^1$, respectively, which are consistent with the Nordheim weak rule for the odd-odd nuclei [32], as well as the explicit shell-model calculations in Ref. [33]. Noticeably, the ground state and the first excited state of ¹⁴B are assumed to have the same neutron configuration, and the splitting in their energies corresponds to the hyperfine structure resulting from the spin-spin interaction between the ¹³B core and the valence neutron.

For the neutron-neutron interaction, we adopt the Gogny-Pires-Tourreil (GPT) potential [34]. For the

neutron-core interaction, we adopt a Gaussian form with the spin-dependent interaction given by the Garrido-Fedorov-Jensen (GFJ) ansatz [23, 35],

$$V_{\text{n-core}}(r) = \exp(-r^2/b^2)(V_{\text{C}} + V_{\text{SO}}\boldsymbol{L}\cdot\boldsymbol{s}_{\text{n}} + V_{\text{SS}}\boldsymbol{J}_{\text{n}}\cdot\boldsymbol{s}_{\text{c}}), \quad (4)$$

with L the orbital angular momentum, s_n the spin of the valence neutron, s_c the spin of the ¹³B core, and $J_n = s_n + L$. Naively, there could be other choices for spin-dependent interactions, like

$$V_{\rm SS} \boldsymbol{s}_{\rm n} \cdot \boldsymbol{s}_{\rm c} + V_{\rm SO_n} \boldsymbol{L} \cdot \boldsymbol{s}_{\rm n} + V_{\rm SO_c} \boldsymbol{L} \cdot \boldsymbol{s}_{\rm c},$$
$$V_{\rm SS} \boldsymbol{s}_{\rm n} \cdot \boldsymbol{s}_{\rm c} + V_{\rm SO} \boldsymbol{L} \cdot (\boldsymbol{s}_{\rm n} + \boldsymbol{s}_{\rm c}),$$
$$V_{\rm SS} \boldsymbol{s}_{\rm n} \cdot \boldsymbol{s}_{\rm c} + V_{\rm SO} \boldsymbol{L} \cdot \boldsymbol{s}_{\rm n}, \text{ etc.}$$

However, Ref. [35] shows that choices different from Eq. (4) would probably lead to wrong predictions for the excited states, even if the ground-state properties are forced to be reproduced exactly.

Table 2. Interaction parameters for the neutroncore interaction used in this work. All lengths are in units of fm, while all energies are in units of MeV.

parameter	b	$V_{\rm C}$	$V_{\rm SO}^{(l\neq2)}$	$V_{\rm SO}^{(l=2)}$	$V_{\rm SS}$
value	2.28	-87.4	-13.65	-27.165	-3

Table 3. Three-body interaction parameter sets used in this work, with all lengths in units of fm, and all energies in units of MeV.

parameter	V3BA	V3BB	V3BC
r_{3B}	4.0	5.0	6.0
V_{3B}	-5.69	-3.44	-2.54

The free parameters b, $V_{\rm C}$, $V_{\rm SO}$, and $V_{\rm SS}$ in Eq. (4) are determined by reproducing the following conditions: the root mean square (RMS) radius of ¹³B $r_0 = 1.23A^{1/3} =$ 2.89 fm; the ground state 2^- of ¹⁴B with the valence neutron in the $1s_{1/2}$ orbit has the energy $\epsilon[1s_{1/2}(2^{-})] =$ -0.970 MeV, beneath the ¹³B+n threshold [29]; the first excited state 1^- of ${}^{14}B$ with the valence neutron in the $1s_{1/2}$ orbit has the energy of $\epsilon [1s_{1/2}(1^{-})] = -0.316$ MeV, beneath the ¹³B+n threshold [36]; the last neutron in the $^{13}\mathrm{B}$ ground state $3/2^-$ occupies the $0p_{1/2}$ orbit, and its energy can be estimated by the neutron separation energy of ¹³B, i.e., $\epsilon [0p_{1/2}(1^{-})] = -4.879$ MeV [29]; and the excited state 3^- of ¹⁴B with the valence neutron in the $0d_{5/2}$ orbit has the resonance energy $\epsilon[0d_{5/2}(3^-)] = 0.41$ MeV, above the ${}^{13}B+n$ threshold [29]. The values of b, $V_{\rm C}$, $V_{\rm SO}$, and $V_{\rm SS}$ determined thereby are summarized in Table 2. We also need to introduce a three-body interaction of Gaussian type to account for deviations between computed results with only bare two-body interactions and experimental data, as well as to simulate the effects of core deformations and/or core excitations [17],

$$V_{3b}(\rho) = V_{3B} \exp\left[-\left(\frac{\rho}{r_{3B}}\right)^2\right].$$
 (5)

Explicitly, three three-body interaction parameter sets are considered in this work (see Table 3).

3 Numerical results

Numerical results for the two-body and three-body calculations are discussed as follows. With the interaction parameters in Table 3, the RMS matter radii and single-particle energies of various ¹³B and ¹⁴B states can be found in Table 4. The valence-neutron radius and the total matter radius of the ¹⁴B ground state are given by $r_{\rm n} = 5.51$ fm and $r_{\rm m}[1s_{1/2}(2^{-})] = 3.14$ fm, respectively. The matter radius turns out to be a bit larger than the naive estimation $r_0 = 1.23 A^{1/3} = 2.94$ fm, revealing the existence of a neutron halo in the ¹⁴B ground state. Similar arguments can also be applied to the first excited state 2^{-} . The density distributions of the valence neutron in various ¹³B and ¹⁴B states are shown in Fig. 1. The resonance state 3^{-} of ¹⁴B is located by calculating the two-body scattering process of the valence neutron and ¹³B and determining its resonant peak in the energy curve (see Fig. 2).

Table 4. RMS matter radii and energies of various states in ¹³B and ¹⁴B obtained by parameters in Table 2, with all lengths in units of fm, and all energies in units of MeV. $r_{\rm m}(^{14}{\rm B}) \equiv r_{\rm m}[1s_{1/2}(2^{-})]$ is the RMS matter radius for the ¹³B ground state, while $r_{\rm m}(^{13}{\rm B}) \equiv r_{\rm m}[0p_{1/2}(1^{-})]$ is the RMS matter radius for the ¹⁴B ground state.

$r_{\rm m}(^{13}{\rm B})$	$r_{\rm m}(^{14}{\rm B})$		$r_{\rm m}[1s_{1/2}(1^-)]$
2.88	3.14		3.51
$\epsilon[0p_{1/2}(1^-)]$	$\epsilon[1s_{1/2}(2^-)]$	$\epsilon[1s_{1/2}(1^-)]$	$\epsilon[0d_{5/2}(3^-)]$
-4.919	-0.917	-0.316	0.410

In the three-body calculation of ¹⁵B, one has to first carry out the so-called Pauli blocking procedure to remove the deeply bound states of the neutron-core twobody interaction, which are assumed to be occupied by core neutrons in ¹³B already. This is done through a supersymmetry transformation [37–39]. In our calculation, we use the hyperspherical harmonics expansion heavily, and it is important to check its convergence. The truncation of the hyperspherical harmonics expansion is controlled by the hypermomentum K. In Fig. 3, we show the convergence of the $^{15}\mathrm{B}$ ground state energy E_{gs} in the three-body interaction parameter set V3BA as K_{max} increases. The V3BB and V3BC parameter sets show similar convergence behavior and are not discussed explicitly here. One can see intuitively that for $K_{\rm max} \sim 20$, hyperspherical harmonics expansion already shows good convergence for the ground-state energy. A similar check has also been carried out for the $3/2_2^-$ state. We find that the energy of the $3/2_2^-$ state decreases gradually as K_{\max} increases, affected a bit more significantly by the K truncation compared with the ground-state energy. Therefore, in our calculation, we take $K_{\max} = 22$ to do our best to relieve the impact of the K truncation, which is also consistent with other calculations on core+n+n systems (see, e.g., Refs. [16, 21]).



Fig. 1. (color online) The density distribution of neutrons in the subsystem ¹⁴B. The blue curve is the density distribution of the core neutron in ¹³B and the orange and green curves are the neutron density distributions of the valence neutron in the ground state and first excited state of ¹⁴B, respectively.





We calculate the energies of the ground state $3/2_1^$ and the excited state $3/2_2^-$ (with respect to the $n+{}^{13}B$ threshold), along with their RMS matter radii, using the two-body and three-body interaction models in Tables 2 and 3. The results are listed in Table 5 with all energies given with respect to the ${}^{13}B+n+n$ threshold, and are numerically close to each other, revealing encouraging robustness of our predictions. The three-body interaction parameter sets in Table 3 are chosen to reproduce the ground state energy of ¹⁵B $E_{\rm gs} = -3.747$ MeV exactly. The RMS radius of the ground state $3/2_1^-$ is found to be about $3.085 \sim 3.104$ fm, corresponding to the size of a stable nucleus with $A \approx 16$. In other words, the ground state $3/2_1^-$ is not a halo nucleus, which is consistent with the large two-neutron separation energy. To explicitly see the effects of the three-body interaction, we also do the calculation by switching off the three-body interaction for comparison, and find that, in this case, the ground-state energy turns out to be -2.696 MeV. This shows that the three-body interaction indeed plays an important role in our calculations.



Fig. 3. Convergence of the ${}^{15}B$ ground state energy as a function of the maximum K value in the hyperspherical harmonics expansion.

Table 5. Energies and RMS radii of the ground state $3/2_1^-$ and the excited state $3/2_2^-$, calculated with three different three-body interaction parameter sets. Quantities with * correspond to the excited state. All energies are in units of MeV, while all lengths are in units of fm.

	$E_{\rm gs}$	$r_{ m m}$	E^*	$r_{ m m}^{*}$
V3BA	-3.7475	3.085	-0.1293	4.770
V3BB	-3.7470	3.095	-0.1320	4.763
V3BC	-3.7465	3.104	-0.1481	4.739
Exp.	-3.747(21)	_	-0.267(64)	_

The excited state $3/2_2^-$, on the other hand, is quite interesting. First, its excitation energy is about $E^* =$ $3.598 \sim 3.615$ MeV, which is energetically close to the third excited state observed experimentally at $E_x =$ 3.48(6) MeV. It is thus plausible to identify these two states. This identification is consistent with the WBT and WBT^{*} shell model calculations, which predict that the third excited state has spin/parity $3/2^-$ as well. The tiny difference between our three-body calculation and the experimental measurement may be a result of unresolved experimental errors or theoretical defects in model building. In the following discussions, for simplicity, we shall ignore the difference between theoretical predictions and experimental measurements.

Second, the tiny energy and the large RMS matter radius of the $3/2_2^-$ state indicate that it has a two-neutron halo. Indeed, the matter radius $r_{\rm m}^* \approx 4.763$ fm corresponds to the size of a stable nucleus with $A \approx 58$.

To illustrate the inner structure of the $3/2_2^-$ state further, we calculate the RMS distance between two valence neutrons and that between the ¹³B core and the center of mass of the valence neutron pair, denoted by r_{nn} and $r_{c,nn}$, respectively. The numerical results can be found in Table 6. It is interesting to note that $\frac{r_{nn}}{r_{c,nn}} \approx \frac{r_{nn}^*}{r_{c,nn}^*} \approx 1.86$.

Table 6. RMS distances between two valence neutrons r_{nn} and from the ¹³B core to the valence neutron pair $r_{c,nn}$ in the ¹⁵B ground state $3/2_1^$ and the excited state $3/2_2^-$. Quantities for the excited state are superscribed by an extra *. All lengths are in units of fm.

	$r_{ m nn}$	$r_{ m c,nn}$	$r_{ m nn}^*$	$r_{ m c,nn}^*$
V3BA	5.86	3.15	15.26	8.20
V3BB	5.94	3.19	15.23	8.18
V3BC	6.00	3.23	15.12	8.12



Fig. 4. (color online) Spatial distribution of two valence neutrons in the ground state $3/2_1^-$ of ^{15}B with the three-body interaction parameter set V3BA.

The spatial distributions of the valence neutrons can be calculated by

$$\begin{split} P(r_{\rm nn},r_{\rm c,nn}) &\equiv r_{\rm nn}^2 r_{\rm c,nn}^2 \\ & \times \int \left| \Psi^{JM}(\boldsymbol{r}_{\rm nn},\boldsymbol{r}_{\rm c,nn}) \right|^2 \mathrm{d} \varOmega_{\boldsymbol{r}_{\rm nn}} \mathrm{d} \varOmega_{\boldsymbol{r}_{\rm c,nn}}, \end{split}$$

which are displayed pictorially in Figs. 4 and 5 with the three-body interaction parameter set V3BA. For

the ground state $3/2_1^-$, the spatial distribution function $P(r_{nn}, r_{c,nn})$ is peaked at around $(r_{nn}, r_{c,nn}) = (4.4, 2.4)$ fm, while for the excited state $3/2_2^-$, $P(r_{nn}^*, r_{c,nn}^*)$ has a global maximum at around (12.5, 6.7) fm, as well as a secondary maximum at around (4.1, 2.2) fm. The dominant occupation probabilities of valence neutrons for the ground state and $3/2_2^-$ excited state are given in Table 7, with three different three-body interaction parameter sets. Once again, one observes excellent convergence of numerical results among different interaction parameter sets.



Fig. 5. (color online) Spatial distribution of two valence neutrons in the excited state $3/2_2^-$ of ¹⁵B with the three-body interaction parameter set V3BA.

Table 7. Dominant occupation probabilities of valence neutrons for the ground state and $3/2_2^-$ excited state of ¹⁵B, calculated using three different three-body interaction parameter sets.

$3/2_1^-$	V3BA	V3BB	V3BC
$(1s_{1/2})^2$	96.10%	96.25%	96.28%
$(0d_{5/2})^2$	2.10%	2.01%	1.98%
$(0d_{3/2})^2$	1.40%	1.33%	1.32%
$3/2^{-}_{2}$	V3BA	V3BB	V3BC
$(1s_{1/2})^2$	86.32%	86.32%	86.36%
$(0d_{5/2})^2$	7.23%	7.23%	7.21%
$(0d_{3/2})^2$	4.82%	4.82%	4.81%

4 Conclusions

In summary, in this article we investigate the lowlying bound states of ¹⁵B by assuming it is a three-body system made of an inert ¹³B core and two valence neutrons. It is plausible to identify the excited state $3/2_2^$ appearing in our model with the experimentally observed excited state at $E_x=3.48(6)$ MeV. Such an identification is also consistent with the WBT and WBT^{*} shell model calculations. We then study in detail the properties of the ground state $3/2_1^-$ and the excited state $3/2_2^-$, calculating their energies with respect to the three-body disintegration threshold, RMS matter radii, wave functions, occupation probabilities, etc. Our calculations are carried out with three different three-body parameter sets, and numerical predictions show excellent conver-

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