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Isovector pairing and particle-number fluctuation effects on the spectroscopic factors of one-proton stripping and one-neutron pick-up reactions in proton-rich nuclei

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Abstract: Expressions of the spectroscopic factors (SFs) corresponding to one-particle transfer reactions have been established using a schematic definition. These expressions have been derived by taking into account the isovector neutron-proton (np) pairing correlations and a particle-number projection in the framework of the generalized Sharp-BCS (SBCS) method. Recently proposed expressions of the projected wave-functions of odd-mass nuclei have been used for this purpose. The formalism has first been tested using the single-particle energies of the schematic picket-fence model. It is shown that the np pairing and particle-number fluctuation effects are far from negligible and they depend on the pairing gap parameter values. Their behavior is not the same when the parent nuclei are even-even or odd. Predictions dealing with the SFs corresponding to one-proton stripping and one-neutron pick-up reactions in proton-rich nuclei have then been established within the framework of the realistic Woods-Saxon model. It is shown that the np pairing effect as well as the particle-number projection effect are important and thus have to be included in future calculations of the SF corresponding to these kinds of reactions.

Keywords: neutron-proton pairing, particle-number projection, spectroscopic factor

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1 Introduction

Spectroscopic factors (SFs) were introduced about sixty years ago in the theory of nuclear reactions, since they provide a useful basis of comparison between experiment and the predictions of nuclear models [1–4]. The study of the SFs has been the subject of many works, both on the theoretical and experimental sides (see, e.g. Refs. [5–23]). On the theoretical side, the SF depends essentially on the wave-function, which must then be rigorously chosen. In particular, it must include the pairing correlations, which play a major role in the nuclear structure. Pairing correlations have been included in the evaluation of the SF by several authors. Those most often used are the BCS method (see e.g. Refs. [24–28]) and the pairing-plus-quadrupole model [29]. However, in these papers, the only type of pairing correlations which has been taken into account is the pairing between like-particles. Nevertheless, it is now well established that the neutron-proton (np) pairing effect must be taken into account, in addition to the pairing between like-particles, in nuclei along the $N=Z$ line. Indeed, in this kind of nucleus, the neutron and proton Fermi levels are close to each other. Therefore the np pairing correlations are no longer negligible, as is the case when the neutron excess is important. The study of the np pairing is thus an active area of interest (see e.g. Refs. [30–42]; for reviews, see Refs. [43, 44]). A common approach to treat pairing in $N\simeq Z$ nuclei is the BCS-type one [45]. However, it is well known that the standard BCS wave-function does not conserve the number of particles. The particle-number fluctuations affect several nuclear observables, when one takes into account either the like-particle pairing or the np pairing. Among others, let us cite the energy of the system [46–51], the two-particle separation energy [52, 53], the moment of inertia [54–57], the nuclear radii [58, 59], the electric moments [60, 61] and statistical quantities [62]. The restoration of the particle-number is therefore essential for an accurate determination of the nuclear wave-function. Attempts to solve the problems generated by the use of the BCS wave-function are as old as the BCS theory itself. They are numerous. One of the most important is the projection on the good particle number [63–80]. The projection may be performed before the variation (methods of FBCS type) [67–72] or after it (methods of PBCS type) [73–76]. Some of the projection methods are exact [63, 64] but they present a major drawback due to the complexity of the calculations. This is the reason why some methods that enable one to approximately conserve the particle-number have been

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proposed, like the Lipkin-Nogami method [77–80]. The second approach used to overcome the defect of the BCS wave-function is the inclusion of the interaction which has been neglected in the independent-quasiparticle approximation, such as the quasiparticle random phase approximation (QRPA) and its variants [81–91]. Another possibility is the higher Tamm-Dancoff approximation [92–94] as well as the variation after mean-field projection in realistic model spaces (VAMPIR) [95–97] or the variational approach [98, 99]. Among other methods, the present list not being exhaustive, let us also cite the generalized seniority [100, 101], the density matrix method [102, 103], the nucleon pair approximation [104–106], the shell-model-like approach [107–109] and the methods proposed by Pillet et al. [110] and Molique et al. [111]. The exact number-conserving solution proposed by Feng Pan et al. [112] and the exact pairing method introduced by Volya et al. [113, 114] are also possible approaches. The method introduced by Zeng et al. [115] is also an exact particle-number conserving approach in which a broken pair excited configuration is defined by blocking the real particles which specify the configurations [116, 117]. In the present paper, we will use the Sharp-BCS (SBCS) method [73–75]. It is an exact projection method of PBCS type in which the wave-function appears as a convergent sequence of states.

In two previous papers, the present authors studied the particle-number fluctuation effect on the SF corresponding to one-pair like-nucleon transfer reactions when including [118] or not [119] the isovector pairing correlations. The goal of the present work is to study the effects of the np pairing correlations and the particle-number projection on the SF corresponding to one-particle transfer reactions in proton-rich nuclei. Let us note that for simplicity, we will consider here only the isovector pairing and not the isoscalar pairing.

The paper is organized as follows. The expressions of the wave-functions before and after projection are recalled in Section 2. The expressions of the spectroscopic factors are established in Section 3. The numerical results are presented and discussed in Section 4. Finally, the main conclusions are summarized in last section.

2 Hamiltonian diagonalization

Let us start with the following pure isovector pairing Hamiltonian which describes a system of N neutrons and Z protons if one assumes that the neutrons and the protons occupy the same energy levels [32, 34]

$$H=\sum_{\nu>0,t}\varepsilon_{\nu t}(a^+_{\nu t}a_{\nu t}+a^+_{\tilde{\nu}t}a_{\tilde{\nu}t})$$
$$-\frac{1}{2}\sum_{tt'}G_{tt'}\sum_{\nu,\mu>0}\left(a^+_{\nu t}a^+_{\tilde{\nu}t'}a_{\tilde{\mu}t'}a_{\mu t}+a^+_{\nu t}a^+_{\tilde{\nu}t'}a_{\tilde{\mu}t}a_{\mu t'}\right). \quad (1)$$

In this expression, t is the isospin component $(t=n,p)$, and $a^+_{\nu t}$ and $a_{\nu t}$ respectively represent the creation and annihilation operators of a particle in the $|\nu t\rangle$ state, of energy $\varepsilon_{\nu t}$. The time-reversal of the state $|\nu t\rangle$ is denoted $|\tilde{\nu}t\rangle$. The pairing-strength $G_{tt'}$ is assumed to be constant and such that $G_{pn}=G_{np}$.

H is diagonalized using the generalized Bogoliubov-Valatin transformation [31, 32]

$$\alpha^+_{\nu\tau}=\sum_{t=n,p}(u_{\nu\tau t}a^+_{\nu t}+v_{\nu\tau t}a_{\tilde{\nu}t}) \quad , \quad \tau=1,2, \quad (2)$$

in which $\alpha^+_{\nu\tau}$ is the creation operator of a quasiparticle (qp) of τ type.

2.1 BCS wave-functions

The BCS wave-function $|\psi\rangle$ is obtained by eliminating all the qp from the actual vacuum. It is given by [75]

$$|\psi\rangle=\prod_{j>0}|\psi_j\rangle, \quad (3)$$

with the notations

$$|\psi_j\rangle=\left[B^j_1A^+_{jp}A^+_{jn}+B^j_pA^+_{jp}+B^j_nA^+_{jn}+B^j_4\left(a^+_{\tilde{j}p}a^+_{jn}+a^+_{\tilde{j}n}a^+_{jp}\right)+B^j_5\right]|0\rangle. \quad (4)$$

The coefficients B^j_i are defined by

$$B^j_i=b^j_i/K, \qquad i=1,p,n,4,5, \quad (5)$$

with:

$$\begin{aligned}
b^j_1 &= (v_{j1p}v_{j2n}-v_{j1n}v_{j2p})^2\\
b^j_p &= v^2_{j1p}(u_{j2p}v_{j2p}+u_{j2n}v_{j2n})\\
&\quad +v^2_{j2p}(u_{j1n}v_{j1n}-u_{j1p}v_{j1p})-2u_{j1n}v_{j1p}v_{j2p}v_{j2n}\\
b^j_n &= v^2_{j1n}(u_{j2p}v_{j2p}+u_{j2n}v_{j2n})\\
&\quad -v^2_{j2n}(u_{j1n}v_{j1n}-u_{j1p}v_{j1p})-2u_{j1p}v_{j1n}v_{j2p}v_{j2n}\\
b^j_4 &= v_{j1n}v_{j1p}(u_{j2p}v_{j2p}+u_{j2n}v_{j2n})\\
&\quad -v^2_{j2n}u_{j1n}v_{j1p}-v^2_{j2p}u_{j1p}v_{j1n}\\
b^j_5 &= (u_{j1n}v_{j1n}+u_{j1p}v_{j1p})(u_{j2p}v_{j2p}+u_{j2n}v_{j2n})\\
&\quad -(u_{j1n}v_{j2n}+u_{j1p}v_{j2p})^2,
\end{aligned}$$

with K being the normalization constant given by

$$K=\sqrt{\left(b^j_1\right)^2+\left(b^j_p\right)^2+\left(b^j_n\right)^2+2\left(b^j_4\right)^2+\left(b^j_5\right)^2}.$$

A^+_{jt} is the creation operator of a pair of particles, i.e.,

$$A^+_{jt}=a^+_{\tilde{j}t}a^+_{jt}, \quad t=n,p. \quad (6)$$

The gap equations are given by

$$\Delta_{pp}=-G_{pp}\sum_{j>0}\left(B_1^jB_n^j+B_5^jB_p^j\right), \tag{7}$$

$$\Delta_{nn}=-G_{nn}\sum_{j>0}\left(B_1^jB_p^j+B_5^jB_n^j\right), \tag{8}$$

$$\Delta_{np}=-\frac{1}{2}G_{np}\sum_{j>0}\left(B_1^jB_4^j-B_4^jB_5^j\right), \tag{9}$$

$$\langle\psi|N_p|\psi\rangle=2\sum_{j>0}\left[\left(B_1^j\right)^2+\left(B_p^j\right)^2+\left(B_4^j\right)^2\right], \tag{10}$$

$$\langle\psi|N_n|\psi\rangle=2\sum_{j>0}\left[\left(B_1^j\right)^2+\left(B_n^j\right)^2+\left(B_4^j\right)^2\right], \tag{11}$$

$\Delta_{tt'}$ $(t,t'=n,p)$ being the pairing gap parameters and N_t the particle-number operators.

It may easily be shown that when $G_{np}=0$, the state (3), as well as the gap parameters Δ_{tt} $(t=n,p)$ and the particle-number conservation conditions, reduce to their homologues obtained when only the pairing between like-particles is considered, respectively given by Eqs. (A16), (A3) and (A4).

However, the wave-function (3) cannot describe odd systems. In this kind of system, the wave-function is derived using the blocked-level technique. In the following, it will be assumed that the odd particle is a proton in the ν state, i.e., $Z=2P_p+1$, $N=2P_n$. One then has [120]

$$|\nu P\rangle=a_{\nu p}^+\left(B_n^\nu A_{\nu n}^+ +B_5^\nu\right)\prod_{\substack{j>0\\ j\neq\nu}}|\psi_j\rangle, \tag{12}$$

with $|\psi_j\rangle$ being defined by Eq. (4). For simplicity, in the expression of $|\nu P\rangle$ the dependence of the B_i^j $(i=1,p,n,4,5)$ coefficients on ν has been omitted in the notations.

The corresponding gap equations are given by

$$\Delta_{nn}^{\nu P}=-2G_{nn}\Big\{B_n^\nu B_5^\nu+\left[(B_n^\nu)^2+(B_5^\nu)^2\right] \times\sum_{\substack{j>0\\ j\neq\nu}}\left(B_1^jB_p^j+B_n^jB_5^j\right)\Big\}, \tag{13}$$

$$\Delta_{pp}^{\nu P}=-2G_{pp}\left[(B_n^\nu)^2+(B_5^\nu)^2\right]\sum_{\substack{j>0\\ j\neq\nu}}\left(B_1^jB_n^j+B_p^jB_5^j\right), \tag{14}$$

$$\Delta_{np}^{\nu P}=2G_{np}\left[(B_n^\nu)^2+(B_5^\nu)^2\right]\sum_{\substack{j>0\\ j\neq\nu}}B_4^j\left(B_1^j-B_5^j\right), \tag{15}$$

$$\langle\nu P|N_p|\nu P\rangle=1+2\left[(B_n^\nu)^2+(B_5^\nu)^2\right] \times\sum_{\substack{j>0\\ j\neq\nu}}\left[\left(B_1^j\right)^2+\left(B_p^j\right)^2+\left(B_4^j\right)^2\right], \tag{16}$$

$$\langle\nu P|N_n|\nu P\rangle=2(B_n^\nu)^2+2\left[(B_n^\nu)^2+(B_5^\nu)^2\right] \times\sum_{\substack{j>0\\ j\neq\nu}}\left[\left(B_1^j\right)^2+\left(B_p^j\right)^2+\left(B_4^j\right)^2\right]. \tag{17}$$

In order to obtain the ground-state and the gap equations which correspond to the case where the odd particle is a neutron, one just has to replace the index p by n in Eqs. (12)-(17), and vice versa.

Here again, it may be easily shown that, when $G_{np}=0$, state (12), as well as the gap parameters $\Delta_{tt}^{\nu(N,P)}$ $(t=n,p)$ and the particle-number conservation conditions, reduce to their homologues in the pairing between like-particles case respectively given by Eqs. (A17), (A6), (A7) and (A4).

Nevertheless, neither state (3) nor state (12) are eigenstates of the particle-number operator. This is the main defect of the BCS approach. A particle-number projection is thus necessary.

2.2 Projected wave-functions

In the following, we will briefly recall the expressions of the projected wave-functions obtained using the Sharp-BCS (SBCS) method [75]. For an even-even nucleus, the projected ground-state is given by

$$|\psi_{mm'}\rangle=C_{mm'}\left\{\sum_{k=0}^{m+1}\sum_{k'=0}^{m'+1}\xi_k\xi_{k'}z_k^{-P_n}z_{k'}^{-P_p}|\psi(z_k,z_{k'})\rangle+\mathcal{CC}\right\}, \tag{18}$$

where

$$|\psi(z_k,z_{k'})\rangle=\prod_{j>0}|\psi_j(z_k,z_{k'})\rangle, \tag{19}$$

with the notations

$$|\psi_j(z_k,z_{k'})\rangle=\big[z_kz_{k'}B_1^jA_{jp}^+A_{jn}^+ +z_kB_n^jA_{jn}^+ +z_{k'}B_p^jA_{jp}^+ +\sqrt{z_kz_{k'}}B_4^j\left(a_{\tilde{j}p}^+a_{jn}^+ +a_{\tilde{j}n}^+a_{jp}^+\right)+B_5^j\big]|0\rangle \tag{20}$$

and

$$\xi_k=\begin{cases}\dfrac{1}{2} & \text{if}\quad k=0 \quad\text{or}\quad k=m+1\\ 1 & \text{if}\quad 0<k<m+1\end{cases},\ z_k=\exp\left(\frac{\mathrm{i}k\pi}{m+1}\right). \tag{21}$$

m,m' are non-zero integers and respectively refer to the projection order on the good neutron and proton numbers, and $\mathcal{CC}$ means the summation over the same terms where $(z_k,z_{k'})$ is replaced by $(\overline{z_k},z_{k'})$, then by $(z_k,\overline{z_{k'}})$ and finally by $(\overline{z_k},\overline{z_{k'}})$.

Let us note that state (18) is such that

$$\langle\psi_{mm'}|\mathcal{O}|\psi_{mm'}\rangle=4(m+1)(m'+1)C_{mm'}\langle\psi|\mathcal{O}|\psi_{mm'}\rangle, \tag{22}$$

where $\mathcal{O}$ is any operator which conserves the particle-number.

Using this property, the normalization condition of state (18) then reads

$$1=4(m+1)(m'+1)C_{mm'}^2 \times\left\{\sum_{k=0}^{m+1}\sum_{k'=0}^{m'+1}\xi_k\xi_{k'}z_k^{-P_n}z_{k'}^{-P_p}\prod_{j>0}A_j(z_k,z_{k'})+\mathcal{CC}\right\}, \quad (23)$$

where we set

$$A_j(z_k,z_{k'})=z_kz_{k'}\left(B_1^j\right)^2+z_k\left(B_n^j\right)^2+z_{k'}\left(B_p^j\right)^2 +2\sqrt{z_kz_{k'}}\left(B_4^j\right)^2+\left(B_5^j\right)^2. \quad (24)$$

As soon as

$$\begin{array}{l}2(m+1)>\max(P_n,\Omega-P_n)\\ 2(m'+1)>\max(P_p,\Omega-P_p)\end{array}, \quad (25)$$

all the false components are eliminated in the state $|\psi\rangle$.

In the case of an odd system, the projected ground-state is given by [120]:

$$|(\nu P)_{mm'}\rangle=C_{mm'}^{\nu P}\sum_{k=0}^{m+1}\sum_{k'=0}^{m'+1}\xi_k\xi_{k'}\left\{z_k^{-P_n}z_{k'}^{-P_p}a_{\nu p}^+(B_n^\nu z_kA_{\nu n}^++B_5^\nu)\prod_{\substack{j>0\\ j\neq\nu}}|\psi_j(z_k,z_{k'})\rangle+\mathcal{CC}\right\}, \quad (26)$$

when the odd particle is a proton in the ν state. The condition (25) remains valid in this case. $C_{mm'}^{\nu P}$ is the normalization constant given by

$$1=4(m+1)(m'+1)\left(C_{mm'}^{\nu P}\right)^2\sum_{k=0}^{m+1}\sum_{k'=0}^{m'+1}\xi_k\xi_{k'}\left\{z_k^{-P_n}z_{k'}^{-P_p}\left[z_k(B_n^\nu)^2+(B_5^\nu)^2\right]\prod_{\substack{j>0\\ j\neq\nu}}A_j(z_k,z_{k'})+\mathcal{CC}\right\}. \quad (27)$$

One just has to switch the p and n indexes, as well as z_k and $z_{k'}$, in Eq. (26) to obtain the wave-function which corresponds to the case where the odd particle is a neutron.

3 Spectroscopic factors

In the following, the expressions of the SFs will be derived using a schematic definition similar to the one proposed by Chasman [121] in the case of the np pair transfer. In the case of one-particle stripping reactions, the SF is deduced from the relation

$$\sqrt{S_t^{\rm STR}}=\left\langle\psi^{\rm f}(A+1)\left|\sum_{l>0}\left(a_{lt}^++a_{\tilde{l}t}^+\right)\right|\psi^{\rm i}(A)\right\rangle,\ t=n,p. \quad (28)$$

In the case of one-particle pick-up reactions, it is deduced from the relation

$$\sqrt{S_t^{\rm PIC}}=\left\langle\psi^{\rm f}(A-1)\left|\sum_{l>0}(a_{lt}+a_{\tilde{l}t})\right|\psi^{\rm i}(A)\right\rangle,\ t=n,p, \quad (29)$$

where $|\psi^{\rm i}(A)\rangle$ and $|\psi^{\rm f}(A\pm1)\rangle$ respectively refer to the wave-functions of the initial (i) and final (f) states of the considered nucleus. A corresponds to the total number of nucleons in the initial state.

In the present work, we consider only even-even or odd systems, but not odd-odd ones.

3.1 Before projection

Within the generalized BCS approach, the SFs defined by Eqs. (28)-(29) read, in the case of the transfer of one proton from an even-even nucleus to an odd one,

$$\sqrt{S_p^{\rm STR(1)}}=F_{n5}^{\rm if}(\nu)\prod_{j>0,j\neq\nu}D_j^{\rm if}(\nu), \quad (30)$$

$$\sqrt{S_p^{\rm PIC(1)}}=F_{np}^{\rm if}(\nu)\prod_{j>0,j\neq\nu}D_j^{\rm if}(\nu), \quad (31)$$

where we used the wave-functions defined by Eqs. (3) and (12). The notation STR refers to the stripping reactions whereas PIC refers to pick-up reactions.

In the reciprocal case, the SFs are given by

$$\sqrt{S_p^{\rm STR(2)}}=F_{np}^{\rm fi}(\nu)\prod_{j>0,j\neq\nu}D_j^{\rm fi}(\nu), \quad (32)$$

$$\sqrt{S_p^{\rm PIC(2)}}=F_{n5}^{\rm fi}(\nu)\prod_{j>0,j\neq\nu}D_j^{\rm fi}(\nu), \quad (33)$$

with the notations

$$D_j^{\rm if}(\nu)=B_1^{\rm ji}B_1^{\rm jf}(\nu)+B_p^{\rm ji}B_p^{\rm jf}(\nu)+B_n^{\rm ji}B_n^{\rm jf}(\nu) +2B_4^{\rm ji}B_4^{\rm jf}(\nu)+B_5^{\rm ji}B_5^{\rm jf}(\nu), \quad (34)$$

$$F_{n5}^{\rm if}(\nu)=B_n^{\nu\rm i}B_n^{\nu\rm f}(\nu)+B_5^{\nu\rm i}B_5^{\nu\rm f}(\nu), \quad (35)$$

$$F_{np}^{\rm if}(\nu)=B_1^{\nu\rm i}B_n^{\nu\rm f}(\nu)+B_p^{\nu\rm i}B_5^{\nu\rm f}(\nu). \quad (36)$$

In the latter expressions, one just has to switch between the p and n indexes to obtain the SFs $S_n^{\rm STR(1,2)}$ and $S_n^{\rm PIC(1,2)}$ which correspond to one-neutron transfer reactions.

It may be easily shown that when the np pairing effects vanish, i.e., when the np pairing gap parameters go to zero, Eqs. (30)-(33) reduce to their homologues when only the pairing between like-particles is taken into account, that is, Eqs. (A18)-(A21).

3.2 After projection

After the projection, the SFs are derived using the states defined by Eqs. (18) and (26). One then has, in the case of the transfer of one proton from an even-even nucleus to an odd one,

$$\sqrt{\left(S_p^{\mathrm{STR}(1)}\right)_{mm'}}=4(m+1)(m'+1)C^{\mathrm{i}}_{mm'}C^{(\nu p)f}_{mm'}\sum_{k=0}^{m+1}\sum_{k'=0}^{m'+1}\xi_k\xi_{k'}\left[z_k^{-P_n^{\mathrm{f}}}z_{k'}^{-P_p^{\mathrm{f}}}F^{\mathrm{if}}_{n5}(z_k)\prod_{j\neq\nu}D_j^{\mathrm{if}}(z_k,z_{k'})+\mathcal{CC}\right] \tag{37}$$

$$\sqrt{\left(S_p^{\mathrm{PIC}(1)}\right)_{mm'}}=4(m+1)(m'+1)C^{\mathrm{i}}_{mm'}C^{(\nu p)f}_{mm'}\sum_{k=0}^{m+1}\sum_{k'=0}^{m'+1}\xi_k\xi_{k'}\left[z_k^{-P_n^{\mathrm{f}}}z_{k'}^{-P_p^{\mathrm{f}}}F^{\mathrm{if}}_{np}(z_k,z_{k'})\prod_{j\neq\nu}D_j^{\mathrm{if}}(z_k,z_{k'})+\mathcal{CC}\right]. \tag{38}$$

In the reciprocal case, the SFs are given by

$$\sqrt{\left(S_p^{\mathrm{STR}(2)}\right)_{mm'}}=4(m+1)(m'+1)C^{(\nu p)i}_{mm'}C^{\mathrm{f}}_{mm'}\sum_{k=0}^{m+1}\sum_{k'=0}^{m'+1}\xi_k\xi_{k'}\left[z_k^{-P_n^{\mathrm{f}}}z_{k'}^{-P_p^{\mathrm{f}}}F^{\mathrm{fi}}_{n5}(z_k,z_{k'})\prod_{j\neq\nu}D_j^{\mathrm{fi}}(z_k,z_{k'})+\mathcal{CC}\right], \tag{39}$$

$$\sqrt{\left(S_p^{\mathrm{PIC}(2)}\right)_{mm'}}=4(m+1)(m'+1)C^{(\nu p)i}_{mm'}C^{\mathrm{f}}_{mm'}\sum_{k=0}\sum_{k'=0}^{m'+1}\xi_k\xi_{k'}\left[z_k^{-P_n^{\mathrm{f}}}z_{k'}^{-P_p^{\mathrm{f}}}F^{\mathrm{fi}}_{np}(z_k)\prod_{j\neq\nu}D_j^{\mathrm{fi}}(z_k,z_{k'})+\mathcal{CC}\right], \tag{40}$$

with the notations:

$$D_j^{\mathrm{if}}(z_k,z_{k'})=z_kz_{k'}B_1^{j\mathrm{i}}B_1^{j\mathrm{f}}(\nu)+z_{k'}B_p^{j\mathrm{i}}B_p^{j\mathrm{f}}(\nu)+z_kB_n^{j\mathrm{i}}B_n^{j\mathrm{f}}(\nu)+2\sqrt{z_kz_{k'}}B_4^{j\mathrm{i}}B_4^{j\mathrm{f}}(\nu)+B_5^{j\mathrm{i}}B_5^{j\mathrm{f}}(\nu) \tag{41}$$

$$F^{\mathrm{if}}_{n5}(z_k)=z_k^2B_n^{\nu\mathrm{i}}B_n^{\nu\mathrm{f}}(\nu)+B_5^{\nu\mathrm{i}}B_5^{\nu\mathrm{f}}(\nu) \tag{42}$$

$$F^{\mathrm{if}}_{np}(z_k,z_{k'})=z_k^2z_{k'}B_1^{\nu\mathrm{i}}B_n^{\nu\mathrm{f}}(\nu)+z_{k'}B_p^{\nu\mathrm{i}}B_5^{\nu\mathrm{f}}(\nu). \tag{43}$$

In the latter expressions, one just has to switch the n and p indexes, as well as z_k and $z_{k'}$, to obtain the SFs $\left(S_n^{\mathrm{STR}(1,2)}\right)_{mm'}$ and $\left(S_n^{\mathrm{PIC}(1,2)}\right)_{mm'}$ which correspond to one-neutron transfer reactions. Moreover, one notes a formal similarity between expressions (30)-(33) (i.e., before the projection) and expressions (37)-(40) (i.e., after the projection).

When the np pairing effects vanish, Eqs. (30)-(33) reduce to their homologues in the pairing between like-particles case given by Eqs. (A24)-(A27).

4 Numerical results and discussion

The previously described formalism has been used in order to study numerically the np pairing and projection effects on the SFs corresponding to one-particle transfer reactions. With this aim, two models have been used, the picket-fence schematic model [122] and the realistic Woods-Saxon one [123]. As was the case in Ref. [118], the picket-fence model is used here as a toy model since it does not enable one to obtain the exact values of the SFs.

The convergence of the SBCS method is very rapid (see, e.g., Ref. [118]). Indeed, the convergence is reached as soon as $m,m'\simeq 4-5$ when one uses the picket-fence model and as soon as $m,m'\simeq 5-6$ when one uses the Woods-Saxon model. It is clearly less than the theoretical values predicted by condition (25). In all that follows, we will use the values $m=m'=10$ in order to ensure convergence.

In the following, S_{BCS} and S_{SBCS} mean respectively the SFs evaluated before and after the projection in the pairing between like-particles (i.e., using Eqs. (A18)-(A21) and (A24)-(A27)), and S_{BCS-np} and $S_{SBCS-np}$ are their homologues in the isovector np pairing (i.e., using Eqs. (30)-(33) and (37)-(40)).

4.1 Picket-fence model

In this model, the single-particle levels are equally spaced, that is, $\varepsilon_\nu=\nu$, $\nu=1,2,...,\Omega$, where Ω is the total number of levels.

The values of the pairing gap parameters $\Delta_{tt'}$ $(t,t'=n,p)$ are chosen arbitrarily. The $G_{tt'}$ $(t,t'=n,p)$ values are then deduced using Eqs. (7)-(11) in the case of even-even systems and Eqs. (13)-(17) in the case of odd systems.

Within the picket-fence model, we considered only a one-proton stripping reaction in the case $Z^{\mathrm{i}}=N^{\mathrm{i}}=16$ (Z^{i} and N^{i} being the proton and neutron numbers in the initial state), taken as an example for even-even systems, and $Z^{\mathrm{i}}=15$, $N^{\mathrm{i}}=16$, taken as an example for odd systems.

As a first step, we have studied the variations of the SFs as a function of the np pairing gap parameter in the initial state Δ^{i}_{np}, the values of the other parameters being fixed. The values used are $\Delta^{\mathrm{i}}_{pp}=1.6$ MeV, $\Delta^{\mathrm{f}}_{pp}=1.4$ MeV, $\Delta^{\mathrm{f}}_{nn}=1.3$ MeV, $\Delta^{\mathrm{f}}_{np}=0.2$ MeV and $\Omega=18$. The variations of the SFs, evaluated using the previously cited four approaches, as a function of Δ^{i}_{np} are shown in Fig. 1 for the system $Z^{\mathrm{i}}=N^{\mathrm{i}}=16$ and in Fig. 2 for the system $Z^{\mathrm{i}}=15$, $N^{\mathrm{i}}=16$, in the case $\Delta^{\mathrm{i}}_{nn}=1$ MeV, chosen as an example. S_{BCS} and S_{SBCS} are obviously constant as a function of Δ^{i}_{np}.

As could be foreseen, Figs. 1 and 2 show that the behavior of the SFs is completely different when the initial state is even-even and when it is odd. In particular, the minimum which appears in the S_{BCS-np} and $S_{SBCS-np}$ graphs in Fig. 1 is absent in Fig. 2.

As a second step, we have studied separately the np pairing and projection effects. The np pairing effect, before and after the projection, is evaluated using the relative discrepancies

$$\delta S_{np}=\frac{S_{BCS}-S_{BCS-np}}{S_{BCS}} \tag{44}$$

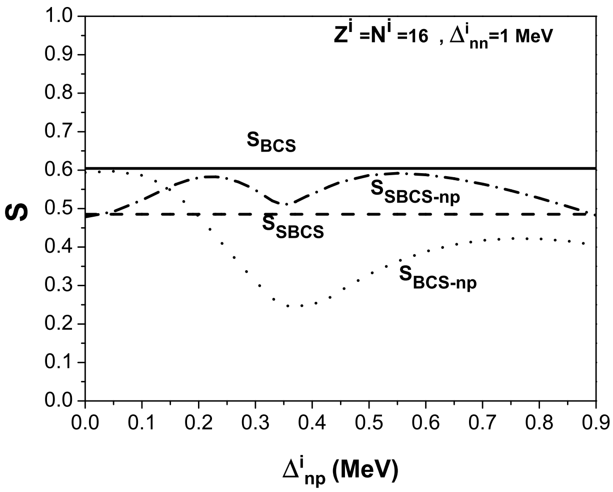

Fig. 1. Variations of the spectroscopic factor corresponding to a one-proton stripping reaction, in the case of the system $Z^{\rm i}=N^{\rm i}=16$, as a function of the np gap parameter $\Delta^{\rm i}_{np}$ of the initial state, when $\Delta^{\rm i}_{nn}=1$ MeV. Solid lines and dashed lines refer to the pairing between like-particles respectively before and after the projection. Dotted and dash-dotted lines refer to the np pairing respectively before and after the projection.

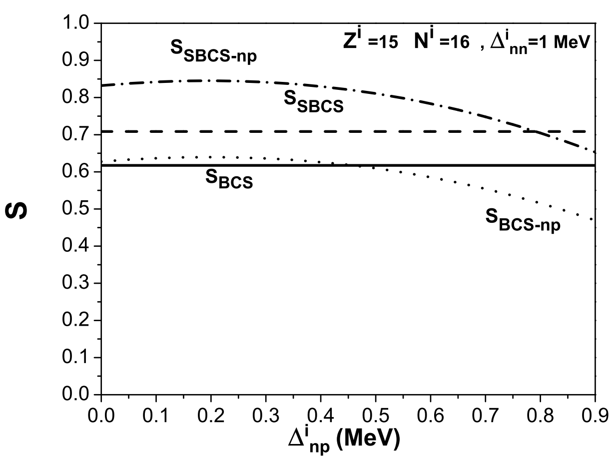

Fig. 2. Variations of the spectroscopic factor corresponding to a one-proton stripping reaction, in the case of the system $Z^{\rm i}=15$, $N^{\rm i}=16$, as a function of the np gap parameter $\Delta^{\rm i}_{np}$ of the initial state, when $\Delta^{\rm i}_{nn}=1$ MeV. Solid lines and dashed lines refer to the pairing between like-particles respectively before and after the projection. Dotted and dash-dotted lines refer to the np pairing respectively before and after the projection.

and

$$\delta S_{np-proj}=\frac{S_{SBCS}-S_{SBCS-np}}{S_{SBCS}}. \tag{45}$$

In the same way, the projection effect, in the pairing between like-particles, as well as in the np pairing, will be evaluated using the relative discrepancies

$$\delta S_{proj}=\frac{S_{BCS}-S_{SBCS}}{S_{BCS}} \tag{46}$$

and

$$\delta S_{proj-np}=\frac{S_{BCS-np}-S_{SBCS-np}}{S_{BCS-np}}. \tag{47}$$

4.1.1 Neutron-proton pairing effect

The influence of the np pairing effect on the SF has been studied by evaluating δS_{np} and $\delta S_{np-proj}$ as a function of the np pairing gap parameter in the initial state $\Delta^{\rm i}_{np}$, for several values of $\Delta^{\rm i}_{nn}$. The corresponding results are displayed in Fig. 3 for the system $Z^{\rm i}=N^{\rm i}=16$ and in Fig. 4 for the system $Z^{\rm i}=15$, $N^{\rm i}=16$. The $\Delta^{\rm i}_{nn}$ values are in the range $1.0\leqslant\Delta^{\rm i}_{nn}\leqslant1.5$ MeV.

The behavior of the δS is obviously different when the initial state is even-even and when it is odd. Indeed, in the graphs in Fig. 3, i.e. when the initial state is even-even, one observes a rapid increase of δS_{np} (i.e. before the projection) until a maximum and then a decrease. δS_{np} is always positive. The np pairing effect thus corresponds in this case to a decrease of the SF values.

After the projection, there is also a maximum in the $\delta S_{np-proj}$ values at the same position as the previous one. These maxima correspond to the minima in the S_{BCS-np} and $S_{SBCS-np}$ graphs in Fig. 1.

Moreover, $\delta S_{np-proj}$ is always negative. The np pairing effect after projection thus corresponds to an increase of the SF values.

It is worth noticing that, contrary to how it appears at first glance, δS_{np} and $\delta S_{np-proj}$ are non-zero when $\Delta^{\rm i}_{np}=0$ (they are of the order of 1%–2% in this case). The fact that δS_{np} and $\delta S_{np-proj}$ do not vanish when $\Delta^{\rm i}_{np}$=0 is due to the $\Delta^{\rm f}_{np}$ value, which remains constant.

One may conclude from Fig. 3 that the np pairing effect on the SF is important for this kind of reaction since $|\delta S_{np}|$ may reach up to 65%, whereas $|\delta S_{np-proj}|$ may reach up to 30%.

The average values of $|\delta S_{np}|$ and $|\delta S_{np-proj}|$ over all the considered values of $\Delta^{\rm i}_{np}$ are reported in Table 1 as a function of $\Delta^{\rm i}_{nn}$. It then appears that the np pairing effect is less important in absolute value after projection than before it. Moreover, $|\delta S_{np}|$ decreases as a function of $\Delta^{\rm i}_{nn}$ whereas $|\delta S_{np-proj}|$ seems to be less sensitive to the $\Delta^{\rm i}_{nn}$ value.

A similar study has been performed in the case of the transfer of one pair of like-particles [118]. The difference in behavior of the curves before or after the projection was less clear in this latter case.

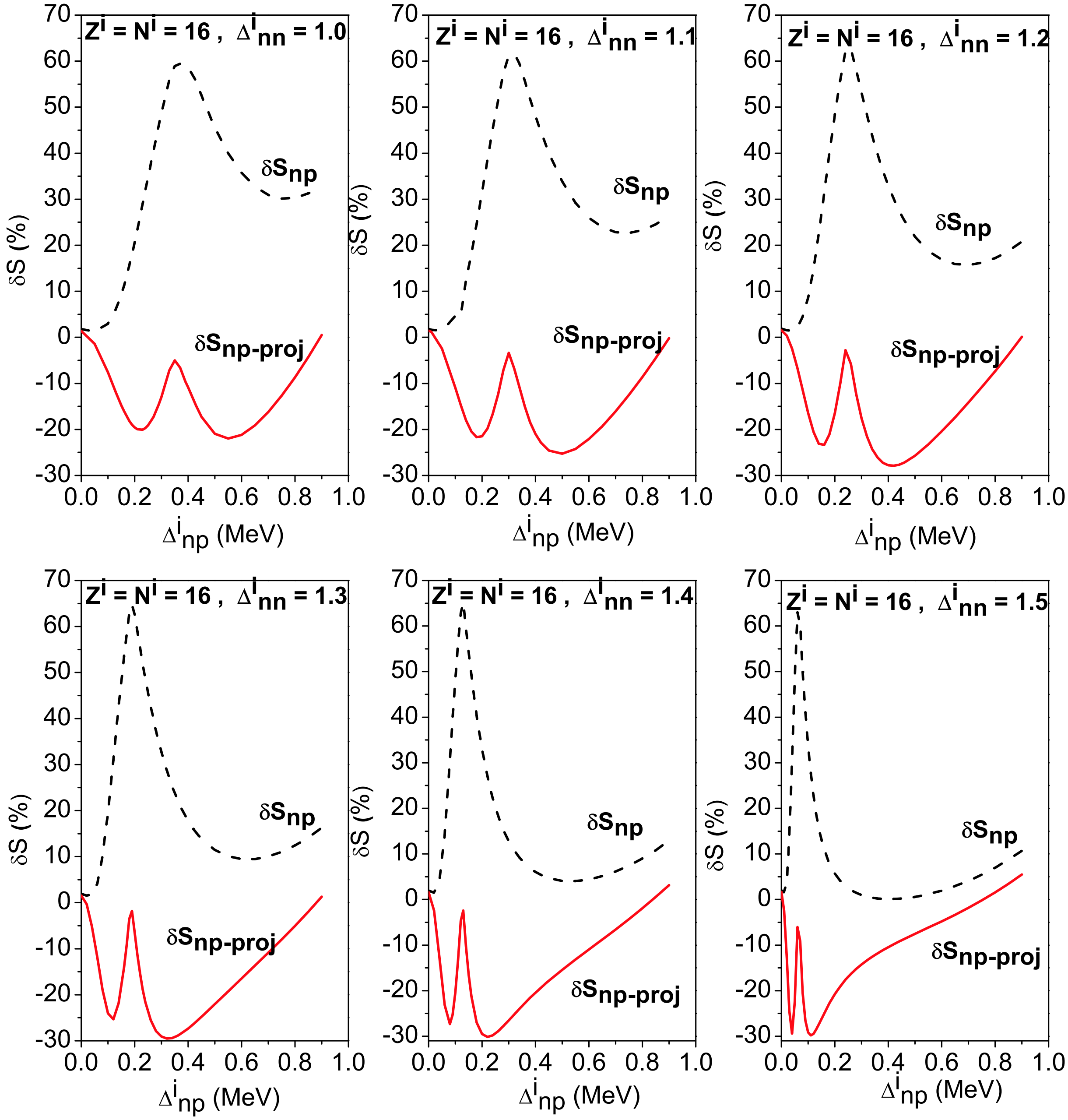

Fig. 3. (color online) Variations of the relative discrepancies of the spectroscopic factors (see the text for notations) corresponding to a one-proton stripping reaction, in the case of the system $Z^{\rm i}=N^{\rm i}=16$, as a function of the np gap parameter $\Delta^{\rm i}_{np}$ of the initial state, for several values of the neutron gap parameter of the initial state $\Delta^{\rm i}_{nn}$ (MeV). Dashed lines show values obtained before the projection and solid lines show those obtained after the projection.

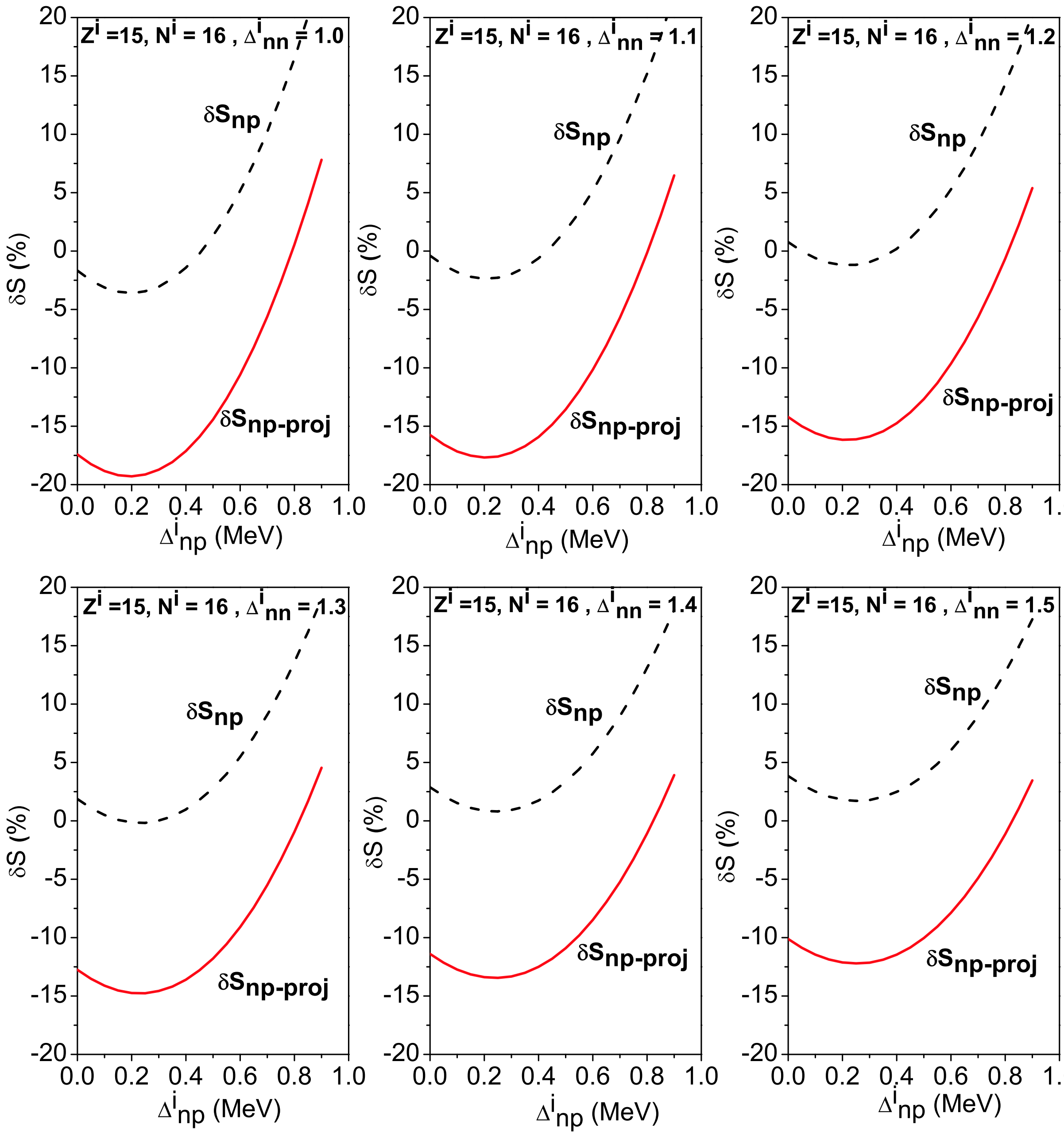

Fig. 4. (color online) Variations of the relative discrepancies of the spectroscopic factors (see the text for notations) corresponding to a one-proton stripping reaction, in the case of the system $Z^{\mathrm{i}}=15$, $N^{\mathrm{i}}=16$ as a function of the np gap parameter Δ^{i}_{np} of the initial state, for several values of the neutron gap parameter of the initial state Δ^{i}_{nn} (MeV). Dashed lines show values obtained before the projection and solid lines show those obtained after the projection.

When the initial state is odd (see Fig. 4), the graphs no longer have a maximum, but a somewhat large minimum appears. For each value of $\Delta^{\rm i}_{nn}$, the behavior of the two graphs is similar: they are quasi-parallel. The np pairing effect is far from negligible, since it may reach up to 20% in absolute value, whether before or after the projection.

In this case also, δS_{np} and $\delta S_{np-proj}$ are non-zero when $\Delta^{\rm i}_{np}=0$ because $\Delta^{\rm f}_{np}\neq 0$.

However, the np pairing effect in absolute value is clearly more important on average after the projection than before it (see Table 1). Unlike what happens when the initial state is even-even, $\overline{|\delta S_{np}|}$ is less than $\overline{|\delta S_{np-proj}|}$ and is practically independent of $\Delta^{\rm i}_{nn}$.

Moreover, the $\overline{|\delta S_{np}|}$ values are clearly less important when $Z^{\rm i}=15$, $N^{\rm i}=16$ than when $Z^{\rm i}=N^{\rm i}=16$. The gap between the $\overline{|\delta S_{np-proj}|}$ values is less clear between the upper and the lower part of Table 1.

Table 1. Average values of $|\delta S|$ (%) as a function of $\Delta^{\rm i}_{nn}$ in the case of one-proton stripping reactions. Columns 2 and 3 of each part show the np pairing effect, and columns 4 and 5 show the projection effect. The gap parameter $\Delta^{\rm i}_{nn}$ values are given in MeV.

system $Z^{\rm i}=16$, $N^{\rm i}=16$				
one-proton stripping				
$\Delta^{\rm i}_{nn}$	$\overline{\lvert\delta S_{np}\rvert}$	$\overline{\lvert\delta S_{np-proj}\rvert}$	$\overline{\lvert\delta S_{proj}\rvert}$	$\overline{\lvert\delta S_{proj-np}\rvert}$
1.0	32.84	13.46	19.73	49.21
1.1	32.54	13.96	19.83	51.90
1.2	27.74	15.89	19.87	43.48
1.3	27.14	16.53	19.88	44.76
1.4	20.97	16.60	19.87	35.62
1.5	13.06	14.97	19.85	26.11
System $Z^{\rm i}=15$, $N^{\rm i}=16$				
One-proton stripping				
$\Delta^{\rm i}_{nn}$	$\overline{\lvert\delta S_{np}\rvert}$	$\overline{\lvert\delta S_{np-proj}\rvert}$	$\overline{\lvert\delta S_{proj}\rvert}$	$\overline{\lvert\delta S_{proj-np}\rvert}$
1.0	6.63	13.08	14.77	33.83
1.1	5.77	12.06	14.62	33.11
1.2	5.19	11.13	14.55	32.59
1.3	5.03	10.23	14.55	32.19
1.4	5.41	9.36	14.56	31.86
1.5	5.86	8.53	14.60	31.58

4.1.2 Projection effect

The projection effect on the SF has been studied by evaluating δS_{proj} and $\delta S_{proj-np}$ as a function of the np pairing gap parameter in the initial state $\Delta^{\rm i}_{np}$, using the same parameters as in the previous section. The corresponding results are displayed in Fig. 5 for the system $Z^{\rm i}=N^{\rm i}=16$ and in Fig. 6 for the system $Z^{\rm i}=15$, $N^{\rm i}=16$. When only the pairing between like-particles is considered, the projection effect (and thus δS_{proj}) is obviously constant as a function of $\Delta^{\rm i}_{np}$. It has been shown here solely for the purpose of comparison.

When the initial state is even-even (see Fig. 5), the projection effect is much more important in the isovector pairing case than when only the pairing between like-particles is considered. Indeed, δS_{proj} is of the order of 20%, whereas $\delta S_{proj-np}$ may reach up to 130% in absolute value. This fact is confirmed in Table 1 where we reported the average values of $|\delta S_{proj}|$ and $|\delta S_{proj-np}|$ over all the considered values of $\Delta^{\rm i}_{np}$ as a function of $\Delta^{\rm i}_{nn}$.

The projection effect may correspond either to an increase or a decrease of the SF values. Moreover a minimum in the $\delta S_{proj-np}$ values appears at the same position as in Fig. 3.

When the initial state is odd (see Fig. 6), the projection effect is also important. When only the pairing between like-particles is considered, this effect is of the order of 15% in absolute value for all the considered values of $\Delta^{\rm i}_{nn}$. In the isovector pairing case, it may reach up to 40% in absolute value. However, δS_{proj} and $\delta S_{proj-np}$ are both negative and thus correspond to an increase of the SF values.

Moreover, the behavior of $\delta S_{proj-np}$ is completely different from that when the initial state is even-even. The minimum in Fig. 5 no longer exists. After a sort of plateau, $\delta S_{proj-np}$ decreases slowly. Whether in the pairing between like-particles or in the isovector pairing, the projection effect is, on average, less important than when the initial state is even-even (see Table 1).

As a conclusion, both the np pairing and projection effects on the SFs are important and must be taken into account. These effects strongly depend on the pairing gap parameter values. The latter must then be carefully chosen. Similar results were obtained in the study of the SFs corresponding to one-pair like-particle transfer reactions [118].

4.2 Woods-Saxon model

In order to study realistic cases, we used the Woods-Saxon model [123] using the parameters described in Ref. [124]. We used a maximal shell number $N_{\rm max}=10$. This value corresponds to a total level degeneracy $\Omega=455$. The ground-state deformation parameters are taken from the tables in Refs. [125] and [126].

When the np pairing correlations are taken into account, the choice of the pairing-strength parameters $G_{tt'}$ $(t,t'=n,p)$ is still an open question and has been the subject of many studies (see e.g. Refs. [31, 34, 35, 121, 127–138]). It has been shown in the previous section that the SF values are very sensitive to the pairing gap parameter values and thus to the $G_{tt'}$ values. We thus prefer, in the present work, to avoid the use of fitted values of the pairing constants. We extract the $G_{tt'}$ values

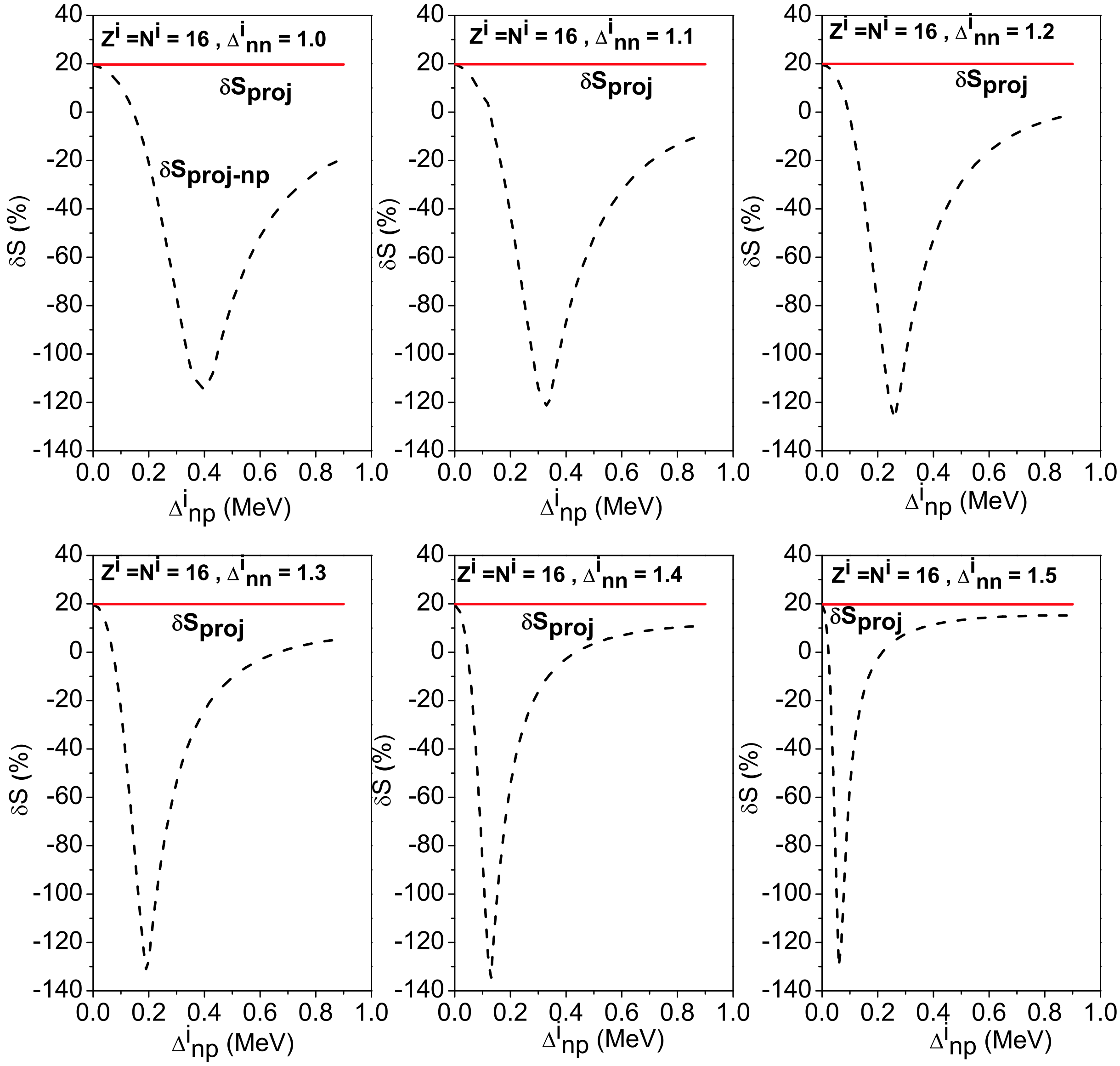

Fig. 5. (color online) Variations of the relative discrepancies of the spectroscopic factors (see the text for notations) corresponding to a one-proton stripping reaction, in the case of the system $Z^{\mathrm{i}}=N^{\mathrm{i}}=16$, as a function of the np gap parameter Δ^{i}_{np} of the initial state, for several values of the neutron gap parameter of the initial state Δ^{i}_{nn} (MeV). Solid lines show values obtained in the pairing between like-particles and dashed lines show those obtained in the np pairing case.

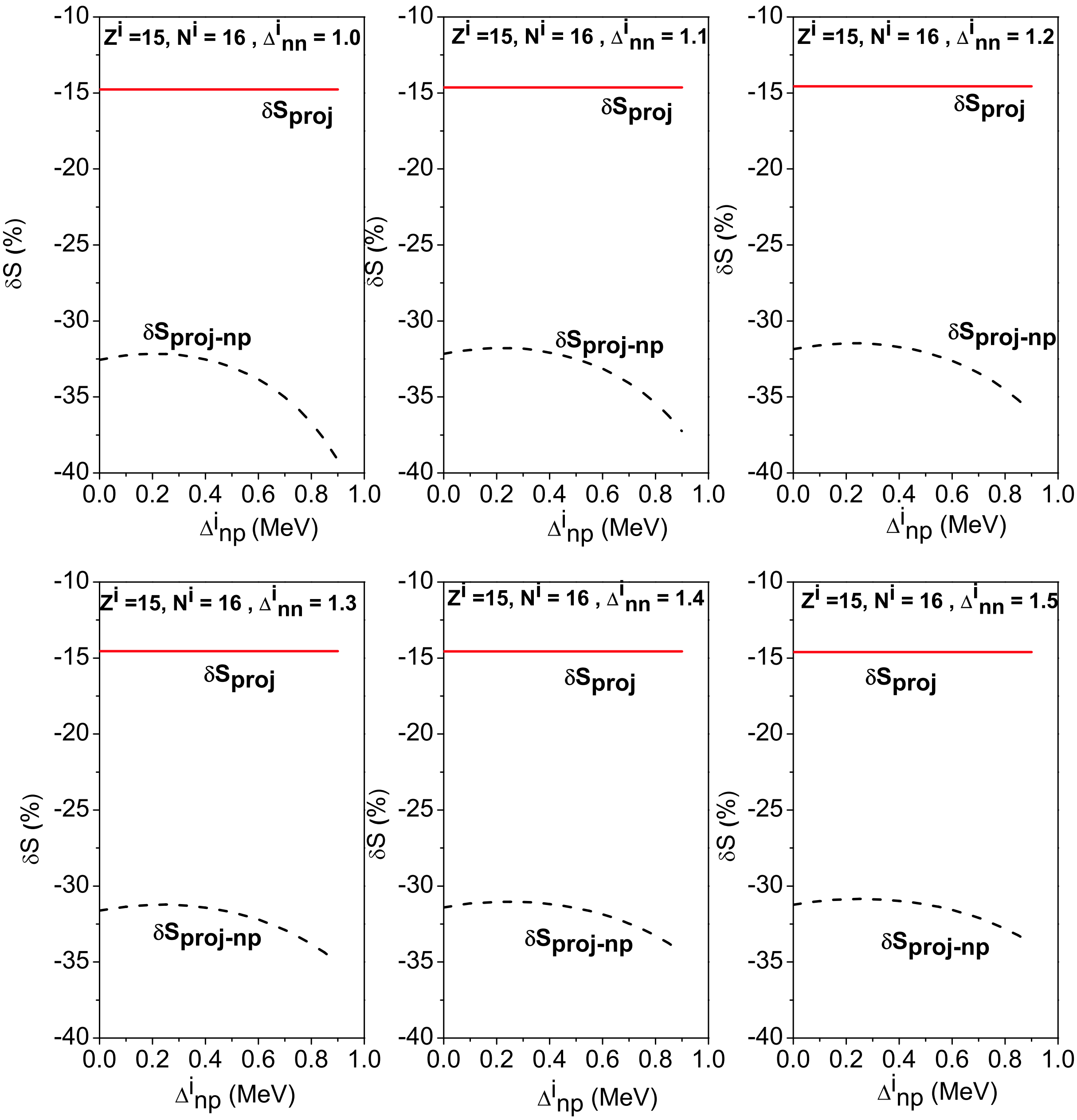

Fig. 6. (color online) Variations of the relative discrepancies of the spectroscopic factors (see the text for notations) corresponding to a one-proton stripping reaction, in the case of the system $Z^{\rm i}=15$, $N^{\rm i}=16$, as a function of the np gap parameter $\Delta^{\rm i}_{np}$ of the initial state, for several values of the neutron gap parameter of the initial state $\Delta^{\rm i}_{nn}$ (MeV). Solid lines show values obtained in the pairing between like-particles and dashed lines show those obtained in the np pairing case.

directly from the odd-even mass differences given by [34]:

$$\Delta_{pp}^{\rm exp}=-\frac{1}{8}[M(Z+2,N)-4M(Z+1,N)+6M(Z,N)\\ -4M(Z-1,N)+M(Z-2,N)], \quad (48)$$

$$\Delta_{nn}^{\rm exp}=-\frac{1}{8}[M(Z,N+2)-4M(Z,N+1)+6M(Z,N)\\ -4M(Z,N-1)+M(Z,N-2)], \quad (49)$$

$$\Delta_{np}^{\rm exp}=\frac{1}{4}\Big\{ 2[M(Z,N+1)+M(Z,N-1)\\ +M(Z-1,N)+M(Z+1,N)]-4M(Z,N)\\ -[M(Z+1,N+1)+M(Z-1,N+1)\\ +M(Z+1,N-1)+M(Z-1,N-1)] \Big\}. \quad (50)$$

In the latter expression, $M(Z,N)$ is the experimental mass value given in the Atomic Mass Evaluation 2012 (AME 2012) [139].

$G_{tt'}$ $(t,t'=n,p)$ are then obtained by solving Eqs. (7)-(11) for even-even systems and Eqs. (13)-(17) for odd ones.

Since the np pairing effect is supposed to be maximal in $N\simeq Z$ nuclei, we considered only systems such as $1\leqslant(N^{\rm i}-Z^{\rm i})\leqslant4$, of which the pairing gap parameters values $\Delta_{tt'}^{\rm exp}$, $t,t'=n,p$, are available in the initial and final states.

In the present work, we consider two kinds of reaction: one-proton stripping and one-neutron pick-up reactions. Note that there is no experimental data dealing with the SF values corresponding to these reactions in these kinds of nuclei.

4.2.1 One-proton stripping reactions

The values of the SF corresponding to one-proton stripping reactions are reported in Table 2. They have been evaluated using the BCS and SBCS approaches (i.e. when only the pairing between like-particles is considered), as well as the BCS-np and SBCS-np approaches (i.e. when the isovector pairing is taken into account). The values of the pairing gap parameters of the parent nucleus $\Delta_{tt'}^{\rm i}$ $(t,t'=n,p)$ are given in the same Table.

Table 2. Values of the pairing gap parameters (MeV) in the initial state (columns (2) to (4)) and the SFs corresponding to one-proton stripping reactions using the conventional BCS (column (5)) and SBCS (column (6)) approaches, as well as the BCS-np (column (7)) and SBCS-np (column (8)) approaches.

nucleus	$\Delta_{pp}^{\rm i}$	$\Delta_{nn}^{\rm i}$	$\Delta_{np}^{\rm i}$	S_{BCS}	S_{SBCS}	S_{BCS-np}	$S_{SBCS-np}$
^{34}S	1.562	1.818	0.244	0.412	0.303	0.482	0.361
^{36}S	1.522	2.226	0.513	0.584	0.437	0.574	0.397
^{35}Cl	1.929	1.375	0.692	0.256	0.271	0.181	0.219
^{37}Cl	1.535	1.513	0.605	0.584	0.704	0.544	0.655
^{38}Ar	1.441	2.100	0.250	0.440	0.319	0.444	0.320
^{40}Ar	1.776	1.767	0.684	0.398	0.291	0.374	0.282
^{39}K	1.875	1.732	0.489	0.678	0.683	0.495	0.514
^{41}K	1.875	1.189	0.549	0.787	0.746	0.744	0.720
^{42}Ca	2.110	1.676	0.524	0.810	0.667	0.594	0.488
^{44}Ca	2.097	1.702	0.630	0.844	0.689	0.683	0.574
^{43}Sc	2.477	0.887	1.359	0.129	0.174	0.001	0.015
^{45}Sc	2.172	1.081	0.723	0.205	0.283	0.166	0.247
^{46}Ti	2.093	1.878	0.898	0.526	0.385	0.331	0.289
^{48}Ti	1.896	1.564	0.585	0.607	0.432	0.201	0.197
^{47}V	2.131	0.709	1.284	0.294	0.351	0.094	0.164
^{49}V	1.834	0.897	0.695	0.450	0.540	0.387	0.467
^{50}Cr	1.697	1.584	0.526	0.374	0.309	0.304	0.231
^{52}Cr	1.578	1.595	0.336	0.534	0.399	0.530	0.389
^{51}Mn	1.787	1.056	0.737	0.362	0.442	0.238	0.323
^{53}Mn	1.540	1.129	0.488	0.678	0.706	0.411	0.410
^{54}Fe	1.497	1.594	0.259	0.798	0.646	0.455	0.348
^{56}Fe	1.572	1.425	0.336	0.749	0.612	0.415	0.357
^{55}Co	1.809	1.248	0.614	0.300	0.364	0.214	0.274
^{57}Co	1.650	1.098	0.290	0.401	0.514	0.345	0.446
^{58}Ni	1.667	1.349	0.232	0.660	0.566	0.532	0.434
^{60}Ni	1.663	1.537	0.334	0.612	0.485	0.327	0.244
^{59}Cu	1.623	0.924	0.648	0.198	0.260	0.073	0.124

Continued on next page

Table 2. (Continued)

nucleus	$\Delta^{\rm i}_{pp}$	$\Delta^{\rm i}_{nn}$	$\Delta^{\rm i}_{np}$	S_{BCS}	S_{SBCS}	S_{BCS-np}	$S_{SBCS-np}$
^{61}Cu	1.485	1.164	0.417	0.265	0.342	0.189	0.247
^{62}Zn	1.459	1.617	0.609	0.479	0.360	0.278	0.197
^{64}Zn	1.429	1.699	0.515	0.574	0.420	0.428	0.329
^{63}Ga	1.716	0.725	1.114	0.307	0.353	0.008	0.055
^{65}Ga	1.542	1.143	0.611	0.523	0.582	0.348	0.407
^{66}Ge	1.607	1.799	0.786	0.524	0.464	0.387	0.348
^{68}Ge	1.592	1.876	0.615	0.470	0.357	0.441	0.333
^{67}As	1.862	0.737	1.233	0.156	0.191	0.014	0.044
^{69}As	1.698	1.276	0.625	0.617	0.657	0.523	0.555
^{70}Se	1.755	1.914	0.764	0.622	0.557	0.363	0.314
^{72}Se	1.743	1.982	0.643	0.762	0.661	0.200	0.136
^{73}Br	1.725	1.259	0.568	0.502	0.568	0.198	0.219
^{75}Br	1.810	1.152	0.573	0.502	0.554	0.103	0.094
^{74}Kr	1.580	1.681	0.649	0.353	0.316	0.301	0.279
^{76}Kr	1.715	1.578	0.547	0.687	0.580	0.100	0.050
^{75}Rb	1.572	0.581	0.944	0.145	0.183	0.083	0.117
^{77}Rb	1.538	0.918	0.527	0.177	0.226	0.095	0.133
^{80}Sr	1.742	1.629	0.632	0.335	0.280	0.133	0.130
^{82}Sr	1.841	1.715	0.597	0.471	0.390	0.169	0.162
^{81}Y	1.692	0.999	0.692	0.400	0.431	0.250	0.287
^{83}Y	1.845	1.119	0.636	0.546	0.594	0.303	0.351
^{82}Zr	1.498	1.671	0.336	0.508	0.454	0.384	0.366
^{84}Zr	1.838	1.762	0.664	0.632	0.558	0.375	0.368
^{85}Nb	1.819	1.168	0.693	0.401	0.458	0.331	0.386
^{87}Nb	1.768	1.114	0.504	0.420	0.480	0.287	0.331
^{86}Mo	1.825	1.784	0.711	0.470	0.398	0.368	0.322
^{88}Mo	1.737	1.600	0.531	0.519	0.429	0.350	0.325
^{90}Tc	1.655	1.053	0.532	0.545	0.605	0.429	0.491
^{91}Tc	1.543	1.040	0.406	0.576	0.639	0.302	0.296
^{90}Ru	1.537	1.577	0.456	0.494	0.407	0.432	0.362
^{92}Ru	1.511	1.431	0.463	0.500	0.411	0.323	0.284
^{93}Rh	1.494	0.948	0.502	0.571	0.633	0.501	0.559
^{94}Pd	1.506	1.430	0.452	0.517	0.438	0.488	0.403
^{96}Pd	1.331	1.679	0.257	0.575	0.474	0.587	0.481
^{97}Ag	1.325	1.369	0.334	0.679	0.713	0.620	0.675
^{100}Cd	1.265	1.174	0.200	0.841	0.689	0.620	0.532

Since it was shown, within the framework of the picket-fence model, that the behavior of the SFs is different when the parent nucleus is even-even and when it is odd, these cases will be studied separately in the following.

1) Neutron-proton pairing effect

As a first step, the np pairing effect has been studied, before and after the projection, using the relative discrepancies δS_{np} and $\delta S_{np-proj}$. Their variations, as a function of the atomic number of the initial state $Z^{\rm i}$, for various values of the neutron excess in the initial state $(N^{\rm i}-Z^{\rm i})$, are reported in the left-hand part of Fig. 7 when the parent nucleus is even-even and that of Fig. 8 when the parent nucleus is odd. The average absolute values of the same quantities are reported in Table 3.

It may be seen from Figs. 7 and 8 that the np pairing effect on the SF values is important whether before or after the projection. The values of $|\delta S_{np}|$ and $|\delta S_{np-proj}|$ may reach up to 90%. It also appears that the np pairing effect is practically the same before and after the projection. Indeed, the values of δS_{np} and $\delta S_{np-proj}$ are close to each other except when $(N^{\rm i}-Z^{\rm i})$=1, i.e., when the np pairing effect is the most important.

From Table 3, one may conclude that the np pairing effect is, on average, more important when the parent nucleus is odd. On the other hand, when the parent nucleus is odd, as could be expected, $\overline{|\delta S_{np}|}$ and $\overline{|\delta S_{np-proj}|}$ clearly decrease as a function of $(N^{\rm i}-Z^{\rm i})$. Indeed, it is well established that Δ_{np}, and thus the np pairing effect, decreases as a function of $(N-Z)$ [31]. The behavior of $\overline{|\delta S_{np}|}$ and $\overline{|\delta S_{np-proj}|}$ as a function of the neutron excess in the initial state is more surprising when the parent nucleus is even-even, since these quantities increase as a function of $(N^{\rm i}-Z^{\rm i})$.

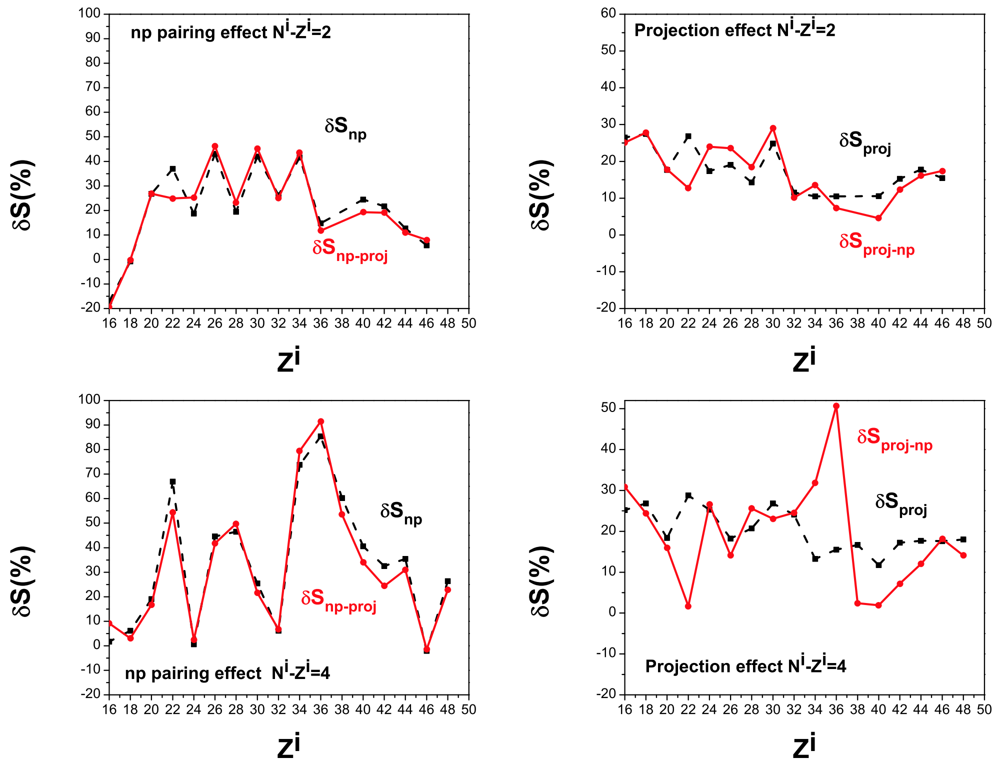

Fig. 7. (color online) np pairing effect (left) and projection effect (right) of the spectroscopic factors corresponding to one-proton stripping reactions, as a function of $Z^{\rm i}$, for $(N^{\rm i}-Z^{\rm i})$=2,4, when the parent nucleus is even-even. See the text for notations.

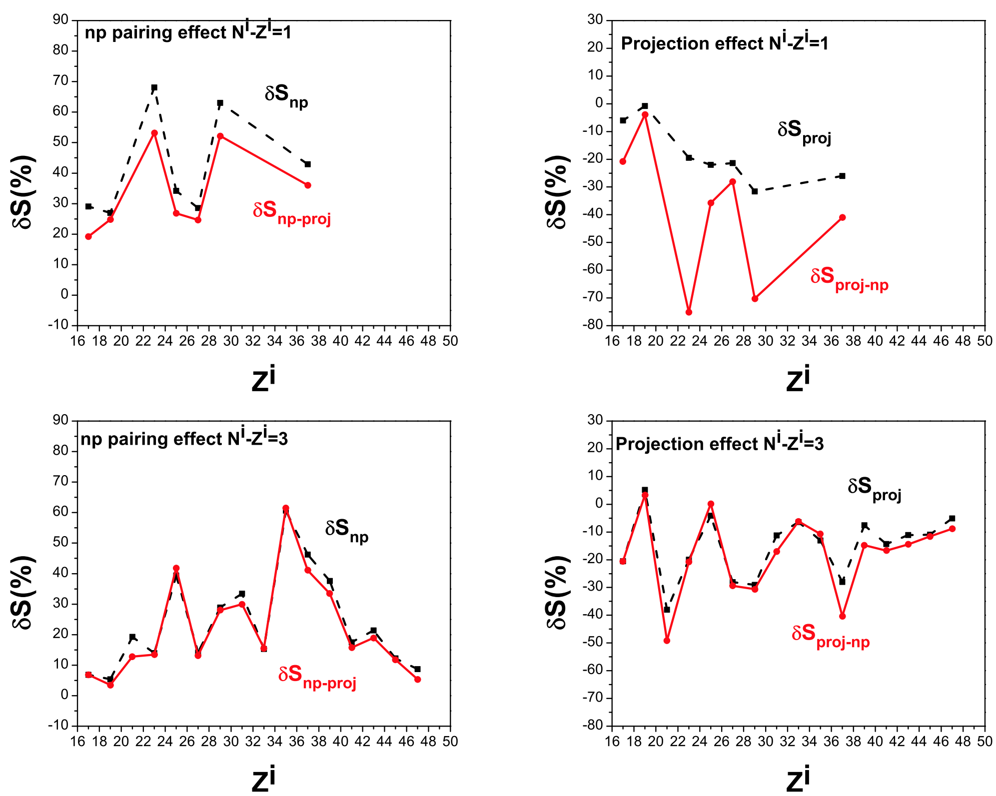

Fig. 8. (color online) np pairing effect (left) and projection effect (right) of the spectroscopic factors corresponding to one-proton stripping reactions, as a function of $Z^{\rm i}$, for $(N^{\rm i}-Z^{\rm i})$=1,3, when the parent nucleus is odd. See the text for notations.

Table 3. Average values of $|\delta S|$ (%) as a function of $(N^{\rm i}-Z^{\rm i})$ in the case of one-proton stripping reactions. Columns 2 and 3 of each part show the np pairing effect, and columns 4 and 5 show the projection effect. The upper part corresponds to the case when the parent nuclei are even-even, the lower part corresponds to the case when the parent nuclei are odd.

even-even parent nuclei				
$N^{\rm i}-Z^{\rm i}$	$\overline{\lvert\delta S_{np}\rvert}$	$\overline{\lvert\delta S_{np-proj}\rvert}$	$\overline{\lvert\delta S_{proj}\rvert}$	$\overline{\lvert\delta S_{proj-np}\rvert}$
2	23,45	23,25	17,70	17,33
4	33,72	31,95	20,11	19,12
total	28.94	27.88	18.98	18.28
odd parent nuclei				
$N^{\rm i}-Z^{\rm i}$	$\overline{\lvert\delta S_{np}\rvert}$	$\overline{\lvert\delta S_{np-proj}\rvert}$	$\overline{\lvert\delta S_{proj}\rvert}$	$\overline{\lvert\delta S_{proj-np}\rvert}$
1	47.98	39.21	18.73	61.40
3	23.81	22.07	15.78	18.40
total	28.60	25.02	16.53	24.55

2) Projection effect

As a second step, the projection effect has been studied when only the pairing between like-particles is considered and when the isovector pairing is taken into account, using the relative discrepancies δS_{proj} and $\delta S_{proj-np}$. Their variations as a function of the atomic number of the initial state $Z^{\rm i}$ for various values of the neutron excess in the initial state $(N^{\rm i}-Z^{\rm i})$ are reported in the right-hand part of Fig. 7 when the parent nucleus is even-even and that of Fig. 8 when the parent nucleus is odd. The average absolute values of the same quantities are reported in Table 3.

From Figs. 7 and 8, it may be seen that the projection effect is far from negligible, since $|\delta S|$ may reach up to 50%.

As can be confirmed in Table 3, except when $(N^{\rm i}-Z^{\rm i})=1$, the projection effect is practically the same in the pairing between like-particles and in the isovector pairing, whether the initial state is even-even or odd. Moreover, $|\delta S_{proj}|$ and $|\delta S_{proj-np}|$ are quasi-independent of the value of the neutron excess.

When $(N^{\rm i}-Z^{\rm i})=1$, the particle fluctuation effect is more important in the isovector pairing case.

Finally, one notices that, on average, the np pairing effect is more important than the projection one, except when $(N^{\rm i}-Z^{\rm i})=1$.

4.2.2 One-neutron pick-up reactions

The values of the SFs corresponding to one-neutron pick-up reactions are reported in Table 4. The pairing gap parameters in the initial state are given in the same table.

Table 4. Values of the pairing gap parameters (MeV) in the initial state (columns (2) to (4)) and the SFs corresponding to one-neutron pick-up reactions using the conventional BCS (column (5)) and SBCS (column (6)) approaches, as well as the BCS-np (column (7)) and SBCS-np (column (8)) approaches.

nucleus	$\Delta^{\rm i}_{pp}$	$\Delta^{\rm i}_{nn}$	$\Delta^{\rm i}_{np}$	S_{BCS}	S_{SBCS}	S_{BCS-np}	$S_{SBCS-np}$
^{33}S	1.316	2.048	0.562	0.606	0.614	0.535	0.580
^{34}S	1.562	1.818	0.244	0.555	0.492	0.562	0.404
^{35}S	1.032	1.978	0.495	0.458	0.425	0.548	0.512
^{36}S	1.522	2.226	0.513	0.756	0.342	0.760	0.369
^{37}Ar	1.065	2.226	0.841	0.374	0.427	0.310	0.416
^{38}Ar	1.441	2.100	0.250	0.798	0.697	0.659	0.580
^{39}Ar	1.019	1.944	0.570	0.901	0.731	0.836	0.697
^{40}Ar	1.776	1.767	0.684	0.219	0.111	0.191	0.129
^{41}Ca	1.346	2.132	0.838	0.716	0.672	0.595	0.655
^{42}Ca	2.110	1.676	0.524	0.223	0.138	0.167	0.116
^{43}Ca	1.397	1.708	0.670	0.643	0.484	0.658	0.479
^{44}Ca	2.097	1.702	0.630	0.483	0.314	0.381	0.253
^{45}Ti	0.933	2.299	1.488	0.455	0.423	0.111	0.065
^{46}Ti	2.093	1.878	0.898	0.383	0.311	0.174	0.142
^{47}Ti	1.352	1.661	0.648	0.533	0.458	0.574	0.483
^{48}Ti	1.896	1.564	0.585	0.663	0.347	0.045	0.067
^{49}Cr	1.090	1.791	0.799	0.243	0.262	0.130	0.205
^{50}Cr	1.697	1.584	0.526	0.428	0.289	0.346	0.210
^{51}Cr	1.062	1.674	0.580	0.593	0.502	0.578	0.454
^{52}Cr	1.578	1.595	0.336	0.290	0.180	0.202	0.142
^{53}Fe	0.950	1.873	0.799	0.515	0.538	0.422	0.533
^{54}Fe	1.497	1.594	0.259	0.342	0.275	0.027	0.035

Continued on next page

Table 4. (Continued)

nucleus	Δ^{i}_{pp}	Δ^{i}_{nn}	Δ^{i}_{np}	S_{BCS}	S_{SBCS}	S_{BCS-np}	$S_{SBCS-np}$
^{55}Fe	1.233	1.429	0.304	0.776	0.656	0.508	0.218
^{56}Fe	1.572	1.425	0.336	0.364	0.250	0.262	0.142
^{57}Ni	1.296	1.694	0.578	0.615	0.562	0.498	0.501
^{58}Ni	1.667	1.349	0.232	0.326	0.262	0.202	0.164
^{59}Ni	1.338	1.445	0.311	0.767	0.666	0.749	0.584
^{60}Ni	1.663	1.537	0.334	0.284	0.240	0.136	0.088
^{61}Zn	0.606	1.731	0.982	0.419	0.385	0.300	0.386
^{62}Zn	1.459	1.617	0.609	0.397	0.351	0.234	0.163
^{63}Zn	0.937	1.643	0.491	0.585	0.542	0.403	0.303
^{64}Zn	1.429	1.699	0.515	0.399	0.346	0.062	0.067
^{65}Ge	0.529	1.905	1.230	0.292	0.318	0.028	0.185
^{66}Ge	1.607	1.799	0.786	0.201	0.225	0.066	0.065
^{67}Ge	0.978	1.852	0.635	0.449	0.409	0.364	0.276
^{68}Ge	1.592	1.876	0.615	0.277	0.209	0.080	0.063
^{69}Se	0.789	2.018	1.207	0.165	0.139	0.557	0.121
^{70}Se	1.755	1.914	0.764	0.159	0.148	0.021	0.018
^{71}Se	1.112	1.956	0.668	0.646	0.577	0.399	0.183
^{72}Se	1.743	1.982	0.643	0.350	0.296	0.004	0.006
^{73}Kr	0.643	1.891	1.213	0.267	0.237	0.152	0.233
^{74}Kr	1.580	1.681	0.649	0.260	0.228	0.129	0.008
^{75}Kr	1.113	1.590	0.532	0.372	0.331	0.269	0.168
^{76}Kr	1.715	1.578	0.547	0.376	0.304	0.005	0.003
^{78}Sr	1.353	1.310	0.212	0.372	0.317	0.018	0.054
^{79}Sr	1.137	1.469	0.381	0.574	0.501	0.282	0.180
^{80}Sr	1.742	1.629	0.632	0.196	0.197	0.010	0.015
^{82}Zr	1.498	1.671	0.336	0.207	0.218	0.132	0.126
^{83}Zr	1.021	1.803	0.662	0.581	0.546	0.438	0.336
^{84}Zr	1.838	1.762	0.664	0.418	0.351	0.181	0.134
^{86}Mo	1.825	1.784	0.711	0.406	0.398	0.271	0.244
^{87}Mo	1.167	1.669	0.611	0.503	0.484	0.413	0.357
^{88}Mo	1.737	1.600	0.531	0.559	0.431	0.285	0.242
^{90}Ru	1.537	1.577	0.456	0.516	0.501	0.349	0.286
^{91}Ru	1.087	1.497	0.418	0.598	0.555	0.710	0.618
^{92}Ru	1.511	1.431	0.463	0.595	0.337	0.109	0.110
^{94}Pd	1.506	1.430	0.452	0.579	0.494	0.458	0.376
^{95}Pd	0.973	1.550	0.443	0.761	0.661	0.738	0.622
^{96}Pd	1.331	1.679	0.257	0.290	0.213	0.202	0.158
^{98}Cd	1.310	1.756	0.290	0.297	0.260	0.215	0.177
^{99}Cd	1.023	1.431	0.292	0.786	0.714	0.780	0.614
^{100}Cd	1.265	1.174	0.200	0.238	0.197	0.242	0.181

1) Neutron-proton pairing effect

In order to study the np pairing effect on the SF corresponding to one-neutron pick-up reactions, before and after the projection, the variations of the relative discrepancies δS_{np} and $\delta S_{np-proj}$ as a function of the atomic number of the initial state Z^{i} for various values of the neutron excess in the initial state $(N^{\mathrm{i}}-Z^{\mathrm{i}})$ are reported in the left-hand part of Fig. 9 when the parent nucleus is even-even and that of Fig. 10 when the parent nucleus is odd. From Figs. 9 and 10, one may conclude that the np pairing effect is also important in this kind of reaction, since $|\delta S_{np}|$ and $|\delta S_{np-proj}|$ may reach up to 90%. From the figures, it may be also seen that δS_{np} and $\delta S_{np-proj}$ behave in a similar way and are somewhat close to each other, except in the case $(N^{\mathrm{i}}-Z^{\mathrm{i}})=1$.

The average absolute values of the same quantities are reported in Table 5. It can be then seen that $\overline{|\delta S_{np}|}$ and $\overline{|\delta S_{np-proj}|}$ are of the same order of magnitude when the parent nucleus is even-even. Furthermore, contrary to what could be expected, the np pairing effect increases as a function of the neutron excess, whether before or after the projection.

When the parent nucleus is odd, not only do the $\overline{|\delta S_{np}|}$ and $\overline{|\delta S_{np-proj}|}$ values clearly differ, but their respective behavior as a function of $(N^{\mathrm{i}}-Z^{\mathrm{i}})$ is different.

Let us note that these results clearly differ from those

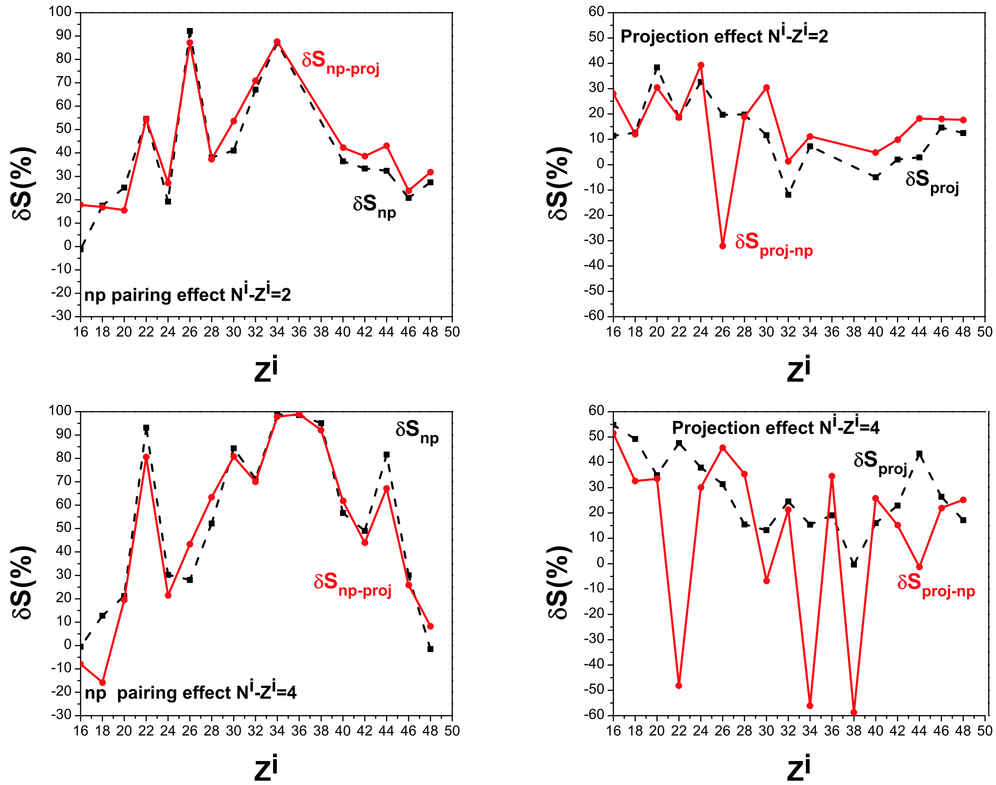

Fig. 9. (color online) np pairing effect (left) and projection effect (right) of the spectroscopic factors corresponding to one-neutron pick-up reactions, as a function of Z^{i}, for $(N^{\mathrm{i}}-Z^{\mathrm{i}})$=2,4, when the parent nucleus is even-even. See the text for notations.

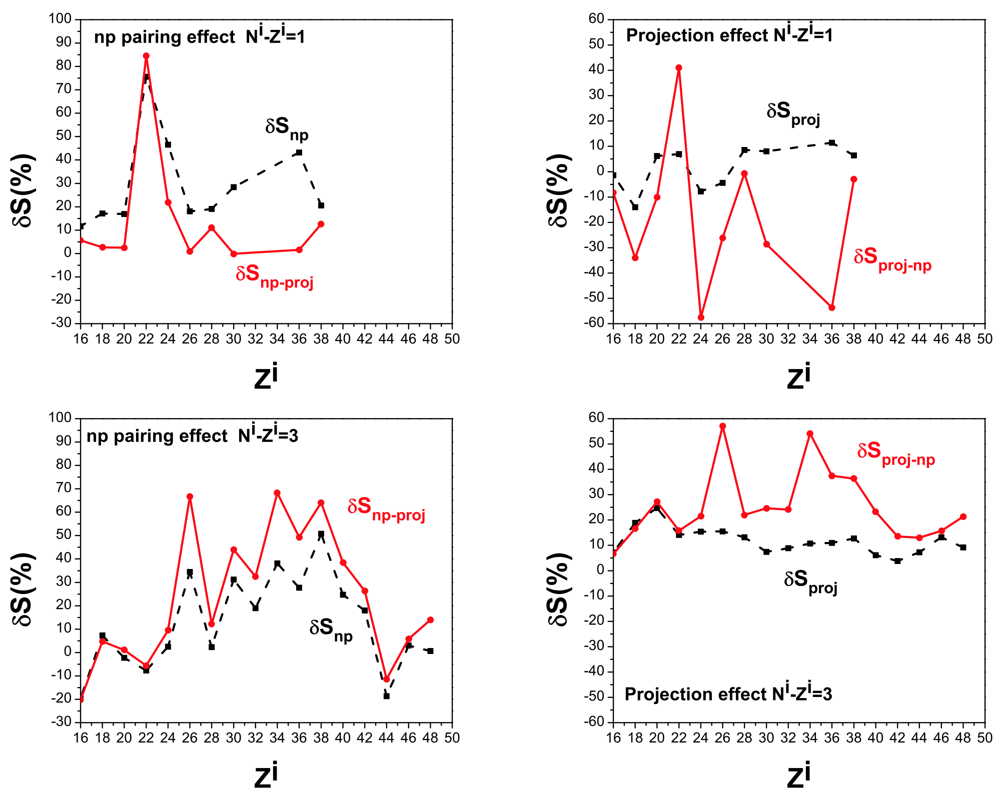

Fig. 10. (color online) np pairing effect (left) and projection effect (right) of the spectroscopic factors corresponding to one-neutron pick-up reactions, as a function of Z^{i}, for $(N^{\mathrm{i}}-Z^{\mathrm{i}})$=1,3, when the parent nucleus is odd. See the text for notations.

obtained in the case of one-proton stripping reactions.

2) Projection effect

In order to study the projection effect on the SF corresponding to one-neutron pick-up reactions, when only the pairing between like-particles is taken into account and when the isovector pairing is included, δS_{proj} and $\delta S_{proj-np}$ have been evaluated as a function of $Z^{\rm i}$. Their variations are shown in the right-hand side of Fig. 9 when the parent nucleus is even-even and in that of Fig. 10 when the parent nucleus is odd. The average absolute values of the same quantities are reported in Table 5.

Table 5. Average values of $|\delta S|$ (%) as a function of $(N^{\rm i}-Z^{\rm i})$ in the case of one-neutron pick-up reactions. Columns 2 and 3 of each part show the np pairing effect, and columns 4 and 5 show the projection effect. The upper part corresponds to the case when the parent nuclei are even-even, the lower part corresponds to the case when the parent nuclei are odd.

even-even parent nuclei				
$N^{\rm i}-Z^{\rm i}$	$\overline{\lvert\delta S_{np}\rvert}$	$\overline{\lvert\delta S_{np-proj}\rvert}$	$\overline{\lvert\delta S_{proj}\rvert}$	$\overline{\lvert\delta S_{proj-np}\rvert}$
2	39.57	43.22	12.50	19.40
4	53.25	52.85	27.65	32.00
total	46.95	49.80	21.32	28.15
odd parent nuclei				
$N^{\rm i}-Z^{\rm i}$	$\overline{\lvert\delta S_{np}\rvert}$	$\overline{\lvert\delta S_{np-proj}\rvert}$	$\overline{\lvert\delta S_{proj}\rvert}$	$\overline{\lvert\delta S_{proj-np}\rvert}$
1	29.70	14.33	7.51	26.29
3	18.12	27.88	11.71	25.30
total	30.09	22.51	10.36	27.55

From Figs. 9 and 10, it appears that the projection effect is as important as in the case of one-proton stripping reactions and may also reach up to 50% in absolute value. However, in this case, the behavior of δS_{proj} is different from that of $\delta S_{proj-np}$. As may be seen in Table 5, the projection effect varies as a function of $(N^{\rm i}-Z^{\rm i})$, unlike in the case of one-proton stripping reactions.

Finally, it is worth noticing that, except when $(N^{\rm i}-Z^{\rm i})=1$, the np pairing effect is, on average, more important than the projection effect.

5 Conclusion

Expressions of the SFs corresponding to one-particle transfer reactions have been established using a schematic definition. These expressions have been derived by taking into account the isovector np pairing correlations and a particle-number projection in the framework of the generalized Sharp-BCS method. Recently proposed expressions of the projected wave-functions of odd-mass nuclei [120] have been used.

In order to test the formalism, numerical calculations have been performed using the single-particle energies of the schematic picket-fence model. The np pairing effect has been evaluated by comparing the results of the formalism of the present work to that of the conventional BCS approach. The particle-number fluctuation effect has been evaluated by comparing the results obtained before and after the particle-number projection. It was shown that the np pairing and particle-number fluctuation effects are far from negligible and they depend on the pairing gap parameter values. It also appears that, as could be foreseen, the behavior of the np pairing and projection effects is not the same when the parent nuclei are even-even or odd.

Predictions dealing with the SF corresponding to one-proton stripping and one-neutron pick-up reactions in proton-rich nuclei have then been established. The single-particle energies and eigen-states used are those of the realistic Woods-Saxon model. Since the np pairing is supposed to be maximal in $N\simeq Z$ nuclei, we considered only nuclei such as the neutron excess in the initial state is in the range $1\leqslant(N^{\rm i}-Z^{\rm i})\leqslant4$. Furthermore, only nuclei of which the gap parameters may be deduced from the experimental odd-even mass differences were considered.

The np pairing and projection effects on the SF values were then studied. It was shown that both effects are important and thus have to be included in future calculations of the SFs corresponding to these kinds of reaction. As was already the case within the picket-fence model, the results are different when the parent nuclei are even-even or odd. Finally, except when $(N^{\rm i}-Z^{\rm i})=1$, the np pairing effect is more important, on average, than the projection effect.

Appendices A: Pairing between like-particles case

Wave-functions

Before projection

Let us consider a system constituted by an even number of paired particles (neutrons or protons). If only the pairing between like-particles is considered, the BCS ground-state is given by [140]

$$|BCS\rangle_t=\prod_{j>0}\left(u_{jt}+v_{jt}a_{jt}^+a_{\tilde{j}t}^+\right)|0\rangle\ ,\quad t=n,p. \tag{A1}$$

a_{jt}^{+} is the creation operator of a particle in the state $|jt\rangle$ of energy ε_{jt}. u_{jt} and v_{jt} respectively represent the inoccupation and occupation probability amplitudes of the state $|jt\rangle$. They are given by

$$\left.\begin{matrix} u_{jt}^2 \\ v_{jt}^2 \end{matrix}\right\} = \frac{1}{2}\left\{1 \pm \frac{\varepsilon_{jt}-\lambda_t}{\sqrt{(\varepsilon_{jt}-\lambda_t)^2+\Delta_t^2}}\right\} \quad t=n,p \quad , \tag{A2}$$

$$\Delta_t = G_t \sum_{j>0} v_{jt} u_{jt} \quad t=n,p \tag{A3}$$

being the pairing gap parameter and λ_t the energy of the Fermi-level.

The particle-number conservation condition reads, in this case,

$$N_t = 2\sum_{j>0} v_{jt}^2 \quad t=n,p. \tag{A4}$$

When the particle-number is odd, the BCS ground-state is defined using the blocked-level technique. It is then given by [140]

$$|\nu t\rangle = a_{\nu t}^{\dagger} \prod_{\substack{j>0 \\ j\neq\nu}} \left(u_{jt}(\nu) + v_{jt}(\nu) a_{jt}^{+} a_{\tilde{j}t}^{+}\right)|0\rangle \quad , \quad t=n,p, \tag{A5}$$

where ν refers to the blocked level and $v_{jt}(\nu)$ et $u_{jt}(\nu)$ are the inoccupation and occupation probability amplitudes of the single-particle state $|jt\rangle$. The pairing gap parameter reads, in this case,

$$\Delta_t^{\nu} = G_t \sum_{\substack{j>0 \\ j\neq\nu}} v_{jt}(\nu) u_{jt}(\nu) \quad t=n,p. \tag{A6}$$

The particle-number conservation condition is given by

$$N_t^{\nu} = 1 + 2\sum_{\substack{j>0 \\ j\neq\nu}} v_{jt}^2(\nu) \quad t=n,p. \tag{A7}$$

After projection

After projection, the ground-state is given, when the particle-number is even, by [73]

$$|\psi_m\rangle_t = C_{mt} \left\{ \sum_{k=0}^{m+1} \xi_k z_k^{-P_t} |\psi(z_k)\rangle_t + cc \right\} \quad , t=n,p \tag{A8}$$

where P_t is the number of paired pairs, ξ_k and z_k are defined by Eq. (21), m is a non-zero integer, cc means the complex conjugate with respect to z_k, and

$$|\psi(z_k)\rangle_t = \prod_{j>0} \left(u_{jt} + z_k v_{jt} a_{jt}^{+} a_{\tilde{j}t}^{+}\right)|0\rangle. \tag{A9}$$

The normalization constant C_{mt} is given by

$$1 = 2(m+1) C_{mt}^2 \left\{ \sum_{k=0}^{m+1} \xi_k z_k^{-P_t} \prod_j \left(u_{jt}^2 + z_k v_{jt}^2\right) + cc \right\}. \tag{A10}$$

A property similar to the one given by Eq. (22) has been used to derive the latter expression, that is

$${}_t\langle\psi_m|\mathcal{O}|\psi_m\rangle_t = 2(m+1) C_{mt} \; {}_t\langle BCS|\mathcal{O}|\psi_m\rangle_t, \tag{A11}$$

where $\mathcal{O}$ is any operator which conserves the particle number.

As soon as

$$2(m+1) > \max(P_t, \Omega_t - P_t) \quad , \quad t=n,p \quad , \tag{A12}$$

all the false components in the state $|\psi_m\rangle_t$ are eliminated, Ω_t being the total degeneracy of states.

When the particle-number is odd, the projected ground-state is given by

$$|(\nu t)_m\rangle = C_{mt}^{\nu} \left\{ \sum_{k=0}^{m+1} \xi_k z_k^{-P_t} a_{\nu t}^{\dagger} |(\nu t(z_k))\rangle + cc \right\} \tag{A13}$$

where we set

$$|(\nu t(z_k))\rangle = \prod_{\substack{j>0 \\ j\neq\nu}} \left(u_{jt}(\nu) + z_k v_{jt}(\nu) A_{jt}^{\dagger}\right)|0\rangle. \tag{A14}$$

In this expression, P_t is the number of paired pairs. Using the property (A11), the normalization C_{mt}^{ν} is given by

$$1 = 2(m+1)\left(C_{mt}^{\nu}\right)^2 \times \left\{ \sum_{k=0}^{m+1} \xi_k z_k^{-P_t} \prod_{j\neq\nu} \left(u_{jt}^2(\nu) + z_k v_{jt}^2(\nu)\right) + cc \right\}. \tag{A15}$$

Spectroscopic factors

As in this case, the proton and neutron systems are considered separately, the wave-function which describes the total system is defined as the product of the neutron and proton system wave-functions. In the following, it will be assumed that the transfer of one neutron or one proton does not affect the proton or neutron system.

Before projection

When the nucleus considered is even-even, the wave-function of the total system is given by

$$\left|\psi^{i(f)}\right\rangle = \left|BCS^{i(f)}\right\rangle_n \left|BCS^{i(f)}\right\rangle_p, \tag{A16}$$

where $|BCS\rangle_t$ ($t=n,p$) is given by Eq. (A1).

When the considered nucleus is odd, the total wave-function is given by

$$\left|\psi^{i(f)}\right\rangle = \left|BCS^{i(f)}\right\rangle_t \left|\nu t'^{i(f)}\right\rangle, \quad t,t'=n,p, \quad t\neq t' \tag{A17}$$

where $|BCS\rangle_t$ ($t=n,p$) is given by Eq. (A1) and $|\nu t'\rangle$ is given by Eq. (A5).

The SFs corresponding to one-particle transfer reactions starting from an even-even nucleus, defined by Eqs.

(28) and (29), then become [141]

$$\sqrt{s_t^{\mathrm{STR}(1)}}=u_{\nu t}^{\mathrm{i}}\prod_{j>0,j\neq\nu}\left(v_{jt}^{\mathrm{i}}v_{jt}^{\mathrm{f}}(\nu)+u_{jt}^{\mathrm{i}}u_{jt}^{\mathrm{f}}(\nu)\right), \qquad \text{(A18)}$$

$$\sqrt{s_t^{\mathrm{PIC}(1)}}=v_{\nu t}^{\mathrm{i}}\prod_{j>0,j\neq\nu}\left(v_{jt}^{\mathrm{i}}v_{jt}^{\mathrm{f}}(\nu)+u_{jt}^{\mathrm{i}}u_{jt}^{\mathrm{f}}(\nu)\right). \qquad \text{(A19)}$$

When the parent nucleus is odd, they are given by

$$\sqrt{s_t^{\mathrm{STR}(2)}}=v_{\nu t}^{\mathrm{f}}\prod_{j>0,j\neq\nu}\left(v_{jt}^{\mathrm{i}}(\nu)v_{jt}^{\mathrm{f}}+u_{jt}^{\mathrm{i}}(\nu)u_{jt}^{\mathrm{f}}\right), \qquad \text{(A20)}$$

$$\sqrt{s_t^{\mathrm{PIC}(2)}}=u_{\nu t}^{\mathrm{f}}\prod_{j>0,j\neq\nu}\left(v_{jt}^{\mathrm{i}}(\nu)v_{jt}^{\mathrm{f}}+u_{jt}^{\mathrm{i}}(\nu)u_{jt}^{\mathrm{f}}\right). \qquad \text{(A21)}$$

After projection

After projection, the total wave-function of an even-even system is given by

$$\left|\psi_m^{\mathrm{i(f)}}\right\rangle=\left|\psi_m^{\mathrm{i(f)}}\right\rangle_n\left|\psi_m^{\mathrm{i(f)}}\right\rangle_p, \qquad \text{(A22)}$$

where $\left|\psi_m\right\rangle_t$ is defined by Eq. (A8).

When the considered nucleus is odd, the total wave-function is given by

$$\left|\psi_m^{\mathrm{i(f)}}\right\rangle=\left|\psi_m^{\mathrm{i(f)}}\right\rangle_t\left|(\nu t')_m\right\rangle,\ \ t,t'=n,p\ \ ,\ \ t'\neq t. \qquad \text{(A23)}$$

In the latter expression, $\left|\psi_m\right\rangle_t$ is defined by Eq. (A8) and $\left|(\nu t')_m\right\rangle$ is given by Eq. (A13).

The SFs corresponding to one-particle transfer reactions, starting from an even-even nucleus, are then given by

$$\sqrt{\left(s_t^{\mathrm{STR}(1)}\right)_m}=2(m+1)C_{mt}^{\mathrm{i}}C_{mt}^{\nu\mathrm{f}}\left\{\sum_{k=0}^{m+1}\xi_k z_k^{-P_t^{\mathrm{f}}}u_{\nu t}^{\mathrm{i}}\prod_{j>0,j\neq\nu}\left(u_{jt}^{\mathrm{i}}u_{jt}^{\mathrm{f}}(\nu)+z_k v_{jt}^{\mathrm{i}}v_{jt}^{\mathrm{f}}(\nu)\right)+cc\right\}, \qquad \text{(A24)}$$

$$\sqrt{\left(s_t^{\mathrm{PIC}(1)}\right)_m}=2(m+1)C_{mt}^{\mathrm{i}}C_{mt}^{\nu\mathrm{f}}\left\{\sum_{k=0}^{m+1}\xi_k z_k^{-P_t^{\mathrm{f}}}v_{\nu t}^{\mathrm{i}}\prod_{j>0,j\neq\nu}\left(u_{jt}^{\mathrm{i}}u_{jt}^{\mathrm{f}}(\nu)+z_k v_{jt}^{\mathrm{i}}v_{jt}^{\mathrm{f}}(\nu)\right)+cc\right\}. \qquad \text{(A25)}$$

When the parent nucleus is odd, they are given by

$$\sqrt{\left(s_t^{\mathrm{STR}(2)}\right)_m}=2(m+1)C_{mt}^{\mathrm{i}}C_{mt}^{\mathrm{f}}\left\{\sum_{k=0}^{m+1}\xi_k z_k^{-P_t^{\mathrm{f}}}v_{\nu t}^{\mathrm{f}}\prod_{j>0,j\neq\nu}\left(u_{jt}^{\mathrm{i}}(\nu)u_{jt}^{\mathrm{f}}+z_k v_{jt}^{\mathrm{i}}(\nu)v_{jt}^{\mathrm{f}}\right)+cc\right\}, \qquad \text{(A26)}$$

$$\sqrt{\left(s_t^{\mathrm{PIC}(2)}\right)_m}=2(m+1)C_{mt}^{\mathrm{i}}C_{mt}^{\mathrm{f}}\left\{\sum_{k=0}^{m+1}\xi_k z_k^{-P_t^{\mathrm{f}}}u_{\nu t}^{\mathrm{f}}\prod_{j>0,j\neq\nu}\left(u_{jt}^{\mathrm{i}}(\nu)u_{jt}^{\mathrm{f}}+z_k v_{jt}^{\mathrm{i}}(\nu)v_{jt}^{\mathrm{f}}\right)+cc\right\}. \qquad \text{(A27)}$$

It has been assumed here that convergence is reached for the same value m of the extraction degrees of the false components of the wave-function in the initial and final states.

One may note a formal similarity between expressions of the various SFs obtained before and after the projection.

References

1 M. H. MacFarlane and J. B. French, Rev. Mod. Phys., **32**: 567-691 (1960)
2 N. K. Timofeyuk, Phys. Rev. C, **81**: 064306 (2010)
3 O. Jensen, G. Hagen, T. Papenbrock et al, Phys. Rev. C, **82**: 014310 (2010)
4 F. Flavigny, *Détermination de facteurs spectroscopiques absolus par réactions de knockout et de transfert*, Ph. D. Thesis (Université Paris-Sud 6, 2011) (in French)
5 P. O. Hess, A. Algora, J. Cseh et al, Phys. Rev. C, **70**: 051303(R) (2004)
6 M. B. Tsang, Jenny Lee, and W. G. Lynch, Phys. Rev. Lett., **95**: 222501 (2005)
7 F. Beck, D. Frekers, P. von Neumann-Cosel et al, Phys. Lett. B, **645**: 128-132 (2007)
8 Jenny Lee, M. B. Tsang, and W. G. Lynch, Phys. Rev. C, **75**: 064320 (2007)
9 D. Y. Pang, F. M. Nunes, and A.M. Mukhamedzhanov, Phys. Rev. C, **75**: 024601 (2007)
10 M. B. Tsang, Jenny Lee, S. C. Su et al, Phys. Rev. Lett., **102**: 062501 (2009)
11 H. T. Fortune and R. Sherr, Phys. Rev. C, **85**: 027305 (2012)
12 M. Horoi, J. Phys.: Conf. Ser., **413**: 012020 (2013)
13 N. K. Timofeyuk, Phys. Rev. C, **88**: 044315 (2013)
14 Sh. Hamada, N. Burtebayev, and N. Amangeldi, Int. J. Mod. Phys. E, **23**: 1450061 (2014).
15 N. Burtebayev, J. T. Burtebayeva, A. Duisebayev et al, Acta Phys. Pol. B, **46**: 1037-1054 (2015)
16 N. V. Gnezdilov, E. E. Saperstein, and S. V. Tolokonnikov, Phys. Atom. Nucl., **78**: 27-37 (2015)
17 P. C. Srivastava and V. Kumar, Phys. Rev. C, **94**: 064306 (2016)
18 Vishal Srivastava, C. Bhattacharya, T. K. Rana et al, Phys. Rev. C, **93**: 044601 (2016)
19 F. Wang, B. H. Sun, Z. Liu et al, Phys. Lett. B, **770**: 83-87 (2017)
20 M. Febbraro, F. D. Becchetti, R. O. Torres-Isea et al, Phys. Rev. C, **96**: 024613 (2017)
21 A. Belhout, S. Ouichaoui, H. Beaumevieille et al, Phys. Rev. C, **96**: 054601 (2017)
22 A. O. Macchiavelli, H. L. Crawford, C. M. Campbell et al,

Phys. Rev. C, **96**: 054302 (2017)
23 A. O. Macchiavelli, H. L. Crawford, C. M. Campbell et al, Phys. Rev. C, **97**: 011302(R) (2017)
24 E. Baranger and T. T. S. Kuo, Nucl. Phys. A, **97**: 289-297 (1967)
25 S. Aberg, P. B. Semmes, and W. Nazarewicz, Phys. Rev. C, **56**: 1762-1773 (1997)
26 C. Basu, Pramana J. Phys., **63**: 1047-1052 (2004)
27 J. M. Yao, B. Sun, P. J. Woods et al, Phys. Rev. C, **77**: 024315 (2008)
28 H. F. Zhang, Y. J. Wang, J. M. Dong et al, J. Phys. G: Nucl. Part. Phys., **37**: 085107 (2010)
29 K. Kumar, J. B. Gupta, and J. H. Hamilton, Nucl. Phys. A, **448**: 36-44 (1986)
30 A. L. Goodman, Adv. Nucl. Phys., **11**: 263-366 (1979)
31 O. Civitarese and M. Reboiro, Phys. Rev. C, **56**: 1179-1182 (1997)
32 A. A. Raduta, L. Pacearescu, P. Sarriguren et al, Nucl. Phys. A, **675**: 503-530 (2000)
33 A. A. Raduta and E. Moya de Guerra, Ann. Phys. (NY), **284**: 134-166 (2000)
34 F. Simkovic, Ch. C. Moustakidis, L. Pacearescu et al, Phys. Rev. C, **68**: 054319 (2003)
35 D. Mokhtari, N. H. Allal, and M. Fellah, Acta Phys. Hung. A: Heavy Ion Phys., **19**: 187-190 (2004)
36 N. Sandulescu, D. Negrea, J. Dukelsky et al, Phys. Rev. C, **85**: 061303(R) (2012)
37 J. Dukelsky, S. Pittel, and C. Esebbag, Phys. Rev. C, **93**: 034313 (2016)
39 H. Sagawa, C. L. Baiy, and G. Colo, Phys. Scrip. , **91**: 083011 (2016)
39 M. Sambataro and N. Sandulescu, Phys. Rev. C, **93**: 054320 (2016)
40 Wenjin Tan, Dongdong Ni, and Zhongzhou Ren, Chin. Phys. C, **41**: 54103 (2017)
41 P. C. Srivastava, S. Aberg, and I. Ragnarsson, Phys. Rev. C, **95**: 11303(R) (2017)
42 M. Saleh Yousef, H.M. Elsharkawy, and N. Rashed, Nucl. Phys. A, **975**: 97-106 (2018)
43 S. Frauendorf and A. O. Machiavelli, Prog. Part. Nucl. Phys., **78**: 24-90 (2014)
44 C. Qi and R. Wyss, Phys. Scr., **91:** 013009 (2016)
45 J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev., **106**: 162-164 (1957)
46 D. C. Zheng, D. W. L. Sprung, and H. Flocard, Phys. Rev. C, **46**: 1355-1363 (1992)
47 P.-G. Reinhard, W. Nazarewicz, M. Bender et al, Phys. Rev. C, **53**: 2776-2785 (1996)
48 R. Capote and Augusto Gonzalez, Phys. Rev. C, **59**: 3477-3480 (1996)
49 J. A. Sheikh, P. Ring, E. Lopes et al, Phys. Rev. C, **66**: 044318 (2002)
70 H. Olofsson, R. Bengtsson, and P. Moller, Nucl. Phys. A, **784**: 104-146 (2007)
51 G. Hupin and Denis Lacroix, Phys. Rev. C, **86**: 024309 (2012)
52 N. Benhamouda, N. H. Allal, M. Fellah et al, Int. J. Mod. Phys. E, **17**: 1357-1365 (2008)
53 S. Kerrouchi, D. Mokhtari, N. H. Allal et al, Int. J. Mod. Phys. E, **18**: 141-160 (2009)
54 N. H. Allal and M. Fellah, Phys. Rev. C, **43**: 2648-2657 (1991)
55 J. Y. Zeng, T.H. Jin, and Z. J. Zhao, Phys. Rev. C, **50**: 1388-1397 (1994).
56 N. H. Allal, M. Fellah, N. Benhamouda et al, Phys. Rev. C, **77**: 054310 (2008)
57 Faiza Hammache, N. H. Allal, M. Fellah et al, Int. J. Mod. Phys. E, **25**: 1650032 (2016)
58 M. Douici, N. H. Allal, M. Fellah, et al, Int. J. Mod. Phys. E, **21**: 1250046 (2012)
59 N. H. Allal, M. Fellah, M. Douici et al, Int. J. Mod. Phys. E, **25**: 1650108 (2016)
60 M. Douici, N.H. Allal, M. Fellah, et al, Int. J. Mod. Phys. E, **22**: 1350029 (2013)
61 M. Borrajo and J. L. Egido, Phys. Lett. B, **764**: 328-334 (2017)
62 P. Fanto, Y. Alhassid, and G. F. Bertsch, Phys. Rev. C, **96**: 014305 (2017)
63 B.F. Bayman, Nucl. Phys., **15**: 33-38 (1960)
64 K. Dietrich, H. J. Mang, and J.H. Pradal, Phys. Rev., **135**: B22-B34 (1964)
65 V. N. Fomenko, J. Phys. A, **3**: 8-20 (1970)
66 C. W. Ma and J. O. Rasmussen, Phys. Rev. C, **16**: 1179-1195 (1977)
67 K. Allaart and E. Boeker, Nucl. Phys. A, **168**: 630-662 (1971)
68 M. R. Oudih, M. Fellah, and N. H. Allal, Int. J. Mod. Phys. E, **12**: 109-123 (2003)
69 M. V. Stoitsov, J. Dobaczewski, R. Kirchner et al, Phys. Rev. C, **76**: 014308 (2007)
70 H. Olofsson, R. Bengtsson, and P. Moller, Nucl. Phys. A, **784**: 104-146 (2007)
71 M. R. Oudih, M. Fellah, N. H. Allal et al, Phys. Rev. C, **76**: 047307 (2007)
72 A. A. Raduta, M. I. Krivoruchenko, and A. Faessler, Phys. Rev. C, **85**: 054314 (2012)
73 M. Fellah, T. F. Hammann, and D. E. Medjadi, Phys. Rev. C,**8**: 1585-1592 (1973)
74 M. Fellah and T. F. Hammann, Phys. Rev. C, **20**: 1560-1571 (1979)
75 N. H. Allal, M. Fellah, M. R. Oudih et al, Eur. Phys. J. A, **27**, s01: 301-306 (2006)
76 N. Sandulescu, B. Erren, and J. Dukelsky, Phys. Rev. C **80**: 044335 (2009)
77 H. J. Lipkin, Ann. Phys. (NY), **12**: 452-462 (1960)
78 Y. Nogami, Phys. Rev., **134**: B313-B321 (1964)
79 W. Satula and R. Wyss, Phys. Lett. B, **393**: 1-6 (1997)
80 K. Sieja and A. Baran, Acta Phys. Pol. B, **35**: 107-114 (2006)
81 D. J. Rowe, Phys. Rev., **175**: 1283-1292 (1968)
82 D. J. Rowe, Rev. Mod. Phys., **40**: 153-166 (1968)
83 D. J. Rowe, *Nuclear Collective Motion* (Methuen, London, 1970)
84 G. Pantis, F. Simkovic, J. D. Vergados et al, Phys. Rev. C, **53**: 695-707(1996)
85 O. Civitarese, F. Montani, and M. Reboiro, Phys. Rev. C , **60**: 024305 (1999)
86 S. G. Frauendorf and J. A. Sheikh, Nucl. Phys. A, **645**: 509-535 (1999)
87 K. Hagino and G. F. Bertsch, Nucl. Phys. A, **679**: 163-174 (2000)
88 D. S. Delion, J. Dukelsky, P. Schuck et al, Phys. Rev. C, **62**: 044311 (2000)
89 N. Paar, T. Niksic, D. Vretenar et al, Phys. Rev. C, **69**: 054303 (2004)
90 Dong-Lian Fang, Phys. Rev. C, **93**: 034306 (2016)
91 J. Terasaki, Phys. Rev. C, **97**: 034304 (2018)
92 K. Sieja, T. L. Ha, P. Quentin et al, Int. J. Mod. Phys. E, **16**: 289-298 (2007)
93 L. Bonneau, P. Quentin, and K. Sieja, Phys. Rev. C, **76**: 014304 (2007)
94 H. Laftchiev , J. Libert , P. Quentin et al, Nucl. Phys. A, **845**: 33-57 (2010)
95 K. W. Schmid, F. Grummer, and A. Faessler, Ann. Rev. (NY), **180**: 1-73 (1987)
96 A. Petrovici, K.W. Schmid, and Amand Faessler, Nucl. Phys. A, **647**: 197-216 (1999)
97 A. Petrovici, K.W. Schmid and Amand Faessler, Nucl. Phys. A, **728**: 396-414 (2003)
98 R. R. Chasman, Phys. Rev. Lett., **95**: 262501 (2005)

99 R. R. Chasman and P. Van Isacker, Phys. Lett. B, **685**: 55-58 (2010)
100 S. Shlomo and I. Talmi, Nucl. Phys. A, **198**: 81-108 (1972)
101 L. Y. Jia, J. Phys. G: Nucl. Part. Phys., **42**: 115105 (2015)
102 L. Y. Jia, Phys. Rev. C, **88**: 044303 (2013)
103 L. Y. Jia, Nucl. Phys. A, **941**: 293-306 (2015)
104 J. Q. Chen, Nucl. Phys. A, **626**: 686-714 (2000)
105 Y. M. Zhao, N. Yoshinaga, S. Yamaji et al, Phys. Rev. C, **62**: 014304 (2000)
106 G. J. Fu, Y. Lei, Y. M. Zhao et al, Phys. Rev. C, **87**: 044310 (2013)
107 J. Meng, J. Y. Guo, L. Liu, and S. Q. Zhang, Front. Phys. China, **1**: 38-46 (2006)
108 M.-J. Cheng, L. Liu, and Y.-X. Zhang, Chin. Phys. C, **39**: 104102 (2015)
109 Z. Shi, Z. H. Zhang, Q. B. Chen et al, Phys. Rev. C, **97**: 034317 (2018)
110 N. Pillet, P. Quentin, and J. Libert, Nucl. Phys. A, **697**: 141-163 (2002)
111 H. Molique and J. Dudek, Phys. Rev. C, **56**: 1795-1813 (1997)
112 Feng Pan, J. P. Draayer, and W. E. Ormand, Phys. Lett. B, **422**: 1-5 (1998)
113 A. Volya, B. A. Brown, and V. Zelevinsky, Phys. Lett. B, **509**: 37-42 (2001)
114 W.C. Chen, J. Piekarewicz, and A. Volya, Phys. Rev. C, **89**: 014321 (2014)
115 J. Y. Zeng and T. S. Cheng, Nucl. Phys. A, **405**: 1-28 (1983).
116 X. M. Fu, F. R. Xu, J. C. Pei et al, Phys. Rev. C, **87**: 044319 (2013)
117 W. Y. Liang, C. F. Jiao, Q. Wu et al, Phys. Rev. C, **92**: 064325 (2015)
118 Y. Benbouzid, N. H. Allal, M. Fellah et al, Chin. Phys. C, **42**: 044103 (2018)
119 Y. Benbouzid, N. H. Allal, and M. Fellah, Rom. Journ. Phys., **61**: 424-434 (2016)
120 M. Fellah, N. H. Allal, Faiza Hammache et al, Int. J. Mod. Phys. E, **24**: 1550097 (2015)
121 R. R. Chasman, Phys. Lett. B, **524**: 81-86 (2002)
122 R. W. Richardson and N. Sherman, Nucl. Phys., **52**: 253-268 (1964)
123 R. D. Woods and D. S. Saxon, Phys. Rev., **95**: 577-578 (1954)
124 N. H. Allal and M. Fellah, Phys. Rev. C, **50**: 1404-1411 (1994)
125 S. Goriely, F. Tondeur, and J. M. Pearson, Atom. Data Nucl. Data Tables, **77**: 311-381 (2001)
126 P. Moller, J. R. Nix, W. D. Myers et al, Atom. Data Nucl. Data Tables, **59**: 185-381 (1995)
127 H. Chen and A. Goswami, Phys. Lett. B, **24**: 257-259 (1967)
128 O. Civitarese, M. Reboiro, and P. Vogel, Phys. Rev. C,**56**: 1840-1843 (1997)
129 W. Satula and R. Wyss, Nucl. Phys. A, **676**: 120-142 (2000)
130 S. Szpikowski, Acta Phys. Pol. B **31** : 1823-1838 (2000).
131 W. Satula, R. Wyss, and M. Rafalski, Eur. Phys. J. A, **25**: 559-562 (2005)
132 R. Wyss, P.J. Davies, W. Satula et al, Phys. Rev. C, **76**: 011301(R) (2007)
133 D. Mokhtari, I. Ami, M. Fellah et al, Int. J. Mod. Phys. E,**17**: 655-667 (2008)
134 F. Hammache, N. H. Allal, and M. Fellah, Int. J. Mod. Phys. E,**21**: 1250100 (2012)
135 H. Sagawa, Y. Tanimura, and K. Hagino, Phys. Rev. C, **87**: 034310 (2013)
136 F. Simkovic, V. Rodin, A. Faessler et al, Phys. Rev. C, **87**: 045501 (2013)
137 N. Hinohara and J. Engel, Phys. Rev. C, **90**: 031301(R) (2014)
138 D. Mokhtari, M. Fellah, and N. H. Allal, Int. J. Mod. Phys. E,**25**: 1650035 (2016)
139 M. Wang, G. Audi, A. H. Wapstra et al, Chin. Phys. C, **36**: 1603 (2012)
140 P. Ring and P. Schuck, *The Nuclear Many Body Problem*, (Berlin, Springer, 1980)
141 A. Sitenko and V. Tartakovskii, *Theory of Nucleus*, (Kluwer Academic Publishers, Boston,1997)