Local probes strongly favor ΛCDM against power-law and $R_h\!=\!ct$ universe*

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Abstract: We constrain three cosmological models – the concordance cold dark matter plus cosmological constant (ΛCDM) model, the power-law (PL) model, and the $R_h\!=\!ct$ model – using the available local probes, which include the JLA compilation of type-Ia supernovae (SNe Ia), the direct measurement of the Hubble constant (H(z)), and the baryon acoustic oscillations (BAO). For the ΛCDM model, we consider two different cases, i.e. zero and non-zero spatial curvature. We find that by using the JLA alone, the ΛCDM and PL models are indistinguishable, but the $R_h\!=\!ct$ model is strongly disfavored. If we combine JLA+H(z), the ΛCDM model is strongly favored over the other two models. The combination of all three datasets supports ΛCDM as the best model. We also use the low-redshift ($z\!<\!0.2$) data to constrain the deceleration parameter using the cosmography method, and find that only the ΛCDM model is consistent with cosmography. However, there is no strong evidence to distinguish between flat and non-flat ΛCDM models by using the local data alone.

Keywords: cosmological parameters, distance scale, supernovae: general

PACS: 98.80.-k, 98.80.Es, 97.60.Bw **DOI:** 10.1088/1674-1137/42/9/095101

1 Introduction

Progress in both experimental and theoretical cosmology in recent decades has led to the foundation of the standard model, i.e. the cold dark matter plus cosmological constant (ΛCDM) model. According to this model, the universe mainly consists of cold dark matter and dark energy (cosmological constant), while ordinary baryonic matter only occupies a small proportion of the total content. The cosmological constant is responsible for the accelerated expansion of the universe, which was first discovered from the luminosity of type-Ia supernovae (SNe Ia) being dimmer than expected [1, 2]. The ΛCDM model is consistent with various local observations such as SNe Ia, the direct measurement of Hubble parameters (H(z)), and the baryon acoustic oscillations (BAO). More importantly, it is consistent with the cosmic microwave background from the WMAP [3, 4] and Planck satellites [5, 6].

Although the ΛCDM model has been very successful, it also faces many challenges, of which the most well-known are the "cosmological constant fine tuning problem" and the "cosmic coincidence problem" [7, 8]. The former asks why the cosmological constant is so close to zero but not exactly zero, and the latter concerns why the densities of dark matter and dark energy are approximately equal today. In addition, it is found that the Hubble constant measured from the local SNe Ia and Cepheids is in more than 3σ tension from that obtained from the CMB [9]. These problems motivate cosmologists to pursue new theories beyond the standard model.

Another problem ΛCDM faces is the horizon problem, which asks why the universe appears statistically homogeneous and isotropic in accordance with the cosmological principle. According to the standard Big Bang model, gravitational expansion does not allows the universe to reach thermal equilibrium, hence it is difficult to explain the homogeneity and anisotropy. Although the horizon problem can be solved by adding an exponential inflation epoch to the very early universe, another problem inevitably arises. That is, why the gravitational horizon is equal to the distance light has traveled since the Big Bang at the current epoch. According to ΛCDM, there is only one time at which the gravitational horizon equals the light travelling distance [10]. It is difficult to explain why this equality happens exactly at the present

Received 8 April 2018, Published online 25 July 2018
∗ Supported by National Natural Science Fund of China (11603005, 11775038, 11647307, 11675182, 11690022)
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day, not earlier or later. To avoid this coincidence, Melia [11] proposed the $R_h = ct$ model, in which the gravitational horizon is always equal to the light travelling distance throughout the whole history of the universe. It was shown that various local data are consistent with the $R_h = ct$ model [12–16]. A detailed analysis on the combined data of local probes, however, showed that the $R_h = ct$ model is strongly disfavored [17–19].

An alternative model more general than $R_h = ct$ is the power-law (PL) model [20, 21], which assumes that the universe expands in a simple power law, i.e. the scale factor of the universe follows $a(t) \propto t^n$. Although it is unlikely that the PL model can describe the whole evolution history of the universe, some investigations have shown that it is consistent with various low-redshift data [21–23]. Especially, it was shown that the PL model with index $n \sim 1.5$ can fit the SNe Ia data as well as ΛCDM model [24]. On the other hand, the validity of the PL model is also questioned by some authors [18, 19].

One of the most important discoveries in modern cosmology is the accelerated expansion of the universe. This phenomenon was first discovered from observations of the luminosity of SNe Ia in the late 1990s, which was later awarded the Nobel Prize [1, 2]. Nowadays the acceleration of the universe and the existence of dark energy are widely accepted by cosmologists. Recently, however, some investigations showed that the evidence for acceleration can be weakened. By using unconventional priors on the SN parameters, Nielsen et al. [25] found that the SNe Ia data are still quite consistent with a constant rate of expansion. Tutusaus et al. [26] found that the non-accelerated power-law model is a good fit to various local data if the cosmological evolution of the intrinsic luminosity of SNe is taken into account. A model-independent way to test the acceleration of the universe is using the cosmography method. We note that the $R_h = ct$ model is a non-accelerating model, thus if the universe is proven to be accelerating, then the $R_h = ct$ model can be ruled out.

In this paper, we use various local probes, including the SNe Ia, H(z) and BAO, to test three cosmological models, i.e. the ΛCDM model, PL model and $R_h = ct$ model. To avoid model-dependence, the cosmography method is also used to constrain the deceleration parameter. The rest of the paper is organized as follows. In Section 2, we briefly review the cosmological models. In Section 3, we introduce the observational datasets that are used to constrain the cosmological models. In Section 4, we use the Markov chain Monte Carlo method to calculate the posterior probability density function of cosmological parameters, and then use the information criteria to pick the model which is best consistent with the data. Finally, discussion and conclusions are given in Section 5.

2 Cosmological models

In this section, we briefly review three cosmological models we are interested in, including the ΛCDM, PL and $R_h = ct$ models.

The ΛCDM model is the standard model and has been proven to be consistent with various observations. It is based on the homogeneous and isotropic Friedmann-Robertson-Walker (FRW) metric,

$$ds^2 = c^2 dt^2 - a^2(t)\left(\frac{dr}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right), \quad (1)$$

where $a(t)$ is the scale factor, and $k = 0, \pm 1$ is the curvature parameter of the universe. Substituting the FRW metric into the Einstein field equations results in the Friedmann equation,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3c^2} - \frac{kc^2}{a^2} \quad (2)$$

and the acceleration equation,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\rho + 3p), \quad (3)$$

where $\rho = \rho_r + \rho_m + \rho_\Lambda$ is the total energy density of the universe, which includes radiation, matter and dark energy. Assuming the equations of state (EoS) $w \equiv p/\rho$ for the radiation and matter components are equal to 1/3 and 0 respectively, we obtain that ρ_r scales as a^{-4} and ρ_m scales as a^{-3}. We further assume that the dark energy is a constant and does not evolve with a.

Defining the Hubble parameter $H = \dot{a}/a$ and the critical energy density $\rho_{c,0} = 3c^2 H_0^2 / 8\pi G$, the Friedmann equation (2) can be rewritten as

$$\left(\frac{H}{H_0}\right)^2 = \Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda, \quad (4)$$

where $\Omega_i \equiv \rho_{0,i}/\rho_{c,0}$ $(i = r, m, \Lambda)$ is the normalized energy density today, $\Omega_k \equiv -kc^2/H_0^2$, and H_0 is called the Hubble constant. The total energy density is normalized to unity, i.e. $\Omega_r + \Omega_m + \Omega_k + \Omega_\Lambda = 1$. Using the relation $a = 1/(1+z)$, the Hubble parameter can be rewritten as a function of redshift,

$$H(z) = H_0\sqrt{\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda}. \quad (5)$$

Another important quantity is the deceleration parameter, which is defined by $q = -\ddot{a}a/\dot{a}^2$. A positive or negative q means that the universe is decelerating or accelerating. From Eqs. (2) and (3) the deceleration parameter can be written as a function of the mass components of the universe,

$$q_0 = \Omega_r + \frac{1}{2}\Omega_m - \Omega_\Lambda. \quad (6)$$

At the present day the radiation component is negligible compared to the other components, so we fix $\Omega_r = 0$. If the universe is spatially flat, i.e. $\Omega_k = 0$, the deceleration parameter only depends on the energy density of matter, $q=(3/2)\Omega_m-1$. Such a model is the so-called concordance cosmological model. Here we consider the flat and non-flat ΛCDM models separately. The comoving distance is given by [27]

$$D_{\rm C}=\frac{c}{H_0}\int_0^z\frac{{\rm d}z}{\sqrt{\Omega_m(1+z)^3+\Omega_k(1+z)^2+\Omega_\Lambda}}. \tag{7}$$

The luminosity distance is related to the comoving distance by

$$D_{\rm L}=\begin{cases}(1+z)\dfrac{c}{H_0}\dfrac{1}{\sqrt{\Omega_k}}\sinh\left(\sqrt{\Omega_k}D_{\rm C}H_0/c\right), & \Omega_k>0,\\ (1+z)D_{\rm C}, & \Omega_k=0,\\ (1+z)\dfrac{c}{H_0}\dfrac{1}{\sqrt{-\Omega_k}}\sin\left(\sqrt{-\Omega_k}D_{\rm C}H_0/c\right), & \Omega_k<0.\end{cases} \tag{8}$$

The power-law model [20, 21] is a toy model and is based on the assumption that the scale factor of the universe expands as a simple power law, namely $a(t)=(t/t_0)^n$, regardless of the contents of the universe, where t_0 is the current age of the universe. In the power-law model, the Hubble parameter reads

$$H(z)=H_0(1+z)^{\frac{1}{n}}. \tag{9}$$

The deceleration parameter is given by $q=1/n-1$, and $n>1$ or $n<1$ means an accelerating or a decelerating universe, respectively.

The $R_h=ct$ universe [11, 28] is based on the assumption that the gravitational horizon R_h is equal to the distance ct light has travelled since the Big Bang throughout the cosmos expansion. In the $R_h=ct$ model, the universe also consists of radiation, matter and dark energy, as the ΛCDM model does. The main difference between $R_h=ct$ and ΛCDM is that the former has no assumption on the EoS of dark energy but requires the EoS of the total contents to be $w\equiv p/\rho=-1/3$. According to the $R_h=ct$ universe, the Hubble parameter is given by

$$H(z)=H_0(1+z). \tag{10}$$

In this model, the universe expands steadily and the deceleration parameter is zero.

In the PL and $R_h=ct$ models, the luminosity distance is given by $D_{\rm L}=(1+z)c\int_0^z[1/H(z)]{\rm d}z$. Therefore, we have

$$D_{\rm L}=\begin{cases}(1+z)\dfrac{c}{H_0}\dfrac{(1+z)^{1-\frac{1}{n}}-1}{1-\frac{1}{n}}, & {\rm PL}, n\neq1,\\ (1+z)\dfrac{c}{H_0}\ln(1+z), & {\rm PL}, n=1\ \&\ R_h=ct.\end{cases} \tag{11}$$

It is convenient to convert the luminosity distance to the dimensionless distance modulus by

$$\mu=5\log\frac{D_{\rm L}}{\rm Mpc}+25, \tag{12}$$

where "log" represents the logarithm of base 10.

One of the model-independent ways to describe the local universe is the so-called cosmography [29]. The main idea of cosmography is to expand the scale factor $a(t)$ and other quantities of interests into Taylor series. In this way the Hubble parameter reads

$$H(z)=H_0[1+(1+q_0)z+\mathcal{O}(z^2)], \tag{13}$$

where H_0 is the Hubble constant and q_0 is the deceleration parameter at the present day. The luminosity distance is given by

$$D_{\rm L}(z)=\frac{cz}{H_0}\left[1+\frac{1}{2}(1-q_0)z+\mathcal{O}(z^2)\right]. \tag{14}$$

Cosmography is only valid when $z\ll1$.

3 Data and methodology

In this section, we use the available local data to constrain the cosmological models. These local data include SNe Ia, H(z) and BAO.

The first local probe used in our paper is SNe Ia. Due to their approximately constant absolute luminosity, SNe Ia are widely used as standard candles to constrain the cosmological parameters. Recently, many SNe Ia samples have been released [30–33]. Here we use the most up-to-date compilation of SNe Ia, i.e. the JLA sample [33]. The JLA consists of 740 SNe Ia in the redshift range [0.01,1.30]. Each SN has well-measured light curve parameters. The distance moduli of SNe can be extracted from the light curves using the empirical relation [33–35]

$$\hat{\mu}=m_B^*-(M_B+\delta\cdot\Delta_M-\alpha X_1+\beta\mathcal{C}). \tag{15}$$

where m_B^* is the observed peak magnitude, M_B is the absolute magnitude, X_1 is the stretch factor, and $\mathcal{C}$ is the supernova color at maximum brightness. Strictly speaking, the absolute magnitude is not a constant, and it depends in a complex way on the host galaxy. Following Ref.[33], we use a simple step function to approximate such a dependence, i.e., we add a term $\delta\cdot\Delta_M$ to M_B and set $\delta=1$ (or $\delta=0$) if the mass of host galaxy is larger (or smaller) than $10^{10}M_\odot$. The two parameters α and β are universal constants and they can be fitted simultaneously with cosmological parameters. The best-fit parameters are the ones which can maximize the likelihood

$$\mathcal{L}_{\rm SN}=\frac{1}{\sqrt{\det(2\pi\boldsymbol{C})}}\exp\left[-\frac{1}{2}(\boldsymbol{\mu}-\hat{\boldsymbol{\mu}})^\dagger\boldsymbol{C}^{-1}(\boldsymbol{\mu}-\hat{\boldsymbol{\mu}})\right], \tag{16}$$

where $\boldsymbol{C}$ is the covariance matrix of $\hat{\boldsymbol{\mu}}$. Note that $\boldsymbol{C}$ not only depends on the light curve parameters, but also depends on the nuisance parameters α and β. Therefore, the normalization factor in Eq. (16) is not a constant and cannot be neglected. In each iteration of the minimization procedure, $\boldsymbol{C}$ should be recalculated using the new parameters. Detailed information on the covariance matrix can be found in Ref. [33].

The second local probe used here is H(z) data, which directly measure the Hubble parameter at different redshifts. Two commonly used methods to measure H(z) are the differential age of galaxies (DAG) method [36–38] and the BAO method [39, 40]. The DAG method measures H(z) by comparing the ages of galaxies at different redshifts, and the BAO method extracts H(z) from the peak of acoustic oscillation of baryons. The H(z) data have an advantage over SNe because the latter rely on the integral of the cosmic expansion history rather than the expansion history itself. After the integration some important information may be lost. However, the H(z) data from the BAO method is more or less model-dependent, and only the DAG method is free of cosmological model. In this paper, we use the 30 H(z) data obtained using the DAG method compiled in Ref. [41]. The likelihood for H(z) data is given by

$$\mathcal{L}_{\rm H}\propto\exp\left[-\frac{1}{2}\sum_{i=1}^{N}\frac{[H(z_i)-\hat{H}_i]^2}{\sigma_{\hat{H}_i}^2}\right], \tag{17}$$

where $H(z_i)$ is the theoretical Hubble parameter at redshift z_i, $\hat{H}_i$ is the observed Hubble parameter, and $\sigma_{\hat{H}_i}$ is the uncertainty of $\hat{H}_i$.

The final local probe used in our paper is the BAO data. BAO are regular, periodic fluctuations in the density of the visible baryonic matter of the universe. As the SNe Ia provide a “standard candle” for astronomical observations, BAO provide a “standard ruler” for length scale in cosmology. This standard ruler is characterized by the sound horizon $r_{\rm d}$ when the baryons are decoupled from the Compton drag of photons at redshift $z_{\rm d}$ [42],

$$r_{\rm d}=\int_{z_{\rm d}}^{\infty}\frac{c_{\rm s}(z){\rm d}z}{H(z)}, \tag{18}$$

where $c_{\rm s}(z)$ is the speed of sound at redshift z. The value of $r_{\rm d}$ strongly depends on the early epoch of the universe, and different models may have very different $r_{\rm d}$. From local data alone we cannot get information about the early universe. Following Ref. [26], we treat $r_{\rm d}$ as a free parameter.

BAO measure the ratio of the effective distance to the sound horizon, i.e. $R(z)=D_{\rm V}(z)/r_{\rm d}$, where

$$D_{\rm V}(z)=\left[\frac{d_{\rm L}^2(z)}{(1+z)^2}\frac{cz}{H(z)}\right] \tag{19}$$

is the effective distance, which takes into consideration the anisotropic expansion in the radial and transverse directions. In this paper, we use the seven BAO data points compiled in Table 1 of Ref. [43]. These data are a compilation of BAO data from the 6dF Galaxy Survey [44], Baryon Oscillation Spectroscopic Survey [45, 46], and the WiggleZ Dark Energy Survey [47], The likelihood of BAO data is given by

$$\mathcal{L}_{\rm BAO}\propto\exp\left[-\frac{1}{2}\sum_{i=1}^{N}\frac{[R_{\rm th}(z_i)-R_{\rm obs}(z_i)]^2}{\sigma_{R_i}^2}\right]. \tag{20}$$

Finally, we combine all the data sets to constrain the cosmological models. The total likelihood of the combined data sets is the product of the individual likelihoods, i.e.

$$\mathcal{L}_{\rm total}=\mathcal{L}_{\rm SN}\cdot\mathcal{L}_{\rm H}\cdot\mathcal{L}_{\rm BAO}. \tag{21}$$

We use the information criteria (IC) to pick the model which can best depict the data. The two most widely used IC are the Akaike information criterion (AIC) [48] and the Bayesian information criterion (BIC) [49]. They are defined by

$$AIC=-2\ln\mathcal{L}_{\rm max}+2k, \tag{22}$$

$$BIC=-2\ln\mathcal{L}_{\rm max}+k\ln N, \tag{23}$$

where $\mathcal{L}_{\rm max}$ is the maximum likelihood, k is the number of free parameters, and N is the number of data points. The model which has the smallest IC is the best one. It is not the absolute value of IC but the difference of IC between different models that is important in the model comparison. We use the flat ΛCDM as the fiducial model, and define the difference of IC of a model with respect to that of flat ΛCDM as

$$\Delta IC_{\rm model}=IC_{\rm model}-IC_{\rm flat-\Lambda CDM}. \tag{24}$$

According to the Jeffreys' scale [50, 51], a model with $\Delta IC>5$ or $\Delta IC>10$ means that there is ‘strong’ or ‘decisive’ evidence against this model with respect to the flat ΛCDM.

4 Results

We use the publicly available Python package emcee [52] to calculate the posterior probability distribution functions of free parameters. A flat prior is used on each parameter. First, we use the JLA data alone to constrain the cosmological parameters. In this case, the Hubble constant h_0 ($h_0=H_0/100$ km s^{-1} Mpc^{-1}) is degenerate with the absolute magnitude M_B, so they cannot be constrained simultaneously. Therefore, we fix $h_0=0.7$ and leave M_B free. The mean and 1σ error of each parameter are reported in Table 1. In the last three rows, we also report the $\ln\mathcal{L}_{\rm max}$, ΔAIC and ΔBIC. According to the IC, there is decisive evidence against the $R_h=ct$ model. However, the flat ΛCDM and PL models are indistinguishable using SNe data alone. According to AIC, the flat and non-flat ΛCDM models fit the data equally well, while according to BIC, the data favors the

Table 1. The best-fit parameters and their 1σ uncertainties from JLA.

	flat ΛCDM	non-flat ΛCDM	PL	$R_h=ct$
Ω_m	0.329±0.034	0.226±0.105	—	—
Ω_Λ	—	0.522±0.163	—	—
n	—	—	1.421±0.116	—
α	0.127±0.006	0.127±0.006	0.126±0.006	0.124±0.006
β	2.633±0.066	2.627±0.067	2.631±0.066	2.606±0.067
$M_{\rm B}$	−19.053±0.023	−19.044±0.025	−19.030±0.024	−18.946±0.017
$\Delta_{\rm M}$	−0.054±0.022	−0.054±0.021	−0.056±0.023	−0.059±0.022
$\ln\mathcal{L}_{\rm max}$	337.873	338.509	338.188	322.076
ΔAIC	0	0.728	-0.630	29.594
ΔBIC	0	5.335	-0.630	24.987

Table 2. The best-fit parameters and their 1σ uncertainties from JLA+H(z).

	flat ΛCDM	non-flat ΛCDM	PL	$R_h=ct$
h_0	0.677±0.019	0.671±0.023	0.683±0.019	0.625±0.014
Ω_m	0.324±0.028	0.245±0.095	—	—
Ω_Λ	—	0.542±0.157	—	—
n	—	—	1.289±0.071	—
α	0.128±0.006	0.127±0.006	0.126±0.006	0.124±0.006
β	2.631±0.066	2.646±0.072	2.629±0.068	2.608±0.070
$M_{\rm B}$	−19.128±0.058	−19.133±0.066	−19.064±0.057	−19.189±0.055
$\Delta_{\rm M}$	−0.052±0.022	−0.058±0.022	−0.056±0.021	−0.059±0.022
$\ln\mathcal{L}_{\rm max}$	330.591	331.055	325.141	313.746
ΔAIC	0	1.072	10.900	31.690
ΔBIC	0	5.718	10.900	27.044

flat ΛCDM model. In the non-flat ΛCDM model, the Ω_m and Ω_Λ values are somewhat smaller than, but are still marginally consistent with, the Planck 2015 results [6] within 1σ uncertainty. In the PL model, $n>1$ means that JLA data favors an accelerating universe.

Then we add H(z) data to the JLA and make a combined analysis. Adding H(z) data breaks the degeneracy between H_0 and M_B, so they can be fitted simultaneously. The best-fit parameters are listed in Table 2. In the last two rows we also list the ΔAIC and ΔBIC values for each model. We see that there is decisive evidence against the PL and $R_h=ct$ models. However, there is weak or strong evidence favoring flat-ΛCDM against non-flat ΛCDM, depending on whether the AIC or BIC criterion is chosen. In the PL model, the best-fit power-law index n is reduced compared wih the JLA-only case. In the ΛCDM (both flat and non-flat) model and PL model, the Hubble constant is more consistent with that of the Planck 2015 results [6] than with the local value from the Cepheids [9]. However, in the $R_h=ct$ model, the Hubble constant is unexpectedly small.

Next, we combine the JLA+H(z)+BAO datasets to make an analysis. The results are given in Table 3. Compared with Table 2, we see that adding the BAO data leaves the best-fit parameters of the ΛCDM and $R_h=ct$ models almost unchanged. The most obvious change happens in the PL model, in which the Hubble constant is reduced to $h_0=0.645\pm0.015$ and the power-law index n is reduced to be consistent with 1. The sound horizons in these three models are consistent with each other. Among these models, the PL and $R_h=ct$ models are decisively disfavored compared with ΛCDM, but there is still no strong evidence to distinguish between flat and non-flat ΛCDM. The deceleration parameters of flat-ΛCDM, non-flat ΛCDM and PL models are $q_0=-0.545\pm0.031$, -0.424 ± 0.122 and -0.080 ± 0.030, respectively.

We also check if the H(z) or BAO data alone disfavor any model or not. By using the 30 H(z) data alone, we find that flat and non-flat ΛCDM models have approximately equal maximum likelihoods, as do the PL and $R_h=ct$ models. The best-fit Hubble constant is $H_0\sim67$ km s^{-1} Mpc^{-1} for the flat and non-flat ΛCDM models, and $H_0\sim62$ km s^{-1} Mpc^{-1} for the PL and $R_h=ct$ models. The ΔAIC values are 2.1, 2.1 and 0.1 for the non-flat ΛCDM, PL and $R_h=ct$ models, respectively. The ΔBIC values are 3.5, 2.1 and -1.3 for the non-flat ΛCDM, PL and $R_h=ct$ models, respectively. Therefore, although there is no strong evidence to favor one model over the others, the non-flat ΛCDM and PL models seem to be marginally disfavored. According to BIC, the H(z) data slightly favor $R_h=ct$ against flat ΛCDM. Since there are only seven BAO data points, most model parameters cannot be tightly constrained by BAO data alone, and no model is preferred over the others.

Finally, to avoid model dependence, we apply the cosmography method, and use SNe Ia data with $z<0.2$ to constrain the deceleration parameter. The best-fit parameters are $q_0=-0.372\pm0.181$, $\alpha=0.133\pm0.008$,

Table 3. The best-fit parameters and their 1σ uncertainties from JLA+H(z)+BAO.

	flat ΛCDM	non-flat ΛCDM	PL	$R_h\!=\!ct$
h_0	0.688±0.018	0.676±0.019	0.645±0.015	0.625±0.014
Ω_m	0.303±0.021	0.214±0.059	—	—
Ω_Λ	—	0.531±0.118	—	—
n	—	—	1.087±0.035	—
α	0.127±0.006	0.127±0.006	0.125±0.006	0.124±0.006
β	2.652±0.069	2.646±0.077	2.612±0.062	2.611±0.070
$M_{\rm B}$	−19.099±0.055	−19.124±0.055	−19.144±0.053	−19.189±0.054
$\Delta_{\rm M}$	−0.054±0.022	−0.052±0.023	−0.060±0.024	−0.062±0.020
$r_{\rm d}$/Mpc	150.012±3.237	150.028±2.998	150.102±2.570	150.640±2.871
$\ln\mathcal{L}_{\rm max}$	327.264	328.327	311.807	308.127
ΔAIC	0	-0.126	30.914	36.274
ΔBIC	0	4.529	30.914	31.619

$\beta = 2.731\pm0.103$, $M_{\rm B} = -19.020\pm0.032$, and $\Delta_{\rm M} = -0.101\pm0.032$. Since there are only six H(z) data points and one BAO data point at redshift $z < 0.2$, adding H(z) and BAO data leaves the result almost unchanged. The deceleration parameter is consistent with that of the ΛCDM model within 1σ uncertainty, and is non-zero at 2σ confidence level. Since the $R_h = ct$ model predicts a null deceleration parameter, it can be ruled out. Note that the PL model may have a consistent q_0 if the PL index $n\sim1.6$. However, this PL index is in conflict with that constrained from JLA+H(z) (+BAO). Only the ΛCDM model is consistent with both SNe alone and the combined data. Therefore, ΛCDM is still the best model compared with the other two models.

5 Discussion and conclusions

In this paper, we combined the publicly available low-redshift data to constrain the ΛCDM model and its two alternatives, i.e. the PL model and $R_h\!=\!ct$ model. For the ΛCDM model, we considered flat and non-flat models separately. It was found that, by using the JLA compilation of SNe Ia alone, the $R_h\!=\!ct$ model is conclusively disfavored against the ΛCDM and PL models. However, the ΛCDM and PL models are indistinguishable based on the JLA alone. The power-law index of the PL model is about 1.4. This supports that the universe is really accelerating. By using the H(z) or BAO data alone, no model is strongly favored against the others. If we combine JLA and H(z) datasets, there is conclusive evidence disfavoring the PL and $R_h\!=\!ct$ models against the ΛCDM model. Finally, the combined data of JLA+H(z)+BAO also conclusively disfavor the PL and $R_h\!=\!ct$ models. In addition, the Hubble constant constrained in the ΛCDM model is consistent with that obtained from the CMB. However, in the PL and $R_h = ct$ models, the Hubble constant is much smaller. Therefore, we conclude that the local probes favor ΛCDM over the other two models. However, there is no strong evidence to distinguish between flat and non-flat ΛCDM models.

Shafer [18] analyzed two different compilations of SNe Ia and BAO data sets, and found that neither ΛCDM model nor PL model is strongly preferred over the other if SNe Ia or BAO data are analyzed separately, but the combined analysis of SNe Ia and BAO data strongly favors the ΛCDM model over the PL model. Furthermore, the $R_h\!=\!ct$ model is conclusively disfavoured by the SNe alone. Our calculations confirm the results of Ref. [18]. By adding the $H(z)$ data to SNe Ia and BAO, we find that the significance of disfavoring the PL and $R_h = ct$ models can be highly improved. In addition, we used the cosmography method to constrain the deceleration parameter, and find that only the ΛCDM model has deceleration parameters consistent with cosmography.

Tutusaus et al. [26] analyzed similar datasets and found that both ΛCDM and PL models can fit the local probes equivalently well. The power-law index of the PL model they obtained is slightly smaller than 1, so they doubted if the cosmic acceleration is really proven by the local probes. The main difference between Ref. [26] and our paper is that, in the former, the authors took into consideration the possible redshift dependence of the absolute luminosity of SNe Ia. They considered four parameterizations of such a dependence, each of which has two parameters, i.e. one amplitude parameter ϵ and one power-law index δ. To avoid degeneracy between parameters, they fixed δ to some arbitrary values. In our paper, we adopted the standard procedure and did not consider such a dependence. This is because that there is no evidence for such a dependence. Especially, there is no reason why such a dependence, if it really exists, can be parameterized in these forms. We tried to constrain ϵ and δ with other parameters simultaneously, but found that these two parameters could not be tightly constrained. This implies that the parameterizations are not appropriate. It is always possible to eliminate the acceleration if we properly parameterize the evolution term. Riess et al. [53] pointed out that the better fit of the PL model than ΛCDM may be due to the small number of SNe at $z\!>\!1$ in JLA. With more high-redshift

SNe, the PL model is no longer as good a fit as ΛCDM even if the evolution of SNe luminosity is considered.

Recently, the simultaneous detection of gravitational waves (GWs) and their electromagnetic counterparts has provided another standard siren to test cosmology. The first GW event detected from a binary neutron star merger, GW 170817 [54], was found to be unambiguously associated with a short gamma-ray burst, GRB 170817 [55, 56]. The follow-up observations of this event led to the identification of NGC 4993 as the host galaxy [57]. The advantage of using GWs as a distance indicator is that they do not rely on other distance ladders and are completely independent of cosmological models. Using the luminosity distance obtained from the GW signals and the redshift of the host galaxy, the Hubble constant was tightly constrained to be $70.0^{+12.0}_{-8.0}$ km s^{-1} Mpc^{-1} [58]. Adding this single GW data point to our JLA+H(z)+BAO sample does not improve the constraint. With the launch of third-generation GW detectors, such as the Einstein Telescope and the Cosmic Explorer, hundreds to thousands of GW events are expected be observed in the future. We expect that in the near future, GW multi-messenger astronomy will provide deep insights into the universe.

We are grateful to Zhe Chang and Z. C. Zhao for useful discussions.

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