Recent progress in multiple chiral doublet bands[*]

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Abstract: Recent progress regarding multiple chiral doublet bands (MχD) is reviewed from the experimental and theoretical perspectives. In particular, the experimental findings, theoretical predictions, selection rule for electromagnetic transitions, MχD with octupole correlations, and some related topics are highlighted. Based on these discussions, it is of great scientific interest to search for the other MχD, as well as possible chiral wobblers, chirality-parity quartet bands, and chirality-pseudospin triplet (or quartet) bands in the nuclear system.

Keywords: chiral doublet bands, MχD, octupole correlations, chirality-parity quartet bands

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1 Introduction

Handedness, also known as chirality, is a well-known phenomenon in chemistry, biology, and particle physics. Many biological molecules occur in identical but left- and right-handed modes. In particle physics, chirality is a dynamical feature of massless particles that distinguishes the parallel or antiparallel orientation of spin and momentum. However, scientists have long believed that atomic nuclei are too symmetrical to exist as left- and right-handed versions. The question about the existence of chiral nuclei is thus of great interest.

The first prediction of chirality in atomic nuclei was made by Frauendorf and Meng in 1997 [1]. They pointed out the existence of this phenomenon in triaxial odd-odd nuclei, where three angular momentum vectors may couple to each other in either a left- or right-handed mode. This chiral geometry may give rise to pairs of nearly degenerate $\Delta I = 1$ bands with the same parity, i.e., chiral doublet bands. To test the theoretical prediction of chirality in atomic nuclei, significant effort has been devoted to further exploring this interesting phenomenon. Thus far, such chiral doublet bands have been reported in the $A \sim$ 80 [2-4], 100 [5-12], 130 [13-29], and 190 [30-36] mass regions of the nuclear chart. For details, see recent reviews on nuclear chirality and related topics [37-45] or data tables [46].

In 2006, based on adiabatic and configuration-fixed constrained triaxial relativistic mean field (RMF) calculations, triaxial shape coexistence with high-j proton-hole and neutron-particle configurations was found in ^{106}Rh, which demonstrates the possible existence of multiple chiral doublet (acronym MχD) bands in this nucleus [47]. In the last decade, the theoretical prediction of MχD has stimulated numerous experimental efforts [3, 12, 28, 48-52]. In this review, we present the recent progress in MχD.

2 Experimental findings of MχD

In this section, our attempt is to provide an overview of experimental findings of MχD. In 2013, two distinct sets of chiral doublet bands, based on the $\pi h_{11/2}^2 \otimes \nu h_{11/2}^{-1}$ and $\pi h_{11/2}(g_{7/2})^{-1} \otimes \nu h_{11/2}^{-1}$ configurations, were identified in the odd-A nucleus ^{133}Ce, which was regarded as strong experimental evidence for the existence of MχD [28]. The experimental observations of MχD represent an important confirmation of triaxial shape coexistence and its geometrical interpretation. Later, a novel type of MχD bands with an identical configuration was found in ^{103}Rh [12], which shows that chiral geometry can be robust against the increase of the intrinsic excitation energy. The MχD with octupole correlations was identified in odd-odd ^{78}Br, which provides the first example of chiral geometry in octupole soft nuclei and indicates that nuclear chirality can be robust with respect to octupole correlations [3]. It also indicates that a simultaneous breaking of chiral and space-reflection symmetries may exist in nuclei. In 2018, the MχD involving 3 and 5 quasiparticle

configurations has been observed in odd-A ^{195}Tl [52], which is the first observation of such bands in the $A \sim$ 190 mass region. Five pairs of nearly degenerate doublet bands were reported in even-even ^{136}Nd [51]. Very recently, a new pair of chiral doublet bands with the $\pi g_{7/2} h_{11/2} \otimes \nu h_{11/2}$ configuration was identified in ^{135}Nd [53], which is the isotone of ^{133}Ce. The newly observed chiral doublet bands along with the previously known chiral bands with the $\pi h_{11/2}^2 \otimes \nu h_{11/2}$ configuration [20] represented a new example of MχD bands.

In fact, the existence of more than one chiral configuration in one nucleus has been noted in 2004. The candidate chiral doublet bands in ^{105}Rh with the $\pi g_{9/2}^{-1} \otimes \nu h_{11/2}^2$ configuration [7], and others with the tentatively suggested $\pi g_{9/2}^{-1} \otimes \nu h_{11/2}(d_{5/2}/g_{7/2})$ configuration [8], were reported resepectively by the two research groups. The triaxial RMF approaches were applied to investigate their triaxial deformations with the corresponding configurations in ^{105}Rh. Two pairs of doublet bands in ^{105}Rh were suggested as the candidate MχD bands [54]. A similar discussion was also applied to ^{107}Ag [55]. Two pairs of doublet bands with the $\pi g_{9/2}^{-1} \otimes \nu h_{11/2}^2$ and $\pi g_{9/2}^{-1} \otimes \nu h_{11/2}(d_{5/2}/g_{7/2})$ configurations in ^{107}Ag have been observed in the experiments [56-58]. The RMF calculations showed that the $\pi g_{9/2}^{-1} \otimes \nu h_{11/2}^2$ and $\pi g_{9/2}^{-1} \otimes \nu h_{11/2}(d_{5/2}/g_{7/2})$ bands in ^{107}Ag exhibit obvious triaxial deformation, $\gamma = 27.2°$ and $\gamma = 28.1°$, respectively. These are favorable deformation parameters for chirality. Using these deformation parameters as input, the multiparticle plus rotor model (MPRM) calculations reproduced the available data well for the two pairs of doublet bands. The chiral geometry of the aplanar rotation for two pairs of doublet bands was further confirmed by analyzing the angular

momentum components [55].

The observed MχD nucleus candidate, the reaction used to generate MχD, the number of pairs for the observed chiral doublet bands in each nucleus, and the single-particle configuration of MχD are all summarized in Table 1. As shown in Table 1, all MχD nucleus candidates were discovered in the fusion evaporation reactions using in-beam γ-ray spectroscopy. For the single-particle configurations of MχD, the high-j intruder orbitals (for instance, $g_{9/2}$, $h_{11/2}$, and $i_{13/2}$) are involved in all observed MχD nucleus candidates. Some low-j orbitals also appeared in the multiparticle configurations, and usually acted as spectators in the formation of chiral geometry. Furthermore, the observed MχD nucleus candidates can be roughly divided into two categories. One is MχD bands, with the distinct configurations that differ in their triaxial deformations and configurations. For example, two distinct chiral doublet bands based on the configurations $\pi h_{11/2}^2 \otimes \nu h_{11/2}$ and $\pi g_{7/2} h_{11/2} \otimes \nu h_{11/2}$ in ^{133}Ce [28]. The second is MχD bands with the identical configuration. A unique example is MχD bands with the $\pi g_{9/2} \otimes \nu h_{11/2} g_{7/2}$ configuration in ^{103}Rh [12].

3   Theoretical predictions of MχD

In theory, MχD has been investigated with the triaxial PRM [59-63], the combination of triaxial PRM and RMF approaches [3, 12, 28, 51, 53, 64], the tilted axis cranking model (TAC) with the collective Hamiltonian [65, 66], the projected shell model [67], etc. In this section, I shall only focus on the theoretical predictions of MχD.

The adiabatic and configuration-fixed constrained tri-

Table 1.   Observed MχD nucleus candidates, reaction used to generate MχD, number of pairs for the observed chiral doublet bands, and single-particle configuration of MχD.

Nucleus	Reaction	Number	Single-particle configuration
^{78}Br [3]	^{70}Zn(^{12}C, $p3n$)	2	$\pi g_{9/2} \otimes \nu g_{9/2}$, $\pi f_{5/2} \otimes \nu g_{9/2}$
^{103}Rh [12]	^{96}Zr(^{11}B, $4n$)	3	$\pi g_{9/2} \otimes \nu h_{11/2}^2$, $\pi g_{9/2} \otimes \nu h_{11/2} g_{7/2}$
^{105}Rh [7, 8]	^{100}Mo(^{11}B, $\alpha 2n$), ^{96}Zr(^{13}C, $p3n$)	2	$\pi g_{9/2} \otimes \nu h_{11/2}^2$, $\pi g_{9/2} \otimes \nu h_{11/2} g_{7/2}$
^{107}Ag [56-58]	^{100}Mo(^{11}B, $4n$), ^{94}Zr(^{17}O, $p3n$)	2	$\pi g_{9/2}^{-1} \otimes \nu h_{11/2}^2$, $\pi g_{9/2}^{-1} \otimes \nu h_{11/2}(d_{5/2}/g_{7/2})$
^{133}Ce [28]	^{116}Cd(^{22}Ne, $5n$)^{133}Ce	2	$\pi h_{11/2}^2 \otimes \nu h_{11/2}$, $\pi g_{7/2} h_{11/2} \otimes \nu h_{11/2}$
^{135}Nd [20, 53]	^{110}Pd(^{30}Si, $5n$), ^{100}Mo(^{40}Ar, $5n$)	2	$\pi h_{11/2}^2 \otimes \nu h_{11/2}$, $\pi g_{7/2} h_{11/2} \otimes \nu h_{11/2}$
^{136}Nd [51]	^{100}Mo(^{40}Ar, $5n$)	5	$\pi h_{11/2}^1 (d_{5/2}, g_{7/2})^{-1} \otimes \nu h_{11/2}^{-1}(s_{1/2}, d_{3/2})^{-1}$, $\pi h_{11/2}^3 (d_{5/2}, g_{7/2})^{-1} \otimes \nu h_{11/2}^{-1}(s_{1/2}, d_{3/2})^{-1}$, $\pi h_{11/2}^2 (d_{5/2}, g_{7/2})^{-2} \otimes \nu h_{11/2}^{-1}(f_{7/2}, h_{9/2})^1$, $\pi h_{11/2}^2 (d_{5/2}, g_{7/2})^{-2} \otimes \nu h_{11/2}^{-1}(s_{1/2}, d_{3/2})^{-1}$ $\pi h_{11/2}^2 \otimes \nu h_{11/2}^{-1}(s_{1/2}, d_{3/2})^{-1}$
^{195}Tl [52]	^{185,187}Re(^{13}C, xn)	2	$\pi h_{9/2} \otimes \nu i_{13/2}^{-2}$, $\pi i_{13/2} \otimes \nu i_{13/2}^{-3}(p_{3/2} f_{5/2})^{-1}$

Table 2.   Calculated triaxial deformation parameters β and γ, corresponding valence nucleon and unpaired nucleon configurations of minima, as well as excitation energies E_x in predicted MχD nuclei.

Nuclei	Configuration		(β,γ)	E_x/MeV
	Valence nucleons	Unpaired nucleons		
^{54}Co [68]	$\pi f_{7/2}^{-1} \otimes \nu(g_{9/2}^{1} f_{7/2}^{-2})$	$\pi f_{7/2}^{-1} \otimes \nu g_{9/2}^{1}$	$(0.26,18.2°)$	8.39
	$\pi(g_{9/2}^{1} f_{7/2}^{-2}) \otimes \nu f_{7/2}^{-1}$	$\pi g_{9/2}^{1} \otimes \nu f_{7/2}^{-1}$	$(0.26,17.3°)$	8.1
^{57}Co [68]	$\pi f_{7/2}^{-1} \otimes \nu g_{9/2}^{1}(fp)^{1}$	$\pi f_{7/2}^{-1} \otimes \nu g_{9/2}^{1}(fp)^{1}$	$(0.20,24.0°)$	5.79
	$\pi f_{7/2}^{-1} \otimes \nu(g_{9/2}^{2})$	$\pi f_{7/2}^{-1} \otimes \nu g_{9/2}^{2}$	$(0.25,36.0°)$	11.82
^{60}Co [68]	$\pi f_{7/2}^{-1} \otimes \nu g_{9/2}^{1}(fp)^{4}$	$\pi f_{7/2}^{-1} \otimes \nu g_{9/2}^{1}$	$(0.28,27.0°)$	2.07
	$\pi f_{7/2}^{-1} \otimes \nu g_{9/2}^{2}(fp)^{3}$	$\pi f_{7/2}^{-1} \otimes \nu g_{9/2}^{2}(fp)^{1}$	$(0.30,15.1°)$	6.75
^{74}Br [69]	$\pi(g_{9/2}^{3} f_{5/2}^{-2} p_{3/2}^{-4} p_{1/2}^{-2}) \otimes \nu(g_{9/2}^{4} f_{5/2}^{-1} p_{3/2}^{-2} p_{1/2}^{-2})$	$\pi g_{9/2}^{1} \otimes \nu f_{5/2}^{-1}$	$(0.43,23.2°)$	0.44
	$\pi(g_{9/2}^{3} f_{5/2}^{-2} p_{3/2}^{-4} p_{1/2}^{-2}) \otimes \nu(g_{9/2}^{5} f_{5/2}^{-2} p_{3/2}^{-2} p_{1/2}^{-2})$	$\pi g_{9/2}^{1} \otimes \nu g_{9/2}^{-1}$	$(0.45,27.5°)$	0.47
^{76}Br [69]	$\pi(g_{9/2}^{3} f_{5/2}^{-4} p_{3/2}^{-2} p_{1/2}^{-2}) \otimes \nu(g_{9/2}^{5} p_{3/2}^{-2} p_{1/2}^{-2})$	$\pi g_{9/2}^{1} \otimes \nu g_{9/2}^{-1}$	$(0.41,20.8°)$	0.08
	$\pi(g_{9/2}^{1} f_{5/2}^{-2} p_{3/2}^{-4}) \otimes \nu(g_{9/2}^{4} p_{3/2}^{-1} p_{1/2}^{-2})$	$\pi g_{9/2}^{1} \otimes \nu p_{3/2}^{-1}$	$(0.36,32.0°)$	0.42
	$\pi(f_{5/2}^{-1} p_{3/2}^{-2} p_{1/2}^{-2}) \otimes \nu(g_{9/2}^{5} p_{3/2}^{-2} p_{1/2}^{-2})$	$\pi f_{5/2}^{-1} \otimes \nu g_{9/2}^{1}$	$(0.28,40.3°)$	2.52
^{80}Br [69]	$\pi(g_{9/2}^{2} f_{5/2}^{-1} p_{3/2}^{-4} p_{1/2}^{-2}) \otimes \nu(g_{9/2}^{7} p_{1/2}^{-2})$	$\pi f_{5/2} \otimes \nu g_{9/2}^{-1}$	$(0.31,23.7°)$	0.96
	$\pi(g_{9/2}^{3} f_{5/2}^{-2} p_{3/2}^{-2} p_{1/2}^{-2}) \otimes \nu(g_{9/2}^{7} p_{3/2}^{-2})$	$\pi g_{9/2}^{1} \otimes \nu g_{9/2}^{-1}{}_{*}$	$(0.34,25.2°)$	1.54
^{82}Br [69]	$\pi(g_{9/2}^{3} f_{5/2}^{-2} p_{3/2}^{-4} p_{1/2}^{-2}) \otimes \nu(g_{9/2}^{7} p_{3/2}^{-2} p_{3/2}^{-2})$	$\pi g_{9/2}^{1} \otimes \nu g_{9/2}^{-1}{}_{*}$	$(0.41,17.5°)$	6.62
	$\pi(g_{9/2}^{1} p_{3/2}^{-4} p_{1/2}^{-2}) \otimes \nu(g_{9/2}^{7})$	$\pi g_{9/2}^{1} \otimes \nu g_{9/2}^{-1}$	$(0.15,33.6°)$	2.77
	$\pi(g_{9/2}^{3} f_{5/2}^{-2} p_{3/2}^{-4} p_{1/2}^{-2}) \otimes \nu(g_{9/2}^{7})$	$\pi g_{9/2}^{1} \otimes \nu g_{9/2}^{-1}{}_{*}$	$(0.27,10.2°)$	3.78
^{78}Rb [70]	$\pi(g_{9/2}^{3} f_{5/2}^{-4} p_{3/2}^{-2}) \otimes \nu(g_{9/2}^{5} f_{5/2}^{-2} p_{3/2}^{-2})$	$\pi g_{9/2}^{1} \otimes \nu g_{9/2}^{-1}$	$(0.37,34.0°)$	0.26
	$\pi(g_{9/2}^{2} f_{5/2}^{-3} p_{3/2}^{-2}) \otimes \nu(g_{9/2}^{5} f_{5/2}^{-2} p_{3/2}^{-2})$	$\pi f_{5/2}^{1} \otimes \nu g_{9/2}^{-1}$	$(0.32,38.8°)$	0.51
	$\pi(g_{9/2}^{3} f_{5/2}^{-4} p_{3/2}^{-2}) \otimes \nu(g_{9/2}^{7} f_{5/2}^{-4} p_{3/2}^{-2})$	$\pi g_{9/2}^{1} \otimes \nu g_{9/2}^{-1}$	$(0.40,44.5°)$	1.46
^{80}Rb [70]	$\pi(g_{9/2}^{1} p_{3/2}^{-2} p_{1/2}^{-2}) \otimes \nu(g_{9/2}^{5} p_{1/2}^{-2})$	$\pi g_{9/2}^{1} \otimes \nu g_{9/2}^{-1}$	$(0.22,45.1°)$	0.31
	$\pi(g_{9/2}^{3} f_{5/2}^{-2} p_{3/2}^{-4}) \otimes \nu(g_{9/2}^{6} p_{3/2}^{-1} p_{1/2}^{-2})$	$\pi g_{9/2}^{1} \otimes \nu p_{3/2}^{-1}$	$(0.33,37.3°)$	1.19
	$\pi(g_{9/2}^{3} p_{3/2}^{-4} p_{1/2}^{-2}) \otimes \nu(g_{9/2}^{7} p_{3/2}^{-2} p_{1/2}^{-2})$	$\pi g_{9/2}^{1} \otimes \nu g_{9/2}^{-1}$	$(0.35,39.1°)$	1.98
^{82}Rb [70]	$\pi(g_{9/2}^{2} p_{3/2}^{-3} p_{1/2}^{-2}) \otimes \nu(g_{9/2}^{7} p_{1/2}^{-2})$	$\pi p_{3/2}^{1} \otimes \nu g_{9/2}^{-1}$	$(0.26,42.7°)$	0.07
	$\pi(g_{9/2}^{1} p_{3/2}^{-2} p_{1/2}^{-2}) \otimes \nu(g_{9/2}^{7} p_{1/2}^{-2})$	$\pi g_{9/2}^{1} \otimes \nu g_{9/2}^{-1}$	$(0.22,36.9°)$	0.24
	$\pi(g_{9/2}^{3} f_{5/2}^{-2} p_{3/2}^{-4}) \otimes \nu(g_{9/2}^{7} p_{1/2}^{-2})$	$\pi g_{9/2}^{1} \otimes \nu g_{9/2}^{-1}$	$(0.29,45.5°)$	0.56
^{106}Rh [47, 71–73]	$\pi g_{9/2}^{-3} \otimes \nu h_{11/2}^{1}(d_{5/2}^{2} or d_{3/2}^{2})$	$\pi g_{9/2}^{-1} \otimes \nu h_{11/2}^{1}{}_{*}$	$(0.25,23.3°)$	0.636
	$\pi g_{9/2}^{-3} \otimes \nu h_{11/2}^{3}$	$\pi g_{9/2}^{-1} \otimes \nu h_{11/2}^{3}$	$(0.30,22.9°)$	1.219
^{110}Rh [71]	$\pi g_{9/2}^{-3} \otimes \nu h_{11/2}^{3}$	$\pi g_{9/2}^{-1} \otimes \nu h_{11/2}^{1}$	$(0.26,40.6°)$	0
	$\pi g_{9/2}^{-3} \otimes \nu h_{11/2}^{5}$	$\pi g_{9/2}^{-1} \otimes \nu h_{11/2}^{1}$	$(0.31,18.7°)$	0.51
^{105}Ag [74]	$\pi(g_{9/2}^{-1} p_{1/2}^{-2}) \otimes \nu(h_{11/2}^{1} d_{5/2}^{2} g_{7/2}^{5})$	$\pi g_{9/2}^{-1} \otimes \nu h_{11/2}^{1} g_{7/2}^{-1}{}_{*}$	$(0.23,34.9°)$	2.46
	$(g_{9/2}^{-1} p_{1/2}^{-2}) \otimes \nu(h_{11/2}^{1} d_{5/2}^{1} g_{7/2}^{6})$	$\pi g_{9/2}^{-1} \otimes \nu h_{11/2}^{1} d_{5/2}^{1}$	$(0.23,26.5°)$	3.10
	$(g_{9/2}^{-1} p_{1/2}^{-2}) \otimes \nu(h_{11/2}^{2} g_{7/2}^{6})$	$\pi g_{9/2}^{-1} \otimes \nu h_{11/2}^{2}$	$(0.25,38.5°)$	3.94
^{125}Cs [75]	$\pi(g_{7/2}^{4} h_{11/2}^{1}) \otimes \nu(sd)^{5} h_{11/2}^{7}$	$\pi h_{11/2}^{1} \otimes \nu h_{11/2}^{-1}(sd)^{1}$	$(0.25,26.3°)$	2.75
	$\pi(g_{7/2}^{4} h_{11/2}^{1}) \otimes \nu(sd)^{4} h_{11/2}^{8}$	$\pi h_{11/2}^{1} \otimes \nu h_{11/2}^{-2}$	$(0.26,24.3°)$	4.78
^{129}Cs [75]	$\pi(g_{7/2}^{4} h_{11/2}^{1}) \otimes \nu(sd)^{7} h_{11/2}^{9}$	$\pi h_{11/2}^{1} \otimes \nu h_{11/2}^{-1}(sd)^{1}$	$(0.21,13.4°)$	2.78
	$\pi(g_{7/2}^{4} h_{11/2}^{1}) \otimes \nu(sd)^{8} h_{11/2}^{8}$	$\pi h_{11/2}^{1} \otimes \nu h_{11/2}^{-2}$	$(0.21,22.7°)$	2.31
^{131}Cs [75]	$\pi(g_{7/2}^{4} h_{11/2}^{1}) \otimes \nu(sd)^{9} h_{11/2}^{9}$	$\pi h_{11/2}^{1} \otimes \nu h_{11/2}^{-1}(sd)^{1}$	$(0.18,22.0°)$	2.11
	$\pi(g_{7/2}^{4} h_{11/2}^{1}) \otimes \nu(sd)^{10} h_{11/2}^{8}$	$\pi h_{11/2}^{1} \otimes \nu h_{11/2}^{-2}$	$(0.17,24.8°)$	3.00

$*$: the configurations have been experimentally observed.

axial RMF approaches were developed for the first time to investigate the triaxial shape coexistence and possible chiral doublet bands in 2006 [47]. The existence of multiple chiral doublets (MχD) was suggested in ^{106}Rh from the examination of the deformation and corresponding configurations. Similar investigations have also been performed for several isotope chains. These calculations predicted that the MχD phenomenon might exist in ^{54,57,60}Co [68], ^{74,76,80,82}Br [69], ^{78,80,82}Rb [70], ^{106,110}Rh [71-73], ^{105}Ag [74], and ^{125,129,131}Cs [75] based on the triaxial deformations of the local minima and the corresponding high-j particle(s) and hole(s) configurations. The predicted multi-chiral nuclei are listed in Table 2, along with the calculated triaxial deformation parameters β and γ, corresponding valence nucleon and unpaired nucleon configurations of minima, as well as excitation energies. The configurations of experimentally observed chiral doublet bands are marked with an asterisk. In Table 2, the excitation energies of most chiral configurations are less than 3 MeV. This can be easily populated by experiments. Further experimental explorations are expected to search for the MχD in these nuclei.

It is worthwhile to mention that a three-dimensional tilted axis cranking (3DTAC) method based on covariant density functional theory has been recently established and used to investigate the MχD for the first time in a fully self-consistent and microscopic way [73]. This model reproduced well the available experimental spectra and $B(M1)/B(E2)$ ratios in ^{106}Rh, which exhibited a high predictive power.

4   Selection rule for electromagnetic transitions in MχD

Besides the experimental explorations and theoretical predictions of MχD, it is also interesting to study the fingerprints of MχD. For single chiral doublet bands, the known fingerprints for the ideal chirality are as follows: (i) the nearly degeneracy of doublet bands, (ii) spin independence of $S(I)$, (iii) similar spin alignments, (iv) similar $B(M1)$ and $B(E2)$ values, (v) staggering of $B(M1)$, (vi) vanishing of the interband $E2$ transitions at high spins, and (vii) very small interaction strength [1, 76-79]. Evidently, for the MχD with distinct configurations, the chiral fingerprints are still effective for every pair of chiral doublet bands.

As mentioned above, in contrast with the MχD that differ in their triaxial deformations and configurations, MχD may also exist in a single nucleus with the identical configuration. The chiral bands, including the yrast and yrare bands as well as the higher excited bands, have been studied via the triaxial PRM [59-62], which has been extensively used in studies of chiral doublet bands

and yielded numerous successes [64, 77, 80-89]. The PRM calculations [59-62] showed that the properties of the two higher excited bands, including the excitation energies and selection rule for electromagnetic transitions, were very similar to those of the yrast and yrare bands, which indicated that excited doublet bands could be a pair of chiral partners as well. However, we noted the existence of a number of linking transitions between the lowest-lying chiral bands and higher excited bands. It is necessary to study the properties of these linking transitions.

In Ref. [63], the selection rule of electromagnetic transitions for these linking transitions between the lowest and excited chiral doublet bands in MχD was also studied based on the triaxial PRM. The calculated level scheme for two pairs of chiral doublet bands based on the configuration $\pi h_{11/2} \otimes \nu h_{11/2}^{-1}$ coupled with the $\gamma = 90°$ rotor is shown in Fig. 1. These bands are labeled as 1, 2, 3, and 4. Bands 1 & 2 and bands 3 & 4 form the chiral doublet bands A and B, respectively. In the calculations, the deformation parameters $\beta = 0.22$, $\gamma = 90°$, moment of inertia $\mathcal{J}_0 = 30$ MeV$^{-1}\hbar^2$, intrinsic quadrupole moment $Q_0 = (3/\sqrt{5\pi})R_0^2 Z\beta = 3.5$, and $g_p(g_n) - g_R = 0.7(-0.7)$ are adopted. As shown in Fig. 1, for the in-band $M1$ transitions, the same odd-even spin staggering is clearly observed in the four bands, in which transitions from odd spin to even spin states are allowed. For the interband $M1$ transitions, transitions from band 1 (3) to 2 (4) and band 2 (4) to 1 (3) are allowed for even spin states decaying to odd spin states. The linking $M1$ transitions between chiral doublet bands A and B are allowed from the states of band 3 decaying alternatively to those of band 1 or 2, and then the same behavior for band 4 is exhibited. Based on these calculated results, whole selection rules of electromagnetic transitions including in-band, interband, and linking transitions were summarized in Ref. [63] (see the Tab. 3 in Ref. [63]). Apart from the whole selection rules, the quantitative relations of electromagnetic transitions probabilities were also obtained in Ref. [63]. The $B(M1)$ and $B(E2)$ values in the excited chiral doublet bands have the same order of magnitude as those in the lowest chiral doublet bands. However, the $B(M1)$ and $B(E2)$ values of transitions, which link the excited to the lowest chiral doublet bands, are two orders of magnitude smaller than those in the lowest (or excited) chiral doublet bands. The selection rules and quantitative relations of electromagnetic transition probabilities would be helpful for confirming the existence of MχD bands with the identical configuration in real nuclei.

In Fig. 1, the interband $E2$ transitions in chiral doublet bands A (or B) are forbidden, which is consistent with the fingerprint (vi) of ideal chiral doublet bands. However, there are a number of $E2$ linking transitions between chiral doublet bands A and B. This implies that these bands, 2 and 3, cannot be defined as a pair of ideal
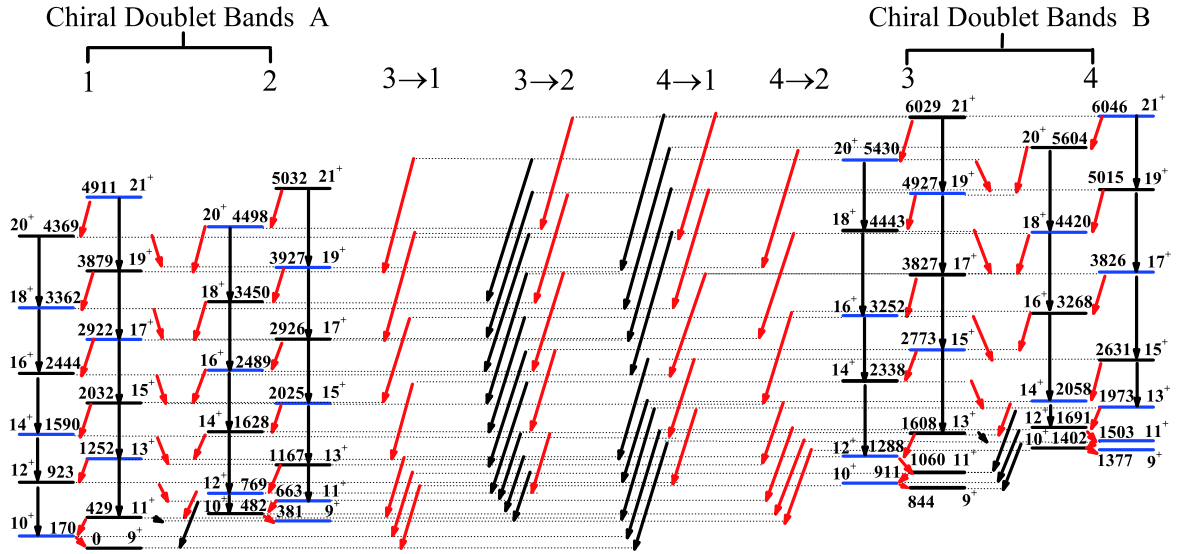
Fig. 1.   (color online) Calculated level scheme for MχD based on configuration $\pi h_{11/2} \otimes \nu h_{11/2}^{-1}$ coupled with $\gamma = 90°$ rotor from Ref. [63]. Red and black arrows represent $M1(I \rightarrow I-1)$ and $E2(I \rightarrow I-2)$ transitions, respectively.

chiral partners. The existence of $E2$ linking transitions allows us to extract the interaction strength between chiral doublet bands A and B. For the doublet bands with the same configuration and deformation, the interaction strength V can be expressed as [79]

$$V = \frac{\sqrt{R}\Delta E_I \Delta E_{I-2}}{\sqrt{(R+1)^2(\Delta E_I^2 + \Delta E_{I-2}^2) + 2\Delta E_I \Delta E_{I-2}(R^2-1)}}, \quad (1)$$

where $R = \frac{B[E2,I^{\text{yrare}} \rightarrow (I-2)^{\text{yrast}}]}{B[E2,I^{\text{yrare}} \rightarrow (I-2)^{\text{yrare}}]}$. According to Eq. (1), we extracted the interaction strength between chiral doublet bands A and B. The values of $B(E2)$ and ΔE arise from the calculated results of PRM [63]. The calculations show that the average interaction strength $\overline{V}$ between chiral doublet bands A and B is approximately equal to 200 keV in the chiral range, which implies that the chiral geometry is mixed by the vibrational component [79, 90]. It should be noted that Eq. (1) is deduced from the two-band mixing picture. Thus, the present calculated V is an approximate solution to MχD bands.

To present the picture more clearly, the mixing ratio and the percentage of $E2$ mixing for the $\Delta I = 1$ linking transitions are calculated in the present work by the PRM with the ideal case, i.e., the configuration $\pi h_{11/2} \otimes \nu h_{11/2}^{-1}$ with $\gamma = 90°$. The calculations show a $\sim 1\%$ and 2% $E2$ admixture for the $\Delta I = 1$ transitions in chiral doublet bands A and B, respectively, suggesting that these transitions are essentially of a pure $M1$ character. Moreover, relatively large $E2$ fractions ($\sim 20\%$) are obtained for the $\Delta I = 1$ linking transitions between chiral doublet bands A and B. The enhanced $E2$ component is a characteristic feature of the wobbling phonon excitation [91]. The similar conjecture has been obtained in Ref. [92] based on the same model calculations. Instead of analyzing the in-

teraction strength between chiral doublet bands A and B, Ref. [92] studied the expectation value $\sqrt{\langle R_3^2 \rangle}$ of the core rotation along the quantization axis, and found that components of the core rotation along the long and short axis for the excited doublet bands are larger than those for the lowest chiral bands. This implied that the excited pair exhibited a chiral geometry, which is realized with a wobbling motion of the core. Hence, the excited pair was claimed as the chiral wobblers [92]. Further detailed studies beyond the scope of this paper are needed to study whether the excited doublet bands are associated with the chiral wobblers. The possible coexistence of chirality and other rotational modes are discussed in the following section.

5   MχD with octupole correlations and some related topics

If another spontaneous breaking of discrete symmetry takes place in addition to chirality, the degree of energy degeneracy will accordingly be increased, resulting in multiple degenerate $\Delta I = 1$ rotational bands. For instance, in the case of the chiral and reflection symmetry breakings, parity and chiral doubling bring about a set of four degenerate $\Delta I = 1$ bands. The first example, though rather a soft breaking of reflection symmetry, has been found in ^{78}Br [3]. In Ref. [3], two pairs of positive- and negative-parity doublet bands, along with eight strong electric dipole transitions linking their yrast positive- and negative-parity bands in ^{78}Br, have been found. This provided the evidence for MχD bands with octupole correlations, reporting the first example of chiral geometry in octupole soft nuclei, and indicated that nuclear
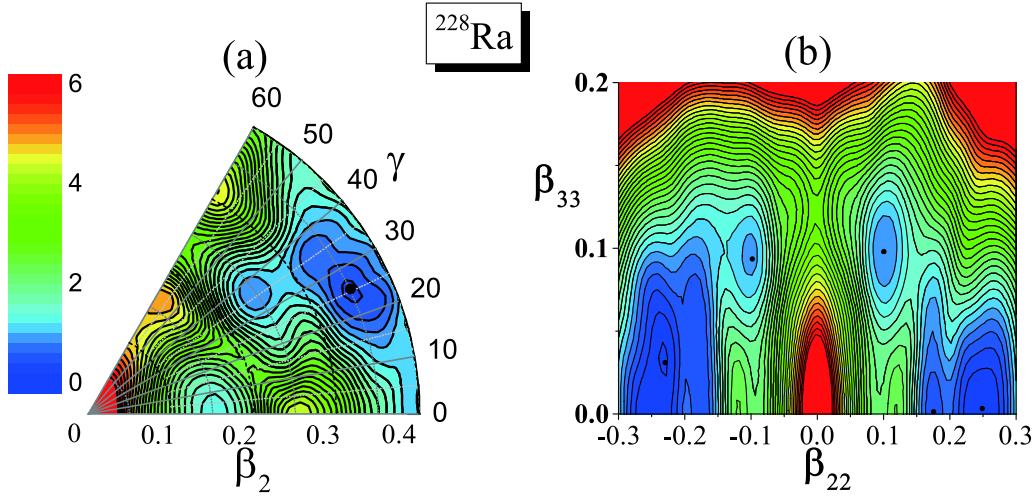
Fig. 2.   (color online) Calculated PES of ^{228}Ra in $\beta_2 - \gamma$ (a) and $\beta_{22} - \beta_{33}$ (b) planes. Energies are normalized with respect to the ground state. The contour separation is 0.2 MeV.

chirality can be robust with respect to octupole correlations. This observation also pointed to the exciting possibility of observing chirality-parity quartet (CPQ) bands, i.e., four $\Delta I = 1$ alternating-parity rotational bands with the same configuration in a single nucleus with both stable triaxial and octupole deformations. Thus far, CPQ bands have not been experimentally observed.

Notably, a pair of positive-parity doublet bands and several E1 transitions linking yrast positive- and negative-parity bands in ^{124}Cs have been reported by Ref. [93] and Refs. [94, 95], respectively. Based on the TPRM calculations, Ref. [86] suggested that the positive-parity doublet bands in ^{124}Cs might correspond to a typical chiral vibration pattern. Recently, lifetime measurements have been carried out using the Doppler shift attenuation method (DSAM) for the yrast positive- and negative-parity bands in ^{124}Cs [96]. The measured results [96] show that the $B(E1)$ rates are on the order of 10^{-4} W.u., thereby indicating the coexistence of a pair of chiral doublet bands and octupole correlations in ^{124}Cs.

To search for the possible candidate cores to construct CPQ bands, the potential energy surfaces (PES) of the even-even Se, Ba, and Ra isotopes were calculated using the macroscopic-microscopic method in a multidimensional space $\{\alpha_{\lambda,\mu}\}$ including quadrupole ($\lambda = 2$, $\mu = 0, 2$) and octupole ($\lambda = 3$, $\mu = 0, 1, 2, 3$) degrees of freedom [97]. The calculated results showed that the even-even isotopes ^{92}Se, $^{112,114,144-150}$Ba, and $^{220-228}$Ra can exhibit the coexistence of triaxial and octupole deformations. It is therefore expected that CPQ bands can be observed experimentally in these even-even nuclei and their neighboring odd-A/odd-odd nuclei. As an example, the calculated PES of ^{228}Ra in the $\beta_2 - \gamma$ and $\beta_{22} - \beta_{33}$ planes using the macroscopic-microscopic method [98-100] are shown in Fig. 2(a) and 2(b), respectively. One can observe from Fig. 2 that ^{228}Ra has obvious triaxial

and octupole deformations.

Here, one must note that in Table 1, the configurations of some MχD involve the orbits of pseudospin doublet states (e.g. $1g_{7/2}$, $2d_{5/2}$). Thus, a competing interpretation of these doublet bands would include the pseudospin doublet bands. Pseudospin symmetry in atomic nuclei was introduced in 1969 [101, 102]. A pair of nearly degenerate doublet bands with the configuration involving pseudospin doublet states have been observed and suggested as the pseudospin doublet bands in several nuclei [103-111].

A specific calculation [74] for the nearly degenerate triplet bands with the $\pi g_{9/2}^{-1} \otimes \nu h_{11/2}(d_{5/2}/g_{7/2})$ configuration in ^{105}Ag was performed using the RMF theory and the MPRM. The configuration-fixed constrained triaxial RMF calculations yielded the pseudospin symmetry in single particle spectra and triaxial shape coexistence. The experimental excitation energies and electromagnetic transition probabilities for the triplet bands were well reproduced by the MPRM calculations. Thus, the first and second lowest energy bands, as well as the second and third bands were interpreted as the pseudospin doublet bands and chiral doublet bands, respectively. This work also motivated the investigation to search for the chirality-pseudospin triplet (or quartet) bands in the nuclei.

6   Summary and perspectives

The recent progress in MχD is reviewed in terms of both experiment and theory. In particular, the experimental findings, theoretical predictions, selection rule of electromagnetic transitions, MχD with octupole correlations, and some related topics are highlighted. Based on the above discussion, it is of great scientific interest to search for other MχD, as well as the possible chiral wobblers,

chirality-parity quartet, and chirality-pseudospin triplet (or quartet) bands in nuclear system. On the other hand, these exotic nuclear phenomena have brought great challenges to current nuclear models and, thus, require the development of novel approaches. Very recently, in the study of the MχD with octupole correlations in ^{78}Br, a reflection-asymmetric triaxial PRM with a quasi-proton and a quasi-neutron coupled with a reflection-asymmetric triaxial rotor has been developed [112]. According to the present review, we would also like to attract more experimental and theoretical efforts in the investigation of chirality or multiple chirality in atomic nuclei.

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