

Accurate correction of arbitrary spin fermion quantum tunneling from non-stationary Kerr-de Sitter black hole based on corrected Lorentz dispersion relation*

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Abstract: According to a corrected dispersion relation proposed in the study on the string theory and quantum gravity theory, the Rarita-Schwinger equation was precisely modified, which resulted in the Rarita-Schwinger-Hamilton-Jacobi equation. Using this equation, the characteristics of arbitrary spin fermion quantum tunneling radiation from non-stationary Kerr-de Sitter black holes were determined. A number of accurately corrected physical quantities, such as surface gravity, chemical potential, tunneling probability, and Hawking temperature, which describe the properties of black holes, were derived. This research has enriched the research methods and enabled increased precision in black hole physics research.

Keywords: black hole, fermions, tunneling radiation, Lorentz dispersion relation

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1 Introduction

Hawking used the quantum theory to postulate that a black hole radiates particles outside its event horizon. Presently, this radiation is known as the Hawking radiation. The Hawking black hole radiation theory profoundly reveals intimate connections among the gravitation theory, the quantum theory, and the thermodynamic statistical physics. Hawking did not use the concepts of quantum tunneling and potential barrier in his specific calculation, nor did he consider the influence of the particles' emission on the black hole's mass or its event horizon, although he had used the quantum tunneling effect when proposing the black hole radiation [1, 2]. Such a calculation resulted in the Hawking radiation having a precise form of the Planck blackbody radiation, but also led to the paradox of the black hole "information loss".

Such an information loss (non-conservation) would lead to a major crisis in quantum theory.

Hawking's theory of tunneling radiation spurred researchers to study black holes. Kraus, Parikh, Wilczek *et al.* studied the quantum tunneling radiation of black holes by considering the energy conservation conditions in the radiation process, and explained Hawking's thermal radiation satisfactorily. This theory not only revised Hawking's pure thermal radiation theory, but also promoted the research and development of black hole physics [3-10]. Then, the semi-classical Hamilton-Jacobi method was proposed for studying the black holes' tunneling radiation [11, 12]. Using this method, the Klein-Gordon equation describing the behavior of "spin 0" particles was rewritten as action, and then was expanded using the Wentzel-Kramers-Brillouin (WKB) approximation. The terms containing \hbar (which was considered as a small

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quantity) were ignored, and the semi-classical Hamilton-Jacobi equation (herein referred to as the H-J equation) was obtained. Subsequently, the semi-classical H-J method was extensively developed to study Hawking radiation and black hole temperature. Kenner and Mann *et al.* firstly used the semi-classical method to study the quantum tunneling of fermions with spin-1/2 [13, 14]. They decomposed the field equation into the "spin up" and "spin down" cases, and obtained the tunneling rate of Dirac particles and the temperature of black holes. Banerjee *et al.* proposed an extended Hamilton-Jacobi method, beyond the semi-classical approximation, by considering all the terms in the expansion of the one-particle action [15]. Yang and Lin developed the semi-classical method and derived the H-J equation from the Dirac equation for spin-1/2 fermions by selecting the gamma matrix in higher-dimensional and lower-dimensional curved space-time [16, 17]. Later, this semi-classical H-J method was generalized to the Rarita-Schwinger equation (herein referred to as the R-S equation) for spin-3/2 fermions, from which the H-J equation was also obtained [18-21]. That is to say, the H-J equation can uniformly describe the dynamic behavior of arbitrary spin fermions in the curved space-time. Chen, Gecim *et al.* studied the effects of quantum gravity on black holes utilizing the semi-classical H-J method and introducing the generalized uncertainty principle (GUP) [22-32]. Through this semi-classical H-J method, the calculation process was simplified, and so the amount of calculations was greatly reduced.

Recent research on the quantum gravity theory suggested that the Lorentz dispersion relation needed to be modified for high energies [33-38]. Accordingly, the quantum tunneling radiation at the event horizon of a black hole has to be modified, owing to the conservation of energy and momentum. Since the H-J equation in the curved space-time corresponds to the Lorentz relation of energy and momentum in the curved space-time, the correction of the Lorentz dispersion relation will inevitably correct the H-J equation of particles in a strong gravitational field, which will lead to the correction of physical quantities such as the quantum tunneling rates of fermions or bosons and the black hole temperature. Although the modification of the Lorentz dispersion relation is only a small correction on the Planck scale, it significantly impacts the quantum tunneling radiation properties of black holes [39-43]. Accurately correcting the tunneling of arbitrary spin fermions at the event horizon of a black hole is a subject that merits further studies.

Some studies have been done on the correction of the tunneling radiation of static and stationary black holes [19-21, 44-51]. Actually, real black holes in the universe appear as non-stationary black holes, owing to evaporation, accretion, or mergers. For non-stationary black holes, Zhao *et al.* proposed a method for determining the

black hole temperature at the event horizon – the so-called tortoise coordinate transformation [52]. Subsequently, a series of studies were conducted on non-stationary black holes [53-57]. However, only a few studies have been reported on the correction of the quantum tunneling radiation from non-stationary black holes based on the corrected Lorentz dispersion relation, which merits further studies. In this paper, the arbitrary spin fermion quantum tunneling from a non-stationary Kerr-de Sitter black hole is studied, using the corrected Lorentz dispersion relation.

The remainder of this paper is organized as follows. In section 2, the dynamics equation of fermions in the non-stationary Kerr-de Sitter space-time is derived, based on the corrected Lorentz dispersion relation. In section 3, some precisely corrected, important physical quantities describing the quantum tunneling characteristics of arbitrary spin fermions are obtained, subsequently yielding the black hole temperature. In section 4, the study conclusions and some discussions, including that on the black hole entropy, are presented.

2 Correction of the dynamic equation for arbitrary spin fermions in non-stationary Kerr-de Sitter space-time

The modified Lorentz dispersion relation proposed in string theory and quantum gravity theory is [36-38, 58, 59]

$$p_0^2 = p^2 + m^2 - (Lp_0)^\alpha p^2, \quad (1)$$

where L is a constant on the Planck scale. When $\alpha = 2$ is considered, the general fermions' dynamic equation—R-S equation [60, 61] can be extended to the Kerr-de Sitter curved space-time as [19-21]

$$\left(\gamma^\mu D_\mu + \frac{m}{\hbar} - \sigma \hbar \gamma^t D_t \gamma^j D_j \right) \psi_{\alpha_1 \dots \alpha_k} = 0, \quad (2)$$

where γ^μ is the gamma matrix in curved space-time, satisfying the following condition:

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} I, \quad (3)$$

and D_μ is the covariant derivative operation symbol of curved space-time, that is

$$D_\mu = \partial_\mu + \Omega_\mu + \frac{i}{\hbar} e A_\mu, \quad (4)$$

where Ω_μ is the rotational connection in curved space-time. As quantum scale corrections, $0 < \sigma \ll 1$, so $\sigma \hbar \gamma^t D_t \gamma^j D_j \psi_{\alpha_1 \dots \alpha_k}$ is a small term. This matrix equation can only be solved in the specific curved space-time, so the fermions' wave function is set first as

$$\psi_{\alpha_1 \dots \alpha_k} = \xi_{\alpha_1 \dots \alpha_k} e^{\frac{i}{\hbar} S}, \quad (5)$$

where S is the action of fermions with mass m in the space-time of the Kerr-de Sitter black hole.

In order to solve Eq. (2), we rewrite it as

$$(i\gamma^\mu \partial_\mu S + m + \sigma \gamma^\nu \partial_\nu S \gamma^j \partial_j S) \xi_{\alpha_1 \dots \alpha_k} = 0, \quad (6)$$

where $\mu = 0, 1, 2, 3; j = 1, 2, 3$. For a non-stationary black hole, we use the advanced Eddington coordinate ν to represent the dynamic characteristics. To solve Eq. (6), define

$$\begin{aligned} \Gamma^\mu &= i\gamma^\mu + \sigma \partial_\nu S \gamma^\nu \gamma^\mu, & \Gamma^\nu &= i\gamma^\nu + \sigma \partial_\nu S \gamma^\nu \gamma^\nu, \\ m_D &= m - \sigma g^{\nu\nu} (\partial_\nu S)^2, \end{aligned} \quad (7)$$

so Eq. (6) becomes

$$(\Gamma^\mu \partial_\mu S + m_D) \xi_{\alpha_1 \dots \alpha_k} = 0. \quad (8)$$

Multiplying both sides of Eq. (8) by $\Gamma^\mu \partial_\mu S$, we obtain

$$(\Gamma^\nu \Gamma^\mu \partial_\nu S \partial_\mu S - m_D^2) \xi_{\alpha_1 \dots \alpha_k} = 0, \quad (9)$$

$$(\Gamma^\mu \Gamma^\nu \partial_\mu S \partial_\nu S - m_D^2) \xi_{\alpha_1 \dots \alpha_k} = 0. \quad (10)$$

Eqs. (9) and (10) are equivalent. Adding Eqs. (9) and (10) and considering Eqs. (7) and (3), we obtain

$$[g^{\mu\nu} \partial_\mu S \partial_\nu S - 2i\sigma \partial_\nu S g^{\nu\nu} \partial_\nu S \gamma^\mu \partial_\mu S - \sigma^2 (\partial_\nu S g^{\nu\nu} \partial_\nu S)^2 + m^2 - 2m\sigma g^{\nu\nu} (\partial_\nu S)^2 + \sigma^2 (g^{\nu\nu})^2 (\partial_\nu S)^4] \xi_{\alpha_1 \dots \alpha_k} = 0. \quad (11)$$

Dividing both sides of Eq. (11) by $-2\partial_\nu S g^{\nu\nu} \partial_\nu S$, we obtain

$$\left\{ i\sigma \gamma^\mu \partial_\mu S - \frac{g^{\mu\nu} \partial_\mu S \partial_\nu S - \sigma^2 (\partial_\nu S g^{\nu\nu} \partial_\nu S)^2 + m^2 - 2m\sigma g^{\nu\nu} (\partial_\nu S)^2 + \sigma^2 (g^{\nu\nu})^2 (\partial_\nu S)^4}{2\partial_\nu S g^{\nu\nu} \partial_\nu S} \right\} \xi_{\alpha_1 \dots \alpha_k} = 0. \quad (12)$$

Defining

$$m_l = -\frac{g^{\mu\nu} \partial_\mu S \partial_\nu S - \sigma^2 (\partial_\nu S g^{\nu\nu} \partial_\nu S)^2 + m^2 - 2m\sigma g^{\nu\nu} (\partial_\nu S)^2 + \sigma^2 (g^{\nu\nu})^2 (\partial_\nu S)^4}{2\partial_\nu S g^{\nu\nu} \partial_\nu S}, \quad (13)$$

Eq. (11) becomes

$$(i\sigma \gamma^\mu \partial_\mu S + m_l) \xi_{\alpha_1 \dots \alpha_k} = 0. \quad (14)$$

Multiplying both sides of Eq. (14) by $i\sigma \gamma^\nu \partial_\nu S$, we obtain

$$(\sigma^2 \gamma^\mu \gamma^\nu \partial_\mu S \partial_\nu S + m_l^2) \xi_{\alpha_1 \dots \alpha_k} = 0. \quad (15)$$

In Eq. (15), μ and ν are interchanged, yielding Eq. (16) as

$$(\sigma^2 \gamma^\nu \gamma^\mu \partial_\nu S \partial_\mu S + m_l^2) \xi_{\alpha_1 \dots \alpha_k} = 0. \quad (16)$$

Adding Eq. (15) and Eq. (16) and combining with Eq. (3), we obtain

$$(\sigma^2 g^{\mu\nu} \partial_\mu S \partial_\nu S + m_l^2) \xi_{\alpha_1 \dots \alpha_k} = 0. \quad (17)$$

Eq. (17) is a matrix equation. In fact, it is an eigenvalue equation, which has a non-zero solution when its coefficient determinant is zero. That is

$$\sigma^2 g^{\mu\nu} \partial_\mu S \partial_\nu S + \left\{ -\frac{g^{\mu\nu} \partial_\mu S \partial_\nu S - \sigma^2 (\partial_\nu S g^{\nu\nu} \partial_\nu S)^2 + m^2 - 2m\sigma g^{\nu\nu} (\partial_\nu S)^2 + \sigma^2 (g^{\nu\nu})^2 (\partial_\nu S)^4}{2\partial_\nu S g^{\nu\nu} \partial_\nu S} \right\}^2 = 0. \quad (18)$$

Taking notice of $g^{\mu\nu} \partial_\mu S \partial_\nu S = -m^2$, Eq. (18) becomes

$$g^{\mu\nu} \partial_\mu S \partial_\nu S + m^2 - 2m\sigma g^{\nu\nu} (\partial_\nu S)^2 - 2m\sigma \partial_\nu S g^{\nu\nu} \partial_\nu S - \sigma^2 (\partial_\nu S g^{\nu\nu} \partial_\nu S)^2 + \sigma^2 (g^{\nu\nu})^2 (\partial_\nu S)^4 = 0. \quad (19)$$

We have maintained the correction item of σ in Eq. (19), so Eq. (19) is the precisely corrected R-S equation considering the corrected Lorentz dispersion relation. Obviously, it is actually also an accurately corrected H-J equation; we call Eq. (19) the Rarita-Schwinger-Hamilton-Jacobi equation (R-S-H-J equation). In the following, we will use this precisely corrected R-S-H-J equation to study the dynamic behavior of arbitrary spin fermions in the non-stationary Kerr-de Sitter space-time; thus, we will study the properties of black holes.

3 Accurate correction of arbitrary spin fermion tunneling from the non-stationary Kerr-de Sitter black hole

In the advanced Eddington-Finkelstein coordinate, the line element of the non-stationary Kerr-de Sitter black hole can be written as [62, 63]

$$\begin{aligned}
 ds^2 = & A \frac{1}{\Sigma} (\Delta_\lambda - \Delta_\theta a^2 \sin^2 \theta) dv^2 \\
 & - 2\sqrt{A} (dv - a \sin^2 \theta d\varphi) dr - \frac{\Sigma}{\Delta_\theta} d\theta^2 \\
 & + A \frac{2a}{\Sigma} [\Delta_\theta (r^2 + a^2) - \Delta_\lambda] \sin^2 \theta dv d\varphi \\
 & - A \frac{1}{\Sigma} [\Delta_\theta (r^2 + a^2) - \Delta_\lambda \sin^2 \theta] \sin^2 \theta d\varphi^2, \quad (20)
 \end{aligned}$$

where

$$\begin{aligned}
 A = & \left(1 + \frac{1}{3} \Lambda a^2\right)^{-2}, \\
 \Sigma = & r^2 + a^2 \cos^2 \theta, \\
 \Delta_\lambda = & r^2 + a^2 - 2Mr - \frac{1}{3} \Lambda r^2 (r^2 + a^2), \\
 \Delta_\theta = & 1 + \frac{1}{3} \Lambda a^2 \cos^2 \theta, \\
 M = & M(v), \\
 a = & a(v). \quad (21)
 \end{aligned}$$

M is the mass of the black hole, and a is the angular momentum per unit mass of the black hole. From Eq. (20), the non-zero components of the contravariant metric tensor of the black hole are

$$\begin{aligned}
 g^{00} = & -\frac{a^2 \sin^2 \theta}{\Delta_\theta A \Sigma}, \\
 g^{11} = & -\frac{\Delta_\lambda}{\Sigma}, \\
 g^{22} = & -\frac{\Delta_\theta}{\Sigma}, \\
 g^{33} = & -\frac{1}{\Delta_\theta A \Sigma \sin^2 \theta}, \\
 g^{01} = g^{10} = & -\frac{r^2 + a^2}{\sqrt{A} \Sigma}, \\
 g^{03} = g^{30} = & -\frac{a}{\Delta_\theta A \Sigma}, \\
 g^{13} = g^{31} = & -\frac{a}{\sqrt{A} \Sigma}. \quad (22)
 \end{aligned}$$

According to the zero hypersurface condition, the event horizon equation satisfies

$$g^{\mu\nu} \frac{\partial f}{\partial x_\mu} \frac{\partial f}{\partial x_\nu} = 0, \quad (23)$$

where f is a hypersurface. Since the Kerr-de Sitter space-time is axisymmetric, Eq. (23) is independent of φ , so $\frac{\partial f}{\partial \varphi} = 0$, and the equation of the event horizon of the Kerr-de Sitter black hole can be expressed as

$$g^{00} \left\{ \frac{\partial f}{\partial v} \right\}^2 + g^{11} \left\{ \frac{\partial f}{\partial r} \right\}^2 + g^{22} \left\{ \frac{\partial f}{\partial \theta} \right\}^2 + 2g^{01} \frac{\partial f}{\partial v} \frac{\partial f}{\partial r} = 0. \quad (24)$$

Its zero hypersurface is

$$f = f(r, v, \theta) = 0, \quad (25)$$

where r is the function of v and θ , $r = r(v, \theta)$. In order to obtain the position of the event horizon of the black hole, we need to calculate the rate of change of f with respect to each of its components. Taking the partial derivative of Eq. (25), we obtain

$$\begin{aligned}
 \frac{\partial f}{\partial v} = & -\frac{\partial f}{\partial r} \frac{\partial r}{\partial v}, \\
 \frac{\partial f}{\partial \theta} = & -\frac{\partial f}{\partial r} \frac{\partial r}{\partial \theta}, \quad (26)
 \end{aligned}$$

so Eq. (24) becomes

$$g^{00} \left\{ \frac{\partial r}{\partial v} \right\}^2 + g^{11} + g^{22} \left\{ \frac{\partial r}{\partial \theta} \right\}^2 + 2g^{01} \frac{\partial r}{\partial v} = 0. \quad (27)$$

Substituting the expressions of $g^{00}, g^{11}, g^{22}, g^{01}$ in Eq. (22) into Eq. (27), when $r \rightarrow r_H$, the equation of the event horizon is obtained as follows

$$a^2 \sin^2 \theta \dot{r}_H^2 - 2\Delta_\theta \sqrt{A} (r_H^2 + a^2) \dot{r}_H + \Delta_\lambda \Delta_\theta A + \Delta_\theta^2 A r_H'^2 = 0, \quad (28)$$

where

$$\begin{aligned}
 \dot{r}_H = & \frac{\partial r_H}{\partial v}, \\
 r'_H = & \frac{\partial r_H}{\partial \theta}, \quad (29)
 \end{aligned}$$

which represents the change of the position of the event horizon with time and angle, respectively. For the Kerr-de Sitter black hole, substituting Eq. (22) into Eq. (19) — the accurately corrected R-S-H-J equation, the specific expression of Eq. (19) is

$$\begin{aligned}
 & g^{00} \left\{ \frac{\partial S}{\partial v} \right\}^2 + g^{11} \left\{ \frac{\partial S}{\partial r} \right\}^2 + g^{22} \left\{ \frac{\partial S}{\partial \theta} \right\}^2 + g^{33} \left\{ \frac{\partial S}{\partial \varphi} \right\}^2 \\
 & + 2g^{01} \frac{\partial S}{\partial v} \frac{\partial S}{\partial r} + 2g^{03} \frac{\partial S}{\partial v} \frac{\partial S}{\partial \varphi} + 2g^{13} \frac{\partial S}{\partial r} \frac{\partial S}{\partial \varphi} \\
 & + m^2 - 2m\sigma \frac{\partial S}{\partial v} \left\{ g^{00} \frac{\partial S}{\partial v} + g^{01} \frac{\partial S}{\partial r} + g^{03} \frac{\partial S}{\partial \varphi} \right\} \\
 & - 2m\sigma g^{00} \left\{ \frac{\partial S}{\partial v} \right\}^2 + \sigma^2 (g^{00})^2 \left\{ \frac{\partial S}{\partial v} \right\}^4 \\
 & - \sigma^2 \left\{ \frac{\partial S}{\partial v} \right\}^2 \left\{ \left\{ g^{00} \frac{\partial S}{\partial v} \right\}^2 + \left\{ g^{01} \frac{\partial S}{\partial r} \right\}^2 + \left\{ g^{03} \frac{\partial S}{\partial \varphi} \right\}^2 \right\} = 0. \quad (30)
 \end{aligned}$$

Since the event horizon of the black hole varies with time, it requires using a generalized tortoise coordinate transformation to solve the tunneling probability of fermions from the event horizon of the black hole. The Kerr-de Sitter black hole is an axisymmetric black hole, so we perform the following transformation

$$\begin{aligned}
 r_* = & r + \frac{1}{2\kappa} \ln \frac{r - r_H(v, \theta)}{r_H(v_0, \theta_0)}, \\
 v_* = & v - v_0, \\
 \theta_* = & \theta - \theta_0. \quad (31)
 \end{aligned}$$

Taking the partial derivative of Eq. (31), we obtain

$$\begin{aligned} \frac{\partial S}{\partial r} &= \frac{\partial S}{\partial r_*} \frac{2\kappa[r-r_H(v,\theta)]+1}{2\kappa[r-r_H(v,\theta)]}, \\ \frac{\partial S}{\partial v} &= \frac{\partial S}{\partial v_*} - \frac{\partial S}{\partial r_*} \frac{\dot{r}_H}{2\kappa[r-r_H(v,\theta)]}, \\ \frac{\partial S}{\partial \theta} &= \frac{\partial S}{\partial \theta_*} - \frac{\partial S}{\partial r_*} \frac{\dot{r}'_H}{2\kappa[r-r_H(v,\theta)]}, \end{aligned} \quad (32)$$

where

$$\begin{aligned} \dot{r}_H(v,\theta) &= \frac{\partial r_H(v,\theta)}{\partial v}, \\ \dot{r}'_H(v,\theta) &= \frac{\partial r_H(v,\theta)}{\partial \theta}, \end{aligned} \quad (33)$$

which implies that the position of the event horizon varies with time and angle, respectively. In the Kerr-de Sit-

ter space-time, the action of arbitrary spin fermions can be expressed as follows:

$$S = S(v_*, r_*, \theta_*, \varphi). \quad (34)$$

Although S cannot be separated, it is certain that

$$\begin{aligned} \frac{\partial S}{\partial v_*} &= -\omega, \\ \frac{\partial S}{\partial \theta_*} &= n, \end{aligned} \quad (35)$$

and let

$$\frac{\partial S}{\partial \theta_*} = P_\theta. \quad (36)$$

Eq. (30) is further simplified by writing the second-order small quantity as $O(\sigma^2)$, which is not considered in the separation of variables; thus, we obtain

$$\begin{aligned} (1-4m\sigma)g^{00} \left\{ \frac{\partial S}{\partial v} \right\}^2 + g^{11} \left\{ \frac{\partial S}{\partial r} \right\}^2 + g^{22} \left\{ \frac{\partial S}{\partial \theta} \right\}^2 + g^{33} \left\{ \frac{\partial S}{\partial \varphi} \right\}^2 \\ + 2(1-2m\sigma)g^{01} \frac{\partial S}{\partial v} \frac{\partial S}{\partial r} + 2(1-m\sigma)g^{03} \frac{\partial S}{\partial v} \frac{\partial S}{\partial \varphi} + 2g^{13} \frac{\partial S}{\partial r} \frac{\partial S}{\partial \varphi} + m^2 + O(\sigma^2) = 0. \end{aligned} \quad (37)$$

Substituting Eqs. (32), (33), (35), and (36) into Eq. (37), we obtain (for compactness, the symbol (v, θ) of the independent variables is omitted in the following.)

$$\begin{aligned} \frac{(1-4m\sigma)g^{00}\dot{r}_H^2 + g^{11}\{2\kappa[r-r_H]+1\}^2 + g^{22}r_H'^2 - 2(1-m\sigma)g^{01}\{2\kappa[r-r_H]+1\}\dot{r}_H}{2\kappa[r-r_H]} \left\{ \frac{\partial S}{\partial r_*} \right\}^2 \\ - \{-2(1-4m\sigma)g^{00}\dot{r}_H + 2(1-m\sigma)g^{01}\{2\kappa[r-r_H]+1\}\} \omega \frac{\partial S}{\partial r_*} \\ + \{-2(1-m\sigma)g^{03}\dot{r}_H n + 2g^{13}\{2\kappa[r-r_H]+1\}n - 2g^{22}r_H'^2 P_\theta\} \frac{\partial S}{\partial r_*} \\ + [(1-4m\sigma)g^{00}\omega^2 - 2(1-m\sigma)g^{03}\omega n + g^{22}P_\theta^2 + g^{33}n^2 + m^2 - O(\sigma^2)]2\kappa[r-r_H] = 0. \end{aligned} \quad (38)$$

Substituting Eq. (22) into Eq. (38), and using $r \rightarrow r_H$ to simplify the obtained equation, we obtain

$$\begin{aligned} \frac{(1-4m\sigma)a^2 \sin^2 \theta \dot{r}_H^2 + \Delta_\lambda \Delta_\theta A \{2\kappa[r-r_H]+1\}^2 + \Delta_\theta^2 r_H'^2 - 2(1-m\sigma)\Delta_\theta \sqrt{A}(r^2+a^2)\{2\kappa[r-r_H]+1\}\dot{r}_H}{2\kappa[r-r_H]} \left\{ \frac{\partial S}{\partial r_*} \right\}^2 \\ - 2\{-(1-4m\sigma)a^2 \sin^2 \theta \dot{r}_H + (1-m\sigma)\Delta_\theta \sqrt{A}(r^2+a^2)\{2\kappa[r-r_H]+1\}\} \omega \frac{\partial S}{\partial r_*} \\ + 2\{-a(1-m\sigma)\dot{r}_H n + a\Delta_\theta \sqrt{A}\{2\kappa[r-r_H]+1\}n - \Delta_\theta^2 A r_H'^2 P_\theta\} \frac{\partial S}{\partial r_*} = 0. \end{aligned} \quad (39)$$

Let

$$\begin{aligned} A_0 &= \frac{(1-4m\sigma)a^2 \sin^2 \theta \dot{r}_H^2 + \Delta_\lambda \Delta_\theta A \{2\kappa[r-r_H]+1\}^2 + \Delta_\theta^2 r_H'^2 - 2(1-m\sigma)\Delta_\theta \sqrt{A}(r^2+a^2)\{2\kappa[r-r_H]+1\}\dot{r}_H}{2\kappa[r-r_H]}, \\ B_0 &= \{-(1-4m\sigma)a^2 \sin^2 \theta \dot{r}_H + (1-m\sigma)\Delta_\theta \sqrt{A}(r^2+a^2)\{2\kappa[r-r_H]+1\}\}, \\ C_0 &= \{-a(1-m\sigma)\dot{r}_H n + a\Delta_\theta \sqrt{A}\{2\kappa[r-r_H]+1\}n - \Delta_\theta^2 A r_H'^2 P_\theta\}. \end{aligned} \quad (40)$$

Dividing both sides of Eq. (39) by B_0 , Eq. (39) becomes

$$\frac{A_0}{B_0} \left\{ \frac{\partial S}{\partial r_*} \right\}^2 - 2\omega \frac{\partial S}{\partial r_*} + 2 \frac{C_0}{B_0} \frac{\partial S}{\partial r_*} = 0. \quad (41)$$

When $r \rightarrow r_H$, the limit of the coefficient of $\left\{\frac{\partial S}{\partial r}\right\}^2$ at the event horizon should be equal to one, that is

$$\lim_{r \rightarrow r_H} \frac{A_0}{B_0} = \frac{(1 - 4m\sigma)a^2 \sin^2 \theta \dot{r}_H^2 + \Delta_\lambda \Delta_\theta A \{2\kappa[r - r_H] + 1\}^2 + \Delta_\theta^2 r_H^2 - 2(1 - m\sigma)\Delta_\theta \sqrt{A}(r^2 + a^2)\{2\kappa[r - r_H] + 1\}\dot{r}_H}{2\kappa[r - r_H]\{- (1 - 4m\sigma)a^2 \sin^2 \theta \dot{r}_H + (1 - m\sigma)\Delta_\theta \sqrt{A}(r^2 + a^2)\{2\kappa[r - r_H] + 1\}\}} = 1. \quad (42)$$

In Eq. (42), the limit of the denominator is zero when $r \rightarrow r_H$; thus, the limit of the numerator should also be zero when $r \rightarrow r_H$. Using L'hospital's rule, we can work out κ as

$$\kappa = \frac{\left(r_H - M - \frac{2}{3}\Lambda r_H^3 - \frac{1}{3}\Lambda a^2 r_H\right)\Delta_\theta A - 2(1 - m\sigma)\sqrt{A}\Delta_\theta r_H \dot{r}_H}{(r_H^2 + a^2)\sqrt{A}\Delta_\theta + 2(r_H^2 + a^2)\sqrt{A}\Delta_\theta \dot{r}_H - a^2 \sin^2 \theta \dot{r}_H - 2\Delta_\theta A \Delta_\lambda} [1 + \sigma \tilde{m} - (\sigma \tilde{m})^2 + \dots], \quad (43)$$

where

$$\tilde{m} = \frac{m[(r_H^2 + a^2)\sqrt{A}\Delta_\theta + 2(r_H^2 + a^2)\sqrt{A}\Delta_\theta \dot{r}_H - 4a^2 \sin^2 \theta \dot{r}_H]}{(r_H^2 + a^2)\sqrt{A}\Delta_\theta + 2(r_H^2 + a^2)\sqrt{A}\Delta_\theta \dot{r}_H - a^2 \sin^2 \theta \dot{r}_H - 2\Delta_\theta A \Delta_\lambda}. \quad (44)$$

For the Schwarzschild space-time, it can be proved that $\tilde{m} = m$. κ is the precisely corrected surface gravity at the event horizon. Taking $\lim_{r \rightarrow r_H} \frac{C_0}{B_0} = \omega_0$, we obtain

$$\omega_0 = \frac{\Delta_\theta \sqrt{A} a n + \Delta_\theta \dot{r}_H P_\theta - (1 - m\sigma) a \dot{r}_H n}{\Delta_\theta \sqrt{A}(r_H^2 + a^2) - a^2 \sin^2 \theta \dot{r}_H} [1 + \sigma m' - (\sigma m')^2 + \dots], \quad (45)$$

where

$$m' = \frac{m[\Delta_\theta \sqrt{A}(r_H^2 + a^2) - 4a^2 \sin^2 \theta \dot{r}_H]}{\Delta_\theta \sqrt{A}(r_H^2 + a^2) - a^2 \sin^2 \theta \dot{r}_H}. \quad (46)$$

ω_0 is the exact corrected chemical potential. Combined with Eqs. (42) and (45), Eq. (41) can be written as follows when $r \rightarrow r_H$

$$\left\{\frac{\partial S}{\partial r_*}\right\}^2 - (\omega - \omega_0) \left\{\frac{\partial S}{\partial r_*}\right\} = 0. \quad (47)$$

Substituting $\frac{\partial s}{\partial r} = \frac{\partial s}{\partial r_*} \frac{2\kappa(r - r_H) + 1}{\kappa(r - r_H)}$ in Eq. (32) into Eq. (47), we obtain

$$\left\{\frac{\partial S}{\partial r}\right\}_\pm = [(\omega - \omega_0) \pm (\omega - \omega_0)] \frac{\kappa(r - r_H) + 1}{\kappa(r - r_H)}. \quad (48)$$

When $r \rightarrow r_H$, the residue theorem can be applied to ob-

tain

$$S_\pm = \lim_{r \rightarrow r_H} \int [(\omega - \omega_0) \pm (\omega - \omega_0)] \frac{\kappa(r - r_H) + 1}{\kappa(r - r_H)} dr = \pi i \frac{(\omega - \omega_0) \pm (\omega - \omega_0)}{2\kappa}. \quad (49)$$

According to the tunneling theory, we obtain the tunneling probability of an arbitrary spin fermion in the space-time of the Kerr-de Sitter black hole

$$\begin{aligned} \Gamma &= \exp[-2(\text{Im}S_+ - \text{Im}S_-)] \\ &= \exp\left\{-\frac{2\pi(\omega - \omega_0)}{\kappa}\right\} \\ &= \exp\left\{-\frac{\omega - \omega_0}{T_H}\right\}. \end{aligned} \quad (50)$$

where

$$T_H = \frac{1}{2\pi} \frac{\left(r_H - M - \frac{2}{3}\Lambda r_H^3 - \frac{1}{3}\Lambda a^2 r_H\right)\Delta_\theta A - 2(1 - m\sigma)\sqrt{A}\Delta_\theta r_H \dot{r}_H}{(r_H^2 + a^2)\sqrt{A}\Delta_\theta + 2(r_H^2 + a^2)\sqrt{A}\Delta_\theta \dot{r}_H - a^2 \sin^2 \theta \dot{r}_H - 2\Delta_\theta A \Delta_\lambda} [1 + \sigma \tilde{m} - (\sigma \tilde{m})^2 + \dots]. \quad (51)$$

In Eq. (51), T_H is the precisely corrected Hawking temperature at the event horizon of the black hole. This is a new form of the Hawking temperature expression for the Kerr-de Sitter black holes.

From the demonstration in this section, we note that using the modified R-S-H-J equation based on the corrected Lorentz dispersion relation, we have obtained a series of precisely corrected physical quantities of the Kerr-de

Sitter black holes, including the surface gravity, chemical potential, tunneling probability of arbitrary spin fermions, and Hawking temperature. The correction is indicated by the parameter σ . From Eqs. (43), (45), (50), and (51), it is observed that for a non-stationary Kerr-de Sitter black hole, the surface gravity, chemical potential, tunneling probability, and Hawking temperature change with time as the event horizon surface changes. The

chemical potential also varies with angle θ .

4 Discussion and conclusion

In this paper, based on the modified Lorentz dispersion relation on the quantum scale and with the condition $\alpha = 2$ selected, we derived the modified R-S-H-J equation by properly selecting the transformation matrix and using the semi-classical method. By using the derived R-S-H-J equation, we solved the dynamic Kerr-de Sitter black hole using the tortoise coordinate transformation, and obtained the accurately corrected surface gravity, chemical potential, tunneling probability, and Hawking temperature. Although the correction term σ is a small quantity, it is still worth further studying.

Another important physical quantity in the thermodynamics of black holes is the black hole entropy. According to the first law of thermodynamics, the entropy S^s of a black hole can be expressed as

$$dM = TdS^s + VdJ + UdQ. \quad (52)$$

For the Kerr-de Sitter black hole,

$$dS^s = \frac{dM - VdJ}{T}. \quad (53)$$

The exactly corrected entropy at the event horizon $r = r_H$ of the black hole can be expressed as

$$S_{r_H}^s = \int \frac{dM - VdJ}{T_H} = \int \frac{dM - VdJ}{T_0} (1 - \sigma\tilde{m}) [1 - \sigma\tilde{m} + (\sigma\tilde{m})^2 - \dots], \quad (54)$$

where

$$\tilde{m} = \frac{2m\sqrt{A}\Delta_\theta r_H \dot{r}_H}{\left(r_H - M - \frac{2}{3}\Lambda r_H^3 - \frac{1}{3}\Lambda a^2 r_H\right)\Delta_\theta A - 2\sqrt{A}\Delta_\theta r_H \dot{r}_H}. \quad (55)$$

In Eq. (53) and Eq. (54), the relational formula $\frac{dM - VdJ}{T_0} = dS_0^s$ represents the black hole entropy before the correction, and T_0 represents the Hawking temperature before the correction, which is

$$T_0 = \frac{1}{2\pi} \frac{\left(r_H - M - \frac{2}{3}\Lambda r_H^3 - \frac{1}{3}\Lambda a^2 r_H\right)\Delta_\theta A - 2\sqrt{A}\Delta_\theta r_H \dot{r}_H}{(r_H^2 + a^2)\sqrt{A}\Delta_\theta + 2(r_H^2 + a^2)\sqrt{A}\Delta_\theta \dot{r}_H - a^2 \sin^2 \theta r_H - 2\Delta_\theta A \Delta_\lambda}. \quad (56)$$

When not considering the correction and returning to the Schwarzschild space-time, combining Eqs. (21) and (28), Eqs. (51) and (56) revert to the form of $T = \frac{1}{8\pi M}$, which also proves the correctness of the obtained results. Equations (43), (45), (50), and (51) are all new expressions with precise corrections. The Lorentz dispersion re-

lation is a theory worth studying in the field of high energies, and is a relation that needs to be considered in both the theories of strong gravitational fields and gravitational waves. It is worth pointing out that in the corrected Lorentz dispersion relation, we considered $\alpha = 2$, and considering other values is also worth pursuing; this will be addressed in our future work.

References

- 1 S. W. Hawking, *Nature*, **248**: 30-31 (1974)
- 2 S. W. Hawking, *Commun. Math. Phys.*, **43**(2): 199-220 (1975)
- 3 P. Kraus and F. Wilczek, *Nucl. Phys. B*, **433**(2): 403-420 (1995)
- 4 M. K. Parikh and F. Wilczek, *Phys. Rev. Lett.*, **85**(4): 5042-5049 (2000)
- 5 Z. Zhao, G. H. Tian, L. Liu *et al.*, *Chin. Phys. Lett.*, **23**(12): 3165-3168 (2006)
- 6 J. Ren, J. Y. Zhang, and Z. Zhao, *Chin. Phys. Lett.*, **23**(8): 2019-2022 (2006)
- 7 C. Z. Liu, J. Y. Zhang, and Z. Zhao, *Phys. Lett. B*, **639**(6): 670-674 (2009)
- 8 H. Z. Fang, J. Ren, and J. Zhao, *Int. J. Mod. Phys. D*, **14**(10): 1699-1750 (2005)
- 9 K. Lin, X. X. Zeng, and S. Z. Yang, *Chin. Phys. Lett.*, **25**(2): 390-392 (2008)
- 10 R. Banerjee and B. R. Majhi, *Phys. Rev. D*, **79**(6): 064024 (2009)
- 11 K. Srinivasan and T. Padmanabhan, *Phys. Rev. D*, **60**(2): 24007 (1999)
- 12 S. Shankaranarayanan, T. Padmanabhan, and K. Srinivasan, *Class. Quantum Grav.*, **19**: 2671-2687 (2002)
- 13 R. Kenner and R. B. Mann, *Class. Quantum Grav.*, **25**(9): 095014 (2008)
- 14 R. Kenner and R. B. Mann, *Phys. Lett. B*, **665**(4): 277-283 (2008)
- 15 R. Banerjee and B. R. Majhi, *J. High Energy Phys.*, **2008**(6): 095 (2008)
- 16 K. Lin and S. Z. Yang, *Phys. Rev. D*, **79**(6): 064035 (2009)
- 17 K. Lin and S. Z. Yang, *Phys. Lett. B*, **674**(2): 127-130 (2009)
- 18 S. Z. Yang and K. Lin, *Sci. Sin-Phy. Mech. Astron.*, **40**(4): 507-512 (2010)
- 19 S. Z. Yang and K. Lin, *Sci. Sin-Phy. Mech. Astron.*, **49**: 019503 (2019)
- 20 B. Sha, Z. E. Liu, X. Tan *et al.*, *Adv. High Energy Phys.*, **2020**: 3238401 (2020)
- 21 Z. E. Liu, X. Tan, B. Sha *et al.*, *Mod. Phys. Lett. A*, (2019)
- 22 D. Y. Chen, Q. Q. Jiang, and P. Wang *et al.*, *J. High Energy Phys.*, **2013**(11): 176 (2013)
- 23 D. Chen, H. Wu, and H. Yang, *Adv. High Energy Phys.*, **2013**: 432412 (2013)
- 24 D. Chen, H. Wu, H. Yang *et al.*, *Int. J. Mod. Phys. A*, **29**(26): 1430054 (2014)
- 25 I. Sakalli, A. Övgün, and K. Jusufi, *Astrophys. Space Sci.*, **361**: 330 (2016)

- 26 G. Gecim and Y. Sucu, *Phys. Lett. B*, **773**: 391-394 (2017)
- 27 T. Ibungochouba Singh, I. Ablu Meitei, and K. Yugindro Singh, *Int. J. Theor. Phys.*, **56**: 2640-2650 (2017)
- 28 G. Gecim and Y. Sucu, *Adv. High Energy Phys.*, **2018**: 8728564 (2018)
- 29 G. Gecim and Y. Sucu, *Mod. Phys. Lett. A*, **33**(28): 1850164 (2018)
- 30 G. Gecim and Y. Sucu, *Eur. Phys. J. C*, **29**: 882 (2019)
- 31 W. Javed, R. Babar, and A. Övgün, *Mod. Phys. Lett. A*, **34**: 1950057 (2019)
- 32 W. Javed, R. Ali, R. Babar *et al.*, *Chin. Phys. C*, **44**(1): 015104 (2020)
- 33 G. Amelino-Camelia and D. V. Ahluwalia, *Int. J. Mod. Phys. D*, **11**(1): 35-59 (2002)
- 34 J. Magueijo and L. Smolin, *Phys. Rev. Lett.*, **88**(19): 190403 (2002)
- 35 J. Ellis, N. E. Mavromatos, and D. V. Nanopoulos, *Phys. Lett. B*, **293**(1-2): 37-48 (1992)
- 36 J. R. Ellis, N. E. Mavromatos, and A. S. Sakharov, *Astropart. Phys.*, **20**(6): 669-682 (2004)
- 37 T. Jacobson, S. Liberati, and D. Mattingly, *Nature*, **424**(6952): 1019-1021 (2003)
- 38 S. I. Kruglov, *Mod. Phys. Lett. A*, **28**(6): 1350014 (2013)
- 39 Q. G. Bailey, V. A. Kostelecky, and R. Xu, *Phys. Rev. D*, **91**(2): 022006 (2015)
- 40 V. A. Kostelecky and R. Potting, *Nucl. Phys. B*, **359**(2-3): 545-570 (1991)
- 41 V. A. Kostelecky and R. Potting, *Phys. Rev. D*, **51**(7): 3923 (1995)
- 42 V. A. Kostelecky and S. Samuel, *Phys. Rev. D*, **39**(2): 683 (1989)
- 43 V. A. Kostelecky and J. D. Tasson, *Phys. Rev. Lett.*, **102**(1): 010402 (2009)
- 44 T. Damour and R. Ruffini, *Phys. Rev. D*, **14**(2): 332-334 (1976)
- 45 S. Sannan, *Gen. Relativ. Gravit.*, **20**(3): 239-246 (1988)
- 46 D. C. Robinson, *Phys. Rev. Lett.*, **34**(14): 905 (1975)
- 47 G. W. Gibbons and M. J. Perry, *P. Roy. Soc. A*, **358**(1695): 467-494 (1978)
- 48 Z. Zhao and Y. X. Guei, *Chinese Astronomy and Astrophysics*, **7**(3): 201-210 (1983)
- 49 W. G. Unruh, *Phys. Rev. D*, **14**(4): 870 (1976)
- 50 J. B. Hartle and S. W. Hawking, *Phys. Rev. D*, **13**(8): 2188 (1976)
- 51 B. Sha, Z. E. Liu, X. Tan *et al.*, *Advances in Astrophysics*, **5**(1): 51002 (2020)
- 52 Z. Zhao and X. X. Dai, *Chin. Phys. Lett.*, **8**(10): 548-550 (1991)
- 53 Z. Zhao and J. Zhu, *Int. J. Theor. Phys.*, **33**(11): 2147 (1994)
- 54 Z. Zhao, Z. Q. Luo, and C. G. Huang, *Chin. Phys. Lett.*, **9**(5): 269 (1992)
- 55 Z. H. Li and Z. Zhao, *Chin. Phys. Lett.*, **11**(1): 8 (1994)
- 56 M. C. Sun, R. Zhao, and Z. Zhao, *IL Nuovo Cimento B*, **110**(7): 829-837 (1995)
- 57 S. Z. Yang, J. Y. Zhu, and Z. Zhao, *Acta Phys. Sin. (Overseas Edn)*, **4**(2): 147-150 (1995)
- 58 J. Magueijo and L. Smolin, *Phys. Rev. D*, **67**(4): 044017 (2003)
- 59 G. Amelino-Camelia, *New J. Phys.*, **6**: 188 (2004)
- 60 W. Rarita and J. Schwinger, *Phys. Rev.*, **60**: 61 (1941)
- 61 G. W. Gibbons, *J. Phys. A: Math. Gen.*, **9**(1): 145-148 (1976)
- 62 M. Carmeli and M. Kaye, *Ann. Phys.*, **103**(1): 97-120 (1977)
- 63 D. Y. Xu, *Class. Quantum Grav.*, **15**: 153 (1998)