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Gravitational leptogenesis in teleparallel and symmetric teleparallel gravities*

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Abstract: In this study, we investigate the possibilities of generating baryon number asymmetry under thermal equilibrium within the frameworks of teleparallel and symmetric teleparallel gravities. Through the derivative couplings of the torsion scalar and the non-metricity scalar to baryons, baryon number asymmetry is produced in the radiation dominated epoch. For gravitational baryogenesis mechanisms in these two frameworks, the produced baryon-to-entropy ratio is too small to be consistent with observations. However, the gravitational leptogenesis models within both frameworks have the potential to explain the observed baryon-antibaryon asymmetry.

Keywords: cosmology, teleparallel gravity, leptogenesis

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I. INTRODUCTION

All current observations suggest that our universe contains an excess of matter over antimatter. The Planck result [1] showed that the left baryon-to-photon ratio is $n_B/n_\gamma = (6.12\pm 0.04)\times 10^{-10}$ [2]. For theoretical discussions, it is more convenient to use the baryon-to-entropy ratio n_B/s to quantify this asymmetry, which is approximately $n_B/s \simeq 8.7\times 10^{-11}$ as calculated from the observational result, as $s\simeq 7.04 n_\gamma$ at present. The origin of this baryon number asymmetry remains unexplained in cosmology. Conventionally, it was argued that this asymmetry was generated dynamically from an initial symmetric baryon phase at the following conditions [3]: (1) baryon number non-conserving interactions, (2) C and CP violations, (3) a departure from thermal equilibrium.

However, if the CPT symmetry is violated, the baryon number asymmetry could be generated in thermal equilibrium [4]. For example, in Refs. [5-7], an effective interaction,

$$\mathcal{L}_{in} = \frac{c}{M_*}\partial_\mu \phi J^\mu_B \,, \tag{1}$$

between the dynamic dark energy (quintessence) and baryons was introduced, which considers a dimensionless coupling constant c and a cut-off mass scale M_*. As the universe expands, the background evolution of the scalar field ϕ breaks the Lorentz and CPT symmetries spontaneously, providing an effective chemical potential for baryons and the opposite for antibaryons:

$$\frac{c}{M_*}\partial_\mu\phi J^\mu_B \to \frac{c}{M_*}\dot{\phi} n_B = \frac{c}{M_*}\dot{\phi}(n_b - n_{\bar{b}}) \,,$$
$$\mu_b = \frac{c}{M_*}\dot{\phi} = -\mu_{\bar{b}} \,. \tag{2}$$

This creates a difference between the distribution functions of baryons and antibaryons in thermal equilibrium, producing an excess of baryons over antibaryons [8]:

$$n_B = \frac{g_b T^3}{6}\left[\frac{\mu_b}{T} + O\left(\frac{\mu_b}{T}\right)^3\right] \simeq c\frac{\dot{\phi}T^2}{3M_*} \,, \tag{3}$$

where $g_b = 2$ is the internal degrees of freedom of the baryon. In terms of the entropy density $s = (2\pi^2/45) g_{*s} T^3$, the baryon-to-entropy ratio is written as

$$\frac{n_B}{s} = \frac{15c}{2\pi^2}\frac{\dot{\phi}}{g_{*s}M_* T} \,. \tag{4}$$

The parameter g_{*s} denotes the total number of relativistic degrees of freedom in the universe and decreases slightly with cosmic expansion and cooling [8]. In the standard model of particle physics, $g_{*s} = 106.75$ in the early universe during radiation domination, i.e., after reheating but well before the electroweak phase transition. Please note that to generate the baryon number asymmetry (4), there should be baryon number violating interactions in thermal

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equilibrium, otherwise the coupling introduced in Eq. (1) does not make sense, as $\mathcal{L}_{in} \sim \partial_\mu \phi J^\mu_B \to -\phi \nabla_\mu J^\mu_B = 0$ after removing a total derivative term. During this early stage, with B-violating processes in thermal equilibrium, the baryon-to-entropy ratio (4) changes as the universe expands. This occurs until the temperature of the universe cools to T_D when the B-violating interactions decouple from the thermal bath, after which the baryon-to-entropy ratio remains unchanged. Therefore, the relic baryon number asymmetry observed today should be

$$\left.\frac{n_B}{s}\right|_{T_D} = \frac{15c}{2\pi^2}\left.\frac{\dot{\phi}}{g_{*s} M_* T}\right|_{T_D} . \tag{5}$$

In such mechanisms, the scalar field ϕ in the coupling (1) is not necessarily dark energy and can be replaced by other cosmic scalar fields. In Ref. [9], the authors proposed the following interaction,

$$\mathcal{L}_{in} = \frac{1}{M_*^2}\partial_\mu R J^\mu_B , \tag{6}$$

between the curvature scalar and baryons. As a result, the baryon-to-entropy ratio is

$$\frac{n_B}{s} \sim \frac{\dot{R}}{M_*^2 T} . \tag{7}$$

As this baryon number asymmetry is generated by the derivative of the curvature, which accounts for the gravitation, this type of baryogenesis model is termed "gravitational baryogenesis." Unfortunately, within the framework of general relativity (GR), the Einstein equation gives: $R = 8\pi G T^\mu_\mu = 8\pi G(1-3w)\rho$, so that R and $\dot{R}$ vanish in the radiation dominated epoch as $w = 1/3$, and no baryon number asymmetry can be produced. To circumvent this problem, various methods to obtain a non-vanishing $\dot{R}$ have been considered in Ref. [9], such as including significant trace anomaly effects in the radiation dominated time $T^\mu_\mu \neq 0$, baryon number asymmetry produced in the reheating period, during which the universe has $w = 0$, and so on. Later, gravitational baryogenesis has been realized using other modifications, such as considering a gravity theory different from GR [10], or replacing the coupling $\partial_\mu R J^\mu_B$ with $\partial_\mu f(R) J^\mu_B$ [11], where $f(R)$ is a nonlinear function of the curvature scalar.

In this study, we are interested in gravitational baryogenesis within the frameworks of teleparallel gravity (TG) [12] and symmetric teleparallel gravity (STG) [13]. These frameworks provide gravity models in non-Riemannian systems. Both TG and STG can be equivalent to GR but are formulated in flat spacetime where the curvature vanishes. In the TG model, gravity is attributed to the spacetime torsion, while in the STG model, gravity is identified with non-metricity. Within the GR equivalent TG model and the GR equivalent STG model, we will first consider the baryogenesis induced by derivative couplings to the baryon current. These couplings are similar to Eq. (6) except the curvature scalar is replaced by the torsion scalar and the non-metricity scalar, respectively. We show that in these baryogenesis models, the produced baryon-to-entropy ratios are too small to be consistent with the observed value. Subsequently, we employ gravitational leptogenesis, in which the torsion scalar and the non-metricity scalar are coupled derivatively to the current of $B-L$. With appropriate cut-off scales, these gravitational leptogenesis models can generate the required baryon number asymmetry. We would like to point out that the gravitational baryogenesis within the framework of TG has been also studied in Refs. [14-16] in various ways. Gravitational leptogenesis from gravitational waves in inflation models was proposed in Ref. [17]. In the following sections we expand on our investigation and demonstrations. In sections II and III we start with brief introductions of TG and STG. Readers who are not familiar with these models may learn more details from reviews on these subjects, e.g., Ref. [18].

We use the convention of most negative signatures for the metric. The spacetime tensor indices are represented in Greek letters $\mu, \nu, \sigma, ... = 0,1,2,3$, and their spatial components are denoted in Latin letters $i, j, k, ... = 1,2,3$. The tensor indices in the local Minkowski spacetime are represented in capital Latin letters $A, B, C, ... = 0,1,2,3$, and the corresponding spatial components are denoted in small Latin letters $a, b, c, ... = 1,2,3$.

II. GRAVITATIONAL LEPTOGENESIS WITHIN TELEPARALLEL GRAVITY

The TG theory can be considered as a constrained metric-affine theory. It is formulated in a spacetime endowed with a metric $g_{\mu\nu}$ and an affine connection $\hat{\Gamma}^\rho_{\mu\nu}$, which are constrained by the vanishing of curvature and the metric compatibility,

$$\begin{aligned} \hat{R}^\rho_{\sigma\mu\nu} &\equiv \partial_\mu \hat{\Gamma}^\rho_{\nu\sigma} - \partial_\nu \hat{\Gamma}^\rho_{\mu\sigma} + \hat{\Gamma}^\rho_{\mu\alpha}\hat{\Gamma}^\alpha_{\nu\sigma} - \hat{\Gamma}^\rho_{\nu\alpha}\hat{\Gamma}^\alpha_{\mu\sigma} = 0 , \\ \hat{\nabla}_\rho g_{\mu\nu} &= \partial_\rho g_{\mu\nu} - \hat{\Gamma}^\lambda_{\rho\mu} g_{\lambda\nu} - \hat{\Gamma}^\lambda_{\rho\nu} g_{\mu\lambda} = 0 . \end{aligned} \tag{8}$$

Without curvature in this theory, gravity is described with spacetime torsion. The torsion tensor is defined as the antisymmetric part of the affine connection: $\mathcal{T}^\rho{}_{\mu\nu} = 2\hat{\Gamma}^\rho_{[\mu\nu]}$. In terms of the language of tetrad and spin connection and the general relations: $g_{\mu\nu} = \eta_{AB} e^A_\mu e^B_\nu$ and $\hat{\Gamma}^\mu_{\rho\sigma} = e^\mu_A(\partial_\rho e^A_\sigma + \omega^A{}_{B\rho} e^B_\sigma)$, one finds that

$$\mathcal{T}^\rho{}_{\mu\nu} = 2e^\rho_A(\partial_{[\mu} e^A_{\nu]} + \omega^A{}_{B[\mu} e^B_{\nu]}) , \tag{9}$$

and the spin connection under the constraints of Eq. (8) can be expressed as,

$$\omega^A_{\ B\mu} = \left(\Lambda^{-1}\right)^A_{\ C}\partial_\mu \Lambda^C_{\ B}\,, \tag{10}$$

where $\Lambda^A_{\ B}$ is an element of an arbitrary Lorentz transformation matrix which is position dependent and satisfies the relation $\eta_{AB}\Lambda^A_{\ C}\Lambda^B_{\ D} = \eta_{CD}$ for any spacetime point.

The TG model we are most concerned with is the teleparallel equivalent of general relativity (TEGR), in which the action for gravity is

$$S_g = \frac{M_p^2}{2}\int \mathrm{d}^4x\, \|e\|\mathbb{T}\,, \tag{11}$$

where $M_p = 1/\sqrt{8\pi G}$ is the reduced Planck mass, $\|e\| = \sqrt{-g}$ is the determinant of the tetrad, and $\mathbb{T}$ is the torsion scalar, defined as

$$\mathbb{T} = -\mathcal{T}_\mu \mathcal{T}^\mu + \frac{1}{4}\mathcal{T}_{\alpha\beta\mu}\mathcal{T}^{\alpha\beta\mu} + \frac{1}{2}\mathcal{T}_{\alpha\beta\mu}\mathcal{T}^{\beta\alpha\mu}\,, \tag{12}$$

with $\mathcal{T}_\mu = \mathcal{T}^\alpha_{\ \mu\alpha}$ being the torsion vector. This action is diffeomorphism invariant and identical to the Einstein-Hilbert action up to a boundary term,

$$S_g = -\frac{M_p^2}{2}\int \mathrm{d}^4x\, \sqrt{-g}[R(e) + 2\nabla_\mu \mathcal{T}^\mu]\,, \tag{13}$$

where the curvature scalar $R(e)$ is defined by the Levi-Civita connection and considered to be fully constructed from the metric, and in turn from the tetrad. The covariant derivative ∇_μ is also associated with the Levi-Civita connection. The boundary term does not affect the equation of motion, so the TEGR model is equivalent to GR. In addition, it can also be considered as a pure tetrad theory. The spin connection only contributes to the boundary term, so it represents pure gauge in the TEGR action (11), and in practice, we may fix a spin connection (as long as it satisfies Eq. (10)) that does not affect the equation of motion. The simplest choice is to use of the Weitzenböck connection, $\omega^A_{\ B\nu} = 0$, which is frequently adopted in the literature. Using the Weitzenböck connection, the torsion two form is simply expressed as

$$\mathcal{T}^A_{\ \mu\nu} = e^A_{\ \rho}\mathcal{T}^\rho_{\ \mu\nu} = \partial_\mu e^A_{\ \nu} - \partial_\nu e^A_{\ \mu}\,. \tag{14}$$

It deserves stressing that in a general TG theory, fixing a spin connection usually means breaking the local Lorentz symmetry, but this is not the case in the TEGR model (11) as the spin connection in this model only contributes the boundary term as shown in its equivalent form, Eq. (13). One can straightforwardly prove that, when taking the Weitzenböck connection, the action (11) is unchanged under the local Lorentz transformation, $e^A_{\ \mu} \to \Lambda^A_{\ B}(x)e^B_{\ \mu}$, up to a boundary term. However, for the modified TEGR models, taking the Weitzenböck connection indeed breaks the local Lorentz symmetry; see Refs. [19-22] for recent discussions. To avoid such explicit Lorentz violation, it is better to keep the general form (10) for the spin connection and treat both the tetrad $e^A_{\ \mu}$ and the Lorentz matrix element $\Lambda^A_{\ B}$ in Eq. (10) as fundamental variables.

Within the framework of TEGR, a gravitational baryogenesis model similar to that in Ref. [9] can be constructed by considering the derivative coupling of the torsion scalar to the baryon current, $\mathcal{L}_{in} \sim \partial_\mu \mathbb{T} J^\mu_B$. This gives the baryon an effective chemical potential, which is proportional to $\dot{\mathbb{T}}$, and the standard cosmology is unchanged, as TEGR is equivalent to GR[1)]. The key point is that $\mathbb{T} = -R - 2\nabla_\mu \mathcal{T}^\mu$ differs from $-R$ by the term $-2\nabla_\mu\mathcal{T}^\mu$, so that $\mathbb{T}$ and its time derivative do not vanish in the radiation dominated epoch. This means the effective chemical potential μ_b for the baryon is non-vanishing, which is then expected to generate a net baryon number according to Eq. (3).

In this section we consider the full action containing this derivative coupling,

$$S = \int \mathrm{d}^4x\, \sqrt{-g}\left(\frac{M_p^2}{2}\mathbb{T} + \frac{1}{M_*^2}\partial_\mu \mathbb{T} J^\mu\right) + S_m\,. \tag{15}$$

Other matter and non-gravitational interactions, including baryon number non-conserving interactions, are described by S_m. All matter is assumed to couple to the metric (or the tetrad) minimally aside from the introduced derivative coupling. As the derivative coupling depends on the torsion scalar, which accounts for gravity, this model is also classified as a modified TEGR model.

The equations of motion follow from the variation of the action with respect to $e^A_{\ \mu}$ and $\Lambda^A_{\ B}$ separately:

$$\left(1 - \frac{2\theta}{M_*^2 M_p^2}\right)G^{\mu\nu} + \frac{\theta}{M_*^2 M_p^2}\mathbb{T}g^{\mu\nu} - \frac{2}{M_*^2 M_p^2}S^{\mu\nu\sigma}\partial_\sigma\theta = \frac{1}{M_p^2}T^{\mu\nu}\,, \tag{16}$$

$$S^{[\mu\nu]\sigma}\partial_\sigma\theta = 0\,, \tag{17}$$

where $\theta \equiv \nabla_\mu J^\mu$, $G^{\mu\nu}$ is the Einstein tensor, $T^{\mu\nu} = -(2/\sqrt{-g})(\delta S_m/\delta g_{\mu\nu})$ is the energy-momentum

1) In gravitational baryogenesis models, the back reaction of the introduced derivative coupling to the spacetime can be ignored because in the radiation dominated epoch the baryon density is subdominant.

tensor of matter, and $S^{\mu\rho\sigma} = (1/2)\mathcal{T}^{\mu\rho\sigma} - \mathcal{T}^{[\rho\sigma]\mu} + 2g^{\mu[\rho}\mathcal{T}^{\sigma]}$ is the so-called superpotential in TG theory and is antisymmetric under the interchange of its last two indices.

Now, we apply these equations to the spatially-flat Friedmann-Robertson-Walker (FRW) universe with $\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\mathrm{d}\vec{x}^2$ to obtain the values of $\mathbb{T}$ and its time derivative. Given the FRW metric, we can always choose the tetrad as $e^A_\mu = \mathrm{diag}\{1,a,a,a\}$. Then, the spin connection will be solved through equations (16) and (17). We first consider the case in which the current in the derivative coupling is the baryon current $J^\mu = J^\mu_B$, so $\theta = \dot{n}_B + 3Hn_B$ only depends on time but does not vanish during the baryogenesis process (in the radiation dominated epoch) due to the baryon number violation. With these, equations (16) and (17) require that $S^{[\mu\nu]0} = 0$ and $S^{ij0} \propto \delta^{ij}$, which in turn give the following constraints on the spin connections:

$$\omega^a_{\ bj}\delta^j_a\delta^b_i = 0,\ \omega^a_{\ 0i} \propto \delta^a_i\ . \tag{18}$$

The next step is to find the Lorentz matrix elements Λ to satisfy the above constraints according to the expression of the spin connection (10). In the homogeneous and isotropic spacetime, it is natural to consider the homogeneous Λs as the solution to the equations. Indeed, if all the Lorentz matrix elements in Eq. (10) only rely on the time, then the constraints in Eq. (18) are satisfied automatically. These considerations lead to the following result:

$$\mathbb{T} = -R - 2\nabla_\mu\mathcal{T}^\mu = -R - 2(\partial_0 + 3H)\left(3H - \frac{1}{a}\omega^a_{\ 0i}\delta^i_a\right) - 2\partial_i\left(\frac{1}{a}\omega^0_{\ a0}\delta^a_i\right) = -6H^2\ . \tag{19}$$

This result means that the torsion scalar does not vanish during the radiation dominated epoch, and its time derivative $\dot{\mathbb{T}} = -12H\dot{H}$ provides an effective chemical potential for baryons, $\mu_b = \dot{\mathbb{T}}/M_*^2$, which then induces the baryon number asymmetry:

$$\frac{n_B}{s} = \frac{15}{2\pi^2}\frac{\dot{\mathbb{T}}}{g_{*s}M_*^2T} = -\frac{90}{\pi^2}\frac{H\dot{H}}{g_{*s}M_*^2T} = \frac{180}{\pi^2}\frac{H^3}{g_{*s}M_*^2T}\ , \tag{20}$$

in the last step we have employed the relation $\dot{H} = -2H^2$ in the radiation dominated epoch. From standard big bang cosmology, it is well known that the Hubble rate in the radiation dominated epoch is $H \simeq g_*^{1/2}T^2/(3M_p)$, where g_* denotes the total degrees of freedom that contributes to the radiation density, which is equal to g_{*s} in the very early universe. Hence, the baryon number asymmetry is

$$\left.\frac{n_B}{s}\right|_{T_D} = \frac{20}{3\pi^2}g_*^{1/2}\frac{T_D^5}{M_p^3M_*^2} \simeq 0.5\times10^{-54}\left(\frac{T_D}{\mathrm{GeV}}\right)^3\left(\frac{T_D}{M_*}\right)^2\ . \tag{21}$$

In the second step of the above equation, we have employed $g_* = 106.75$ and $M_p \simeq 2.4\times10^{18}$ GeV. The decoupling temperature of the baryon number non-conserving interaction is approximately $T_D \sim 100$ GeV [23] and the cut-off scale M_* should be no less than this. Considering this, one can evaluate that the produced baryon number asymmetry is at most on the order of 10^{-48}, which is too small to be consistent with observations. This disappointing consequence has been also obtained in Ref. [14], in which the authors then turned to the modified TEGR gravity, i.e., the $f(\mathbb{T})$ model, replacing the original derivative coupling with $\partial_\mu f(\mathbb{T})J^\mu_B$. We would like to point out that within the framework of the TEGR model there is another way to circumvent this difficulty: gravitational leptogenesis.

In the leptogenesis scenario [24-27], there should be $B-L$ violating processes at high energy scales, and these can be realized purely by lepton number violations. In our gravitational leptogenesis model, in addition to the $B-L$ violating interactions in thermal equilibrium, we identify the current J^μ in the action (15) with J^μ_{B-L}. This means that the torsion scalar couples derivatively to the $B-L$ current instead of the baryon current. Through similar calculations, we obtain the thermally produced $B-L$ asymmetry:

$$\left.\frac{n_{B-L}}{s}\right|_{T_D} = \frac{20}{3\pi^2}g_*^{1/2}\frac{T_D^5}{M_p^3M_*^2} \simeq 0.5\times10^{-54}\left(\frac{T_D}{\mathrm{GeV}}\right)^3\left(\frac{T_D}{M_*}\right)^2\ , \tag{22}$$

where T_D is the decoupling temperature of the $B-L$ violating interaction and can be much higher than the electroweak scale. At a later time, the previously produced n_{B-L}/s asymmetry will not be washed out by the electroweak Sphaleron processes, which violate $B+L$ but conserve $B-L$ and are in thermal equilibrium when the temperature of the universe is in the range of $100\,\mathrm{GeV} < T < 10^{12}$ GeV [23]. Furthermore, the electroweak Sphaleron processes will partially convert the $B-L$ asymmetry to the baryon number and lepton number asymmetries, respectively [28]:

$$\frac{n_B}{s} = c_s\frac{n_{B-L}}{s}\ ,\ \frac{n_L}{s} = (c_s - 1)\frac{n_{B-L}}{s}\ , \tag{23}$$

where $c_s = (8N_f + 4)/(22N_f + 13)$ and N_f is the number of generations. In the standard model, $N_f = 3$ and $c_s \simeq 0.35$, so the final baryon number asymmetry in this model is

$$\frac{n_B}{s} \simeq 0.18\times 10^{-54}\left(\frac{T_D}{\text{GeV}}\right)^3\left(\frac{T_D}{M_*}\right)^2 . \tag{24}$$

Numerically, if T_D is close to M_*, the current observational result, $n_B/s\sim 8.7\times 10^{-11}$, requires the decoupling temperature T_D to be 0.78×10^{15} GeV. However, T_D should be lower than the inflation scale, otherwise the produced asymmetry n_{B-L}/s would be diluted during inflation. One may evaluate that the Hubble rate at T_D as $H(T_D)\simeq g_*^{1/2}T_D^2/(3M_p)\sim 10^{12}$ GeV, which is lower than the energy scale of the inflation process in the single field inflation models.

III. GRAVITATIONAL LEPTOGENESIS WITHIN SYMMETRIC TELEPARALLEL GRAVITY

Now we turn to the gravitational leptogenesis model within the framework of STG, which has not previously been discussed. The STG theory can also be considered as a constrained metric-affine theory. It is formulated in a spacetime with a metric $g_{\mu\nu}$ and an affine connection $\Gamma^\lambda{}_{\mu\nu}$, with the latter leading to zero curvature and zero torsion,

$$\hat{R}^\rho{}_{\sigma\mu\nu}\equiv\partial_\mu\hat{\Gamma}^\rho_{\nu\sigma}-\partial_\nu\hat{\Gamma}^\rho_{\mu\sigma}+\hat{\Gamma}^\rho_{\mu\alpha}\hat{\Gamma}^\alpha_{\nu\sigma}-\hat{\Gamma}^\rho_{\nu\alpha}\hat{\Gamma}^\alpha_{\mu\sigma}=0 ,$$
$$\mathcal{T}^\rho{}_{\mu\nu}=2\hat{\Gamma}^\rho_{[\mu\nu]}=0 . \tag{25}$$

With these constraints, the affine connection can be expressed generally as

$$\hat{\Gamma}^\lambda_{\mu\nu}(x)=\frac{\partial x^\lambda}{\partial y^\beta}\partial_\mu\partial_\nu y^\beta , \tag{26}$$

with $y^\beta(x)$ being four functions. These functions define a special coordinate system in which the affine connection vanishes. One may use the y-coordinate system to do the remaining calculations. This is a gauge choice, with this "coincident gauge" being adopted extensively in studies on STG theories in the literature. However, taking the coincident gauge will break the diffeomorphism invariance explicitly, which we try to avoid in this study. Therefore, we will keep the general form (26) for the affine connection. The gravity in the STG theory is identical to the non-metricity. As standard, the non-metricity tensor is defined as

$$Q_{\alpha\mu\nu}\equiv\hat{\nabla}_\alpha g_{\mu\nu}=\partial_\alpha g_{\mu\nu}-\hat{\Gamma}^\lambda_{\alpha\mu}g_{\lambda\nu}-\hat{\Gamma}^\lambda_{\alpha\nu}g_{\mu\lambda} , \tag{27}$$

which measures the failure of the affine connection to be metric-compatible. The STG Equivalent of GR (STGR) model has the following action,

$$S_g=\frac{M_p^2}{2}\int d^4x\sqrt{-g}\mathbb{Q}\equiv\frac{M_p^2}{2}\int d^4x\ \sqrt{-g}$$
$$\times\left[\frac{1}{4}Q_{\alpha\mu\nu}Q^{\alpha\mu\nu}-\frac{1}{2}Q_{\alpha\mu\nu}Q^{\mu\nu\alpha}-\frac{1}{4}Q^\alpha Q_\alpha+\frac{1}{2}\tilde{Q}^\alpha Q_\alpha\right] , \tag{28}$$

where $\mathbb{Q}$ is a non-metricity scalar and the vectors $Q_\alpha,\tilde{Q}_\alpha$ are two different traces of the non-metricity tensor, i.e., $Q_\alpha=g^{\sigma\lambda}Q_{\alpha\sigma\lambda}$, $\tilde{Q}_\alpha=g^{\sigma\lambda}Q_{\sigma\alpha\lambda}$. In terms of the constraints (25), one can easily determine that $\mathbb{Q}=-R-\nabla_\mu(Q^\mu-\tilde{Q}^\mu)$, so that the STGR action (28) is equal to the Einstein-Hilbert action up to a boundary term,

$$S_g=-\frac{M_p^2}{2}\int d^4x\sqrt{-g}\left[R+\nabla_\mu(Q^\mu-\tilde{Q}^\mu)\right] . \tag{29}$$

Hence, the STGR model is equivalent to GR.

By introducing the derivative coupling of the non-metricity scalar, the full action we consider is

$$S=\int d^4x\sqrt{-g}\left(\frac{M_p^2}{2}\mathbb{Q}+\frac{1}{M_*^2}\partial_\mu\mathbb{Q}J^\mu\right)+S_m . \tag{30}$$

The equation of motion via the variation with the metric is

$$\left(1-\frac{2\theta}{M_*^2M_p^2}\right)G^{\mu\nu}+\frac{\theta}{M_*^2M_p^2}\mathbb{Q}g^{\mu\nu}+\frac{1}{M_*^2M_p^2}P^{\alpha\mu\nu}\partial_\alpha\theta=\frac{1}{M_p^2}T^{\mu\nu} , \tag{31}$$

where again we have $\theta\equiv\nabla_\mu J^\mu$, which is equal to $\dot{n}+3Hn$ in the FRW universe. The superpotential is defined as $P^{\alpha\mu\nu}=Q^{\alpha\mu\nu}-2Q^{(\mu\nu)\alpha}-(Q^\alpha-\tilde{Q}^\alpha)g^{\mu\nu}+g^{\alpha(\mu}Q^{\nu)}$, which is symmetric under the interchange of the last two indices. Besides the metric, the four functions y^ν from which the affine connection is constructed are independent variables in this model. The corresponding equation of motion is

$$\hat{\nabla}_\alpha\hat{\nabla}_\mu(\sqrt{-g}P^{\alpha\mu}{}_\nu\theta)=0 . \tag{32}$$

Similarly, the aim of this section is to determine the value of $\mathbb{Q}$ and its time derivative by solving the above equations in the FRW universe. As the space of the FRW universe is homogeneous and isotropic, it has six spatial Killing vectors ξ^μ. It is reasonable to require the affine connection is also uniformly distributed in the universe, so that its Lie derivatives along the Killing vectors vanish: $\mathcal{L}_\xi\Gamma^\lambda{}_{\mu\nu}=0$. With this symmetry requirement, the non-vanished components of the affine connection have the following forms [29]:

$$\hat{\Gamma}^0{}_{00} = \mathcal{K}_1(t), \quad \hat{\Gamma}^0{}_{11} = \hat{\Gamma}^0{}_{22} = \hat{\Gamma}^0{}_{33} = \mathcal{K}_2(t),$$
$$\hat{\Gamma}^1{}_{01} = \hat{\Gamma}^2{}_{02} = \hat{\Gamma}^3{}_{03} = \mathcal{K}_3(t)\,, \tag{33}$$

where $\mathcal{K}_1(t), \mathcal{K}_2(t), \mathcal{K}_3(t)$ are three uniform functions. The zero curvature condition requires that

$$\mathcal{K}_3(\mathcal{K}_1 - \mathcal{K}_3) - \dot{\mathcal{K}}_3 = 0\,,\ \mathcal{K}_1\mathcal{K}_2 + \dot{\mathcal{K}}_2 = 0\,,\ \mathcal{K}_2\mathcal{K}_3 = 0\,. \tag{34}$$

Now, we discuss the cases of $\mathcal{K}_2 = 0$ and $\mathcal{K}_3 = 0$ separately. In the first case, $\mathcal{K}_2 = 0$, one can obtain the non-metricity scalar and the simple form of constraint equation (32) as follows:

$$\mathbb{Q} = 3(-2H^2 + 3H\mathcal{K}_3 + \dot{\mathcal{K}}_3)\,,\ \mathcal{K}_3(\ddot{\theta} + 3H\dot{\theta}) = 0\,. \tag{35}$$

As mentioned, $\theta = \dot{n} + 3Hn$ does not vanish due to the non-conservation of the corresponding quantum number. Its change with time depends on the specific particle physics model and the quantum number violation. So in general, we cannot expect a vanishing $\ddot{\theta} + 3H\dot{\theta}$. Then, the constraint equation (32) requires that $\mathcal{K}_3 = 0$, and we obtain $\mathbb{Q} = -6H^2$.

In the second case $\mathcal{K}_3 = 0$, we get the non-metricity scalar and the form of constraint equation (32) as follows:

$$\mathbb{Q} = 3(-2H^2 + H\mathcal{K}_2/a^2 + \dot{\mathcal{K}}_2/a^2)\,,\ \mathcal{K}_2(\ddot{\theta} - 2\mathcal{K}_1\dot{\theta} + H\dot{\theta}) = 0\,. \tag{36}$$

Similarly we cannot expect $\ddot{\theta} - 2\mathcal{K}_1\dot{\theta} + H\dot{\theta} = 0$ in general, and we can only have $\mathcal{K}_2 = 0$. So we again obtain $\mathbb{Q} = -6H^2$.

In all, we obtain $\mathbb{Q} = -6H^2$ for the model (30) within the STG theory framework. Then, we have the same premise regarding the baryon number asymmetry as that we have discussed within the TG theory framework in the previous section. If the current in the derivative coupling $(1/M_*^2)\partial_\mu\mathbb{Q}J^\mu$ is the baryon current J_B^μ, the produced baryon number asymmetry is as low as 10^{-48} and is not consistent with current observations. To solve this, one may consider the modified STGR model, such as the $f(\mathbb{Q})$ model, or by changing the coupling $\partial_\mu\mathbb{Q}J_B^\mu$ to $\partial_\mu f(\mathbb{Q})J_B^\mu$. In this study, we prefer the gravitational leptogenesis mechanism, i.e., besides assuming the existence of $B-L$ violating processes in the very early universe, the introduced derivative coupling of $\mathbb{Q}$ should be $(1/M_*^2)\partial_\mu\mathbb{Q}J_{B-L}^\mu$. Consequently, we have the $B-L$ asymmetry

$$\left.\frac{n_{B-L}}{s}\right|_{T_D} \simeq 0.5\times10^{-54}\left(\frac{T_D}{\text{GeV}}\right)^3\left(\frac{T_D}{M_*}\right)^2, \tag{37}$$

and a similar order asymmetry for baryons: $n_B/s \simeq 0.18\times10^{-54}(T_D/\text{GeV})^3(T_D/M_*)^2$. With the decoupling temperature of $B-L$ violating interactions $T_D \sim M_* \sim 10^{15}$ GeV, the required baryon-to-entropy ratio $n_B/s \sim 10^{-10}$ can be obtained.

IV. CONCLUSION

In this study, we investigated the gravitational baryogenesis and leptogenesis models within the frameworks of teleparallel gravity (TG) and symmetric teleparallel gravity (STG). Both the TG and STG theories can include models equivalent to GR, i.e., TEGR and STGR respectively, but account for gravitational phenomena from different viewpoints. By introducing the derivative couplings of the torsion scalar (in TEGR) or the non-metricity scalar (in STGR) to baryons, the baryon number asymmetry can be produced in thermal equilibrium. In the case of baryogenesis, the produced baryon number asymmetry is too small to be consistent with observations. However, as we have shown, the leptogenesis scenario works for these cases.

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