

The $\bar{B}_s^0 \rightarrow J/\psi\pi^0\eta$ decay and the $a_0(980) - f_0(980)$ mixing*

Jia-Ting Li(李嘉婷)¹ Jia-Xin Lin(林佳欣)¹ Gong-Jie Zhang(张功杰)¹ Wei-Hong Liang(梁伟红)^{1,2†} E. Oset^{1,3‡}

¹Department of Physics, Guangxi Normal University, Guilin 541004, China

²Guangxi Key Laboratory of Nuclear Physics and Technology, Guangxi Normal University, Guilin 541004, China

³Departamento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia-CSIC Institutos de Investigación de Paterna, Aptdo.22085, 46071 Valencia, Spain

Abstract: We study the $\bar{B}_s^0 \rightarrow J/\psi f_0(980)$ and $\bar{B}_s^0 \rightarrow J/\psi a_0(980)$ reactions, and pay attention to the different sources of isospin violation and mixing of $f_0(980)$ and $a_0(980)$ resonances where these resonances are dynamically generated from meson–meson interactions. We find that the main cause of isospin violation is isospin breaking in the meson–meson transition T matrices, and the other source is that the loops involving kaons in the production mechanism do not cancel due to the different masses of charged and neutral kaons. We obtain a branching ratio for $a_0(980)$ production of the order of 5×10^{-6} . Future experiments can address this problem, and the production rate and shape of the $\pi^0\eta$ mass distribution will definitely help to better understand the nature of scalar resonances.

Keywords: strange B meson decay, isospin violation, $a_0(980)$ – $f_0(980)$ mixing, hadronic structure

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I. INTRODUCTION

The B_s decay into J/ψ and two mesons is an excellent source of information on meson dynamics. At the quark level the decay proceeds via internal emission [1], as shown in Fig. 1. The $c\bar{c}$ quarks give rise to the J/ψ and the extra $s\bar{s}$, which appear in the Cabibbo favored decay mode, have isospin $I = 0$. It is a rather clean process and indeed, in the LHCb experiment [2] the $f_0(980)$ resonance was seen as a strong peak in the invariant mass distribution of $\pi^+\pi^-$. The way $\pi^+\pi^-$ are produced is studied in detail in Ref. [3]. The $s\bar{s}$ pair of quarks is hadronized, introducing a $\bar{q}q$ pair with vacuum quantum numbers, and $K\bar{K}$ in $I = 0$ plus $\eta\eta$ are produced, which are allowed to interact within the chiral unitary approach [4–7] to produce the $f_0(980)$ resonance, which is dynamically generated from the interaction of pseudoscalar pairs and couples mostly to $K\bar{K}$. With such a clean process producing $I = 0$, one finds a very interesting place to produce the $a_0(980)$, via isospin violation, and add extra information to the subject of the $f_0(980) - a_0(980)$ mixing that has stimulated much work. Indeed, there are many works devoted to this subject [8–36] and some cases where, due to a triangle singularity, the amount of isospin breaking (we

prefer this language rather than mixing, since there is not a universal mechanism for the mixing and it depends upon the particular reaction) is abnormally large [26, 27, 37]. The way the $a_0(980)$ resonance can be produced in the $B_s \rightarrow J/\psi\pi^0\eta$ decay is tied to the nature of $f_0(980)$ and $a_0(980)$, since the resonances are dynamically generated by the pseudoscalar–pseudoscalar (PP) interaction [4]. It is the meson–meson loops in the Bethe–Salpeter equation, particularly $K\bar{K}$ in the case of f_0 and a_0 , that give rise to the resonances. The K^+K^- and $K^0\bar{K}^0$ loops cancel for $I = 1$ starting from the $I = 0$ combination of the hadronized $s\bar{s}$ quarks, but only if the masses of K^+ and K^0 are taken as equal. When the mass difference is considered, then the isospin is automatically broken and some peaks appear for the isospin-violating decay modes which are rather narrow and are tied to the kaon mass differences. The relation of the $a_0 - f_0$ mixing to this mass difference is shared by most theoretical studies, starting from Ref. [8]. However, as shown in Ref. [38] in the study of $D_s \rightarrow e^+\nu_e a_0(980)$, isospin breaking takes place in the loop for $K\bar{K}$ propagation in the decay but also in the same meson–meson scattering matrix, which enters the evaluation of the process, something already noticed in Ref. [39]. Yet, the two sources of isospin violation are

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† E-mail: liangwh@gxnu.edu.cn, Corresponding author

‡ E-mail: oset@ific.uv.es



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different depending on the reaction studied, hence the importance of studying the isospin violation in different processes to gain information on the way the violation is produced and its dependence on the nature of the $a_0(980)$ and $f_0(980)$ resonances, which has originated much debate in the literature.

We study the process of a_0 and f_0 production, following the lines of Refs. [3] and [38], and by taking experimental information on the $B_s \rightarrow J/\psi\pi^+\pi^-$ reaction, we make predictions for the rate of $B_s \rightarrow J/\psi\pi^0\eta$ production and the shape of the $\pi^0\eta$ mass distribution. The branching fraction obtained for this latter decay is of the order of 5×10^{-6} , well within the range of rates already measured and reported by the PDG [40], which should stimulate its measurement in the future.

II. FORMALISM

The mechanism at the quark level for the $\bar{B}_s^0 \rightarrow$

$$M \rightarrow \mathcal{P} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta + \frac{1}{\sqrt{3}}\eta' & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta + \frac{1}{\sqrt{3}}\eta' & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta + \sqrt{\frac{1}{3}}\eta' \end{pmatrix}, \quad (2)$$

which is often used in chiral perturbation theory [4]. On the other hand, when we consider the Branon $\eta-\eta'$ mixing [41], the matrix M can be written as

$$M \rightarrow \mathcal{P}^{(m)} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & K^0 \\ K^- & \bar{K}^0 & -\frac{1}{\sqrt{3}}\eta + \sqrt{\frac{2}{3}}\eta' \end{pmatrix}. \quad (3)$$

Since the η' is inessential in the dynamical generation of the $f_0(980)$ and $a_0(980)$ resonances [4], we will ignore the η' in the present work.

After hadronization of the $s\bar{s}$ component, we obtain

$$s\bar{s} \rightarrow H = \sum_i s\bar{q}_i q_i \bar{s} = \sum_i \mathcal{P}_{3i} \mathcal{P}_{i3} = (\mathcal{P}^2)_{33}. \quad (4)$$

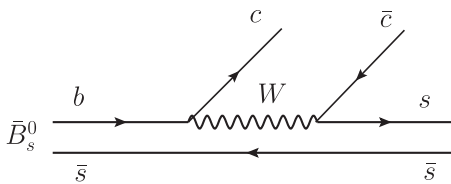


Fig. 1. Diagram for \bar{B}_s^0 decay into J/ψ and a primary $s\bar{s}$ pair.

$J/\psi\pi^+\pi^-(\pi^0\eta)$ reaction is depicted in Fig. 1, having an $s\bar{s}$ pair with isospin $I = 0$ at the end. Note that the light scalars $f_0(980)$ and $a_0(980)$ have $I = 0, 1$, respectively. The production of $f_0(980)$ is isospin conserved, while the production of $a_0(980)$ is isospin forbidden and involves isospin violation.

To obtain $\pi^+\pi^-$ or $\pi^0\eta$ in the final state in Fig. 1, we need to hadronize the $s\bar{s}$ pair by introducing an extra $\bar{q}q$ pair with vacuum quantum numbers. We start with the $q\bar{q}$ matrix M in $SU(3)$,

$$M = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix}. \quad (1)$$

Next, we write the matrix M in terms of pseudoscalar mesons, assuming that the η is η_8 of $SU(3)$,

In the case without $\eta-\eta'$ mixing, the matrix \mathcal{P} of Eq. (2) is used, and then the hadron component H in Eq. (4) is given by

$$H = K^- K^+ + \bar{K}^0 K^0 + \frac{2}{3} \eta\eta. \quad (5)$$

In the case with $\eta-\eta'$ mixing, one uses matrix $\mathcal{P}^{(m)}$ of Eq. (3), and obtains

$$H = K^- K^+ + \bar{K}^0 K^0 + \frac{1}{3} \eta\eta, \quad (6)$$

differing only in the $\eta\eta$ component, which affects the production of f_0 but not the production of a_0 . We define the weight of the PP components in H as

$$h_{K^+K^-} = 1, \quad h_{K^0\bar{K}^0} = 1, \quad h_{\eta\eta} = \frac{2}{3}, \quad h_{\eta\eta}^{(m)} = \frac{1}{3}. \quad (7)$$

One can see that neither Eq. (5) nor Eq. (6) contains $\pi^+\pi^-$ or $\pi^0\eta$, but they can be produced by the final state interaction of the $K\bar{K}$ and $\eta\eta$ components, as depicted in Fig. 2. The transition matrix from the PP state to $\pi^+\pi^-$ or $\pi^0\eta$ is represented by the circle behind the meson–meson loop in Fig. 2, which contains the information of $f_0(980)$ and $a_0(980)$ respectively. According to the method in Ref. [4] (the chiral unitary approach), these resonances are the result of the PP interaction in the coupled channels $K\bar{K}, \pi\pi, \eta\eta$.

By using the unitary normalization [4, 38], the amplitude for the $\bar{B}_s^0 \rightarrow J/\psi\pi^+\pi^-$ decay, as a function of the $\pi^+\pi^-$ invariant mass $M_{\text{inv}}(\pi^+\pi^-)$, is given by [38]

$$t_{\pi^+\pi^-} = C \left[h_{K^+K^-} \cdot G_{K^+K^-}(M_{\text{inv}}(\pi^+\pi^-)) \cdot T_{K^+K^-\pi^+\pi^-}(M_{\text{inv}}(\pi^+\pi^-)) \right. \\ \left. + h_{K^0\bar{K}^0} \cdot G_{K^0\bar{K}^0}(M_{\text{inv}}(\pi^+\pi^-)) \cdot T_{K^0\bar{K}^0\pi^+\pi^-}(M_{\text{inv}}(\pi^+\pi^-)) \right. \\ \left. + h_{\eta\eta} \times 2 \times \frac{1}{2} \cdot G_{\eta\eta}(M_{\text{inv}}(\pi^+\pi^-)) \cdot T_{\eta\eta\pi^+\pi^-}(M_{\text{inv}}(\pi^+\pi^-)) \right], \quad (8)$$

and the amplitude for the $\bar{B}_s^0 \rightarrow J/\psi\pi^0\eta$ decay, as a function of the $\pi^0\eta$ invariant mass $M_{\text{inv}}(\pi^0\eta)$, is given by [38]

$$t_{\pi^0\eta} = C \left[h_{K^+K^-} \cdot G_{K^+K^-}(M_{\text{inv}}(\pi^0\eta)) \cdot T_{K^+K^-\pi^0\eta}(M_{\text{inv}}(\pi^0\eta)) \right. \\ \left. + h_{K^0\bar{K}^0} \cdot G_{K^0\bar{K}^0}(M_{\text{inv}}(\pi^0\eta)) \cdot T_{K^0\bar{K}^0\pi^0\eta}(M_{\text{inv}}(\pi^0\eta)) \right. \\ \left. + h_{\eta\eta} \times 2 \times \frac{1}{2} \cdot G_{\eta\eta}(M_{\text{inv}}(\pi^0\eta)) \cdot T_{\eta\eta\pi^0\eta}(M_{\text{inv}}(\pi^0\eta)) \right], \quad (9)$$

with C an arbitrary normalization constant which is canceled in the ratio of the f_0 and a_0 production rates. For the case with $\eta - \eta'$ mixing, the corresponding amplitudes can be obtained by replacing $h_{\eta\eta}$ with $h_{\eta\eta}^{(m)}$ in Eqs. (8) and (9).

In Eqs. (8) and (9), G_i is the loop function of the two intermediate pseudoscalar mesons, which is regularized with a three momentum cut-off q_{max} [4],

$$G_i(\sqrt{s}) = \int_0^{q_{\text{max}}} \frac{q^2 dq}{(2\pi)^2} \frac{w_1 + w_2}{w_1 w_2 [s - (w_1 + w_2)^2 + i\epsilon]}, \quad (10)$$

with $w_j = \sqrt{m_j^2 + \vec{q}^2}$ and \sqrt{s} the centre-of-mass energy of the two mesons in the loop. $T_{i,j}$ is the total amplitude for the $i \rightarrow j$ transition and can be obtained by solving the Bethe–Salpeter (BS) equation with six PP coupled channels $\pi^+\pi^-, \pi^0\pi^0, K^+K^-, K^0\bar{K}^0, \eta\eta$ and $\pi^0\eta$, in a matrix form,

$$T = [1 - VG]^{-1}V, \quad (11)$$

where the matrix V is the kernel of the BS equation. Its elements V_{ij} are the s -wave transition potentials which can be taken from Eq. (A3) and Eq. (A4) of Ref. [38], corresponding to the cases without and with $\eta - \eta'$ mixing, respectively.

The differential decay width for $\bar{B}_s^0 \rightarrow J/\psi\pi^0\eta$ or $\bar{B}_s^0 \rightarrow J/\psi\pi^+\pi^-$ decay is given by

$$\frac{d\Gamma}{dM_{\text{inv}}(ij)} = \frac{1}{(2\pi)^3} \frac{1}{4M_{\bar{B}_s^0}^2} \frac{1}{3} p_{J/\psi}^2 p_{J/\psi} \tilde{p}_\pi |t_{ij}|^2, \quad (12)$$

where $ij = \pi^+\pi^-$ or $\pi^0\eta$, $M_{\text{inv}}(ij)$ is the invariant mass of the final $\pi^+\pi^-$ or $\pi^0\eta$, $t_{\pi^+\pi^-}$ and $t_{\pi^0\eta}$ are the amplitudes from Eq. (8) and Eq. (9) respectively, $p_{J/\psi}$ is the J/ψ momentum in the \bar{B}_s^0 rest frame, and \tilde{p}_π is the pion momentum in the rest frame of the $\pi^+\pi^-$ or $\pi^0\eta$ system,

$$p_{J/\psi} = \frac{\lambda^{1/2}(M_{\bar{B}_s^0}^2, M_{J/\psi}^2, M_{\text{inv}}^2)}{2M_{\bar{B}_s^0}}, \quad (13)$$

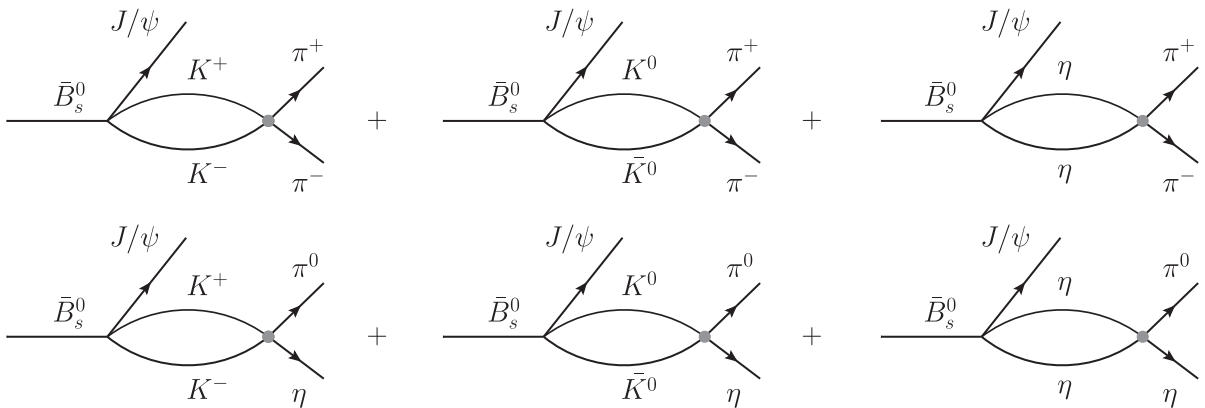


Fig. 2. Final state interaction of the hadron components leading to $\pi^+\pi^-$ or $\pi^0\eta$ in the final state.

$$\tilde{p}_\pi = \begin{cases} \frac{\lambda^{1/2}(M_{\text{inv}}^2, m_\pi^2, m_\pi^2)}{2M_{\text{inv}}}, & \text{for } \pi^+\pi^- \text{ production,} \\ \frac{\lambda^{1/2}(M_{\text{inv}}^2, m_\pi^2, m_\eta^2)}{2M_{\text{inv}}}, & \text{for } \pi^0\eta \text{ production,} \end{cases} \quad (14)$$

with $\lambda(x^2, y^2, z^2) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$ the Källén function. In Eq. (12), the factor $\frac{1}{3}p_{J/\psi}^2$ stems from the fact that we need a p -wave to match angular momentum in the $0^- \rightarrow 1^-0^+$ transition and we take a vertex of type $p_{J/\psi} \cos\theta$.

III. RESULTS

We follow Ref. [38] and take the cut-off $q_{\text{max}} = 600$ MeV and 650 MeV for the cases without $\eta-\eta'$ mixing and with $\eta-\eta'$ mixing respectively, with which the $f_0(980)$ and $a_0(980)$ resonances can be dynamically produced well from the PP interaction. The $\pi^+\pi^-$ and $\pi^0\eta$ mass distributions $\frac{d\Gamma}{dM_{\text{inv}(ij)}}$ are shown in Fig. 3 for the case without $\eta-\eta'$ mixing and in Fig. 4 for the case with $\eta-\eta'$ mixing, respectively. By comparing Fig. 3 and Fig. 4, one finds that the results of the two figures are very similar, and the difference between them can serve as an estimate of the uncertainties of our formalism.

Now, let us look at the $\pi^+\pi^-$ and $\pi^0\eta$ mass distributions in Fig. 4 with $\eta-\eta'$ mixing. One can see a strong peak for $f_0(980)$ production in the $\pi^+\pi^-$ mass distribution and a small peak for $a_0(980)$ production in the $\pi^0\eta$ mass distribution. Here the shape of $a_0(980)$ resonance is quite narrow, considerably different to the standard cusp-like shape (with a width of about 120 MeV) of the ordin-

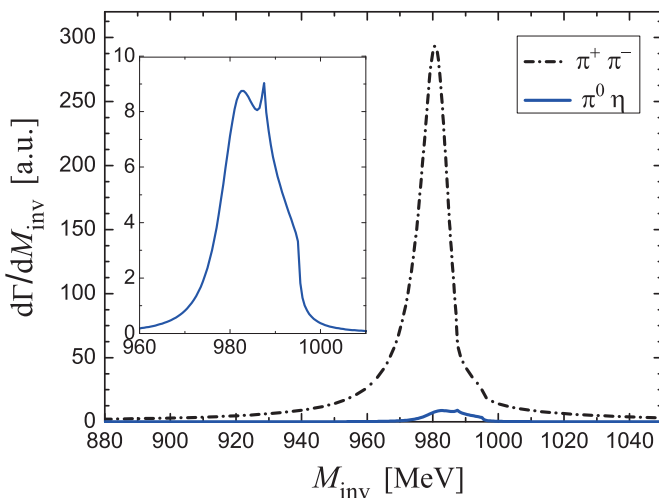


Fig. 3. (color online) $M_{\text{inv}}(\pi^+\pi^-)$ mass distribution for $\bar{B}_s^0 \rightarrow J/\psi f_0(980), f_0(980) \rightarrow \pi^+\pi^-$ decay, and $M_{\text{inv}}(\pi^0\eta)$ mass distribution for $\bar{B}_s^0 \rightarrow J/\psi a_0(980), a_0(980) \rightarrow \pi^0\eta$ decay. Inset: Magnified $\pi^0\eta$. (Without $\eta-\eta'$ mixing).

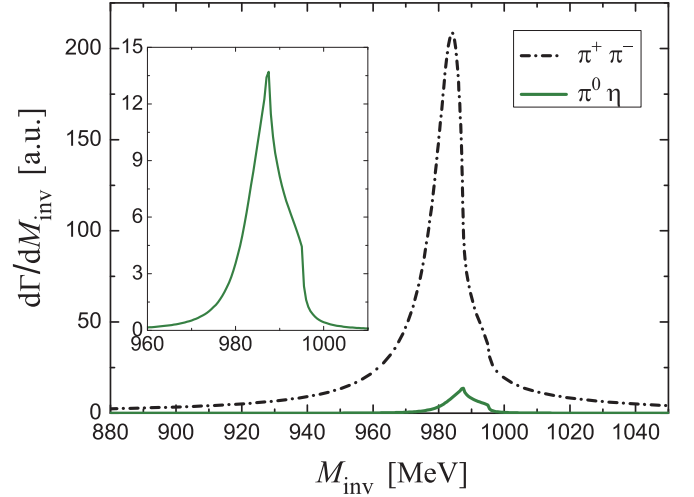


Fig. 4. (color online) $M_{\text{inv}}(\pi^+\pi^-)$ mass distribution for $\bar{B}_s^0 \rightarrow J/\psi f_0(980), f_0(980) \rightarrow \pi^+\pi^-$ decay, and $M_{\text{inv}}(\pi^0\eta)$ mass distribution for $\bar{B}_s^0 \rightarrow J/\psi a_0(980), a_0(980) \rightarrow \pi^0\eta$ decay. Inset: Magnified $\pi^0\eta$. (With $\eta-\eta'$ mixing).

ary production of $a_0(980)$ in an isospin allowed reaction [42]. If isospin were conserved, one would find the $a_0(980)$ production with zero strength. The small peak of $a_0(980)$ in Fig. 4 indicates that isospin violation takes places in the $\bar{B}_s^0 \rightarrow J/\psi \pi^0\eta$ reaction. According to Eq. (A4) of Ref. [38], we have $V_{K^+K^-\pi^0\eta} = -V_{K^0\bar{K}^0\pi^0\eta}$ for the transition potentials. Hence, if we use average masses for kaons, there will be a precise cancellation of the first two terms of the amplitude $t_{\pi^0\eta}$ in Eq. (9), resulting on zero strength for $a_0(980)$ production. On the contrary, using the physical masses for the neutral K^0 and the charged K^+ in the formalism results in the production of the $a_0(980)$ resonance with a narrow shape related to the difference of mass between the charged and neutral kaons. In our picture, there are two sources of isospin violation: one is the K^+, K^0 mass difference for the explicit K^+K^- and $K^0\bar{K}^0$ loops in Fig. 2, and the other is from the T matrix involving rescattering in Fig. 2.

It is interesting to investigate the effects of these two sources of isospin violation. For that, we follow Ref. [38] and define the ratio R , which reflects the amount of the isospin violation, as

$$R = \frac{\Gamma[\bar{B}_s^0 \rightarrow J/\psi a_0(980), a_0(980) \rightarrow \pi^0\eta]}{\Gamma[\bar{B}_s^0 \rightarrow J/\psi f_0(980), f_0(980) \rightarrow \pi^+\pi^-]}, \quad (15)$$

with decay widths $\Gamma[\bar{B}_s^0 \rightarrow J/\psi a_0(980), a_0(980) \rightarrow \pi^0\eta]$ and $\Gamma[\bar{B}_s^0 \rightarrow J/\psi f_0(980), f_0(980) \rightarrow \pi^+\pi^-]$ obtained by integrating Eq. (12) over the invariant mass $M_{\text{inv}(ij)}$.

Under several different assumptions related to the two sources of isospin violation, we evaluate the ratio R . The results are shown in Table 1.

From Table 1, we observe that the ratio R with $\eta-\eta'$ mixing (Case 4) is about 20% bigger than that without

Table 1. Values of R with different assumptions. (In the table, I.V. denotes isospin violation.)

no $\eta - \eta'$ mixing	I.V. both in T matrix and in explicit $K\bar{K}$ loops (Case 1)	3.1×10^{-2}
	I.V. only in T matrix (Case 2)	3.5×10^{-2}
	I.V. only in explicit $K\bar{K}$ loops (Case 3)	7.0×10^{-4}
with $\eta - \eta'$ mixing	I.V. both in T matrix and in explicit $K\bar{K}$ loops (Case 4)	3.7×10^{-2}
	I.V. only in T matrix (Case 5)	4.1×10^{-2}
	I.V. only in explicit $K\bar{K}$ loops (Case 6)	9.7×10^{-4}

$\eta - \eta'$ mixing (Case 1). By comparing the values of R for Case 2 and Case 3 (or, for Case 5 and Case 6), we find that the isospin violation in the T matrix has a more important effect than that in the explicit $K\bar{K}$ loops, being at least one order of magnitude larger. This fact is interesting, since in our picture the $f_0(980)$ and $a_0(980)$ resonances are dynamically generated from the PP interaction with the information on their nature contained in the T matrix. For the $\bar{B}_s^0 \rightarrow J/\psi\pi^+\pi^-(\pi^0\eta)$ decay, neither the $\pi^+\pi^-$ nor the $\pi^0\eta$ can be directly produced from $s\bar{s}$ hadronization [see Eqs. (5) and (6)], hence there is no contribution from the tree level. Instead, they are produced through the rescattering mechanism of Fig. 2, with $f_0(980)$ and $a_0(980)$ resonances as dynamically generated states from the PP interaction. The production rate of the $f_0(980)$ ($a_0(980)$) resonance in the $\bar{B}_s^0 \rightarrow J/\psi\pi^+\pi^-(\pi^0\eta)$ decay is sensitive to the resonance information contained in the T matrix. Therefore, this mode is particularly suitable to test the nature of $f_0(980)$ and $a_0(980)$ resonances and to investigate the isospin violation.

From the PDG [40], the experimental branching ratio of the $\bar{B}_s^0 \rightarrow J/\psi f_0(980), f_0(980) \rightarrow \pi^+\pi^-$ decay reads

$$\text{Br}[\bar{B}_s^0 \rightarrow J/\psi f_0(980), f_0(980) \rightarrow \pi^+\pi^-] = (1.28 \pm 0.18) \times 10^{-4}. \quad (16)$$

By using the ratio R in Table 1 and the branching ratio of Eq. (16), the branching ratio for $a_0(980)$ production can be obtained,

$$\begin{aligned} & \text{Br}[\bar{B}_s^0 \rightarrow J/\psi a_0(980), a_0(980) \rightarrow \pi^0\eta] \\ &= \begin{cases} (3.95 \pm 0.56) \times 10^{-6}, & \text{for Case 1;} \\ (4.74 \pm 0.67) \times 10^{-6}, & \text{for Case 4.} \end{cases} \quad (17) \end{aligned}$$

This branching ratio is of the order of 5×10^{-6} , not too small considering that several rates of the order of 10^{-7} are tabulated in the PDG [40]. The branching ratio and the shape of the $\pi^0\eta$ mass distribution of the $\bar{B}_s^0 \rightarrow J/\psi\pi^0\eta$ decay provide relevant information on the nature of the $a_0(980)$ resonance. Experimental measurements will be very valuable.

IV. CONCLUSIONS

In the present work, we study the isospin allowed decay process $\bar{B}_s^0 \rightarrow J/\psi\pi^+\pi^-$ and the isospin forbidden decay process $\bar{B}_s^0 \rightarrow J/\psi\pi^0\eta$, paying attention to the different sources of isospin violation.

First, we have $J/\psi s\bar{s}$ production in the \bar{B}_s^0 decay, via internal emission as shown in Fig. 1. After the hadronization of $s\bar{s}$ into meson–meson components, we obtain $K\bar{K}$ pairs and $\eta\eta$, while $\pi^+\pi^-$ and $\pi^0\eta$ are not produced at this step. Therefore, to see $\pi^+\pi^-$ or $\pi^0\eta$ in the final state, rescattering of the $K\bar{K}$ or $\eta\eta$ components is needed to produce $\pi^+\pi^-$ and $\pi^0\eta$ at the end. The picture shows that the weak decay amplitudes are proportional to the T matrix of the meson–meson transitions. We can obtain information about the violation of isospin from these magnitudes. In Figs. 3 and 4, we observe a clear signal for $f_0(980)$ production. We also observe that the shape of the $\pi^0\eta$ mass distribution is very different from the shape of the common $a_0(980)$ production in isospin-allowed reactions, and it is related to the difference in mass between the charged and neutral kaons. In the production of $a_0(980)$ we find two sources of isospin violation: one is that the loops containing K^+K^- or $K^0\bar{K}^0$ do not cancel due to the different mass between the charged and neutral kaons, and the other is that the transition T matrix of the meson–meson interaction already contains some isospin violation. In fact, we find that the contribution from isospin violation in the T matrix is far more important than the contribution of the explicit loops in the weak decay, being at least one order of magnitude larger. The study here shows that this reaction is very sensitive to the way the resonances are generated.

The D_s^+ semileptonic decay [38] and \bar{B}_s^0 mesonic decay both produce an $s\bar{s}$ pair at the end, and the two resonances of $f_0(980)$ and $a_0(980)$ are produced dynamically by the interaction of pseudoscalar mesons through the chiral unitary approach. The results of D_s^+ semileptonic decay are consistent with the experimental upper bound. We also calculate the branching ratio of $\bar{B}_s^0 \rightarrow J/\psi a_0(980)$ for $a_0(980)$ production, and the values are not too small, of the order of 5×10^{-6} . Our results provide a reference basis for experiments, which we expect to be carried out in the near future.

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