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QCD sum rule study for hidden-strange pentaquarks*

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Abstract: Inspired by the LHCb observations of hidden-charm $P_{c(s)}$ states, we study their hidden-strange analog P_s states in both the $[udu][\bar{s}s]$ and $[uds][\bar{s}u]$ configurations. We investigate P_s pentaquark states in the $p\eta'$, $p\phi$, ΛK, ΣK, and $\Sigma^* K^*$ structures with $J^P=1/2^-$ and $\Sigma^* K$ and ΣK^* with $J^P=3/2^-$ and calculate their masses in the framework of QCD sum rules. Our numerical results show that the extracted hadron masses for all the $p\eta'$, $p\phi$, ΛK, ΣK, and $\Sigma^* K^*$ structures are significantly higher than the ΣK mass threshold, and the masses for $\Sigma^* K$ and ΣK^* are also higher than the threshold of the corresponding hadron; hence, no bound state exists in such channels, which is consistent with the current experimental status.

Keywords: pentaquark states, QCD sum rules

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I. INTRODUCTION

Quantum chromodynamics (QCD) is the fundamental theory of the strong interaction. Hadrons are $q\bar{q}$ mesons and qqq baryons in the conventional quark model [1, 2]. However, QCD itself allows for the existence of multiquarks. In 2015, the LHCb Collaboration reported two hidden-charm pentaquark states, $P_c(4380)$ and $P_c(4450)$, in the $J/\psi p$ invariant mass spectrum of $\Lambda_b^0 \to J/\psi pK$ decays [3]. $P_c(4450)$ was then further found to be separated into two structures, $P_c(4440)$ and $P_c(4457)$, in the same process, in which a new narrow resonance $P_c(4312)$ was also discovered [4]. Recently, the LHCb Collaboration announced the first evidence of a hidden-charm pentaquark state with strangeness, $P_{cs}^0(4459)$, in the $J/\psi\Lambda$ invariant mass distribution of $\Xi_b^0 \to J/\psi\Lambda K$ decays [5]. Observations of these hidden-charm pentaquarks have immediately garnered extensively theoretical interest, considering them $\bar{D}^{(*)}\Sigma_c^{(*)}$ ($\bar{D}^{(*)}\Xi_c$) hadron molecules or compact pentaquarks with a quark content of $\bar{c}cuud$ ($\bar{c}cuds$) [6–10].

If the above interpretations of P_c are correct, analogous effects could also be expected at the hidden-strange $P_s=\bar{s}suud$ pentaquark. The study of the existence of hidden-strange pentaquarks was conducted considerably earlier than the observations of P_c states. Experiments and analysis of pentaquarks composed of light quarks date back to 1960, when the $N\bar{K}$ molecular state was explained with $\Lambda(1405)$[11–17]. However, there are almost no results for light pentaquarks. From 1994–1999, experiments using the SPHINX spectrometer reported a resonance structure $X(2000)$ as a candidate for the $\bar{s}suud$ state in the proton diffractive reactions $p+N(C)\to X(\Sigma^0K^+)+N(C)$ [18– 21]. Although the extracted resonance had fairly poor statistics, it inspired several theoretical studies on the existence of hidden-strange pentaquarks. In Ref. [22], Williams and Gueye studied the color octet-octet $[q\bar{s}][uds]$ configuration in a non-relativistic quark molecular model and predicted four candidates for P_s states. Using the quark delocalization color screening model (QDCSM), Huang *et al.* calculated the effective potential, masses, and decay widths of $\Sigma^{(*)}K^{(*)}$ molecular states and found that the interactions between $\Sigma^{(*)}$ and $K^{(*)}$ were sufficiently strong to form bound states [23].

Furthermore, a $\phi-N$ bound state was proposed by Gao *et al.*, in which the QCD van der Waals attractive potential is sufficiently strong to bind a ϕ meson to a nucleon inside a nucleus to form a bound state [24]. The calculations in the chiral $SU(3)$ quark model [25, 26], lattice QCD [27], chiral soliton model [28], and QDCSM [29] also support the existence of ϕN bound states. In Ref. [30], the authors proposed a possible ϕp resonance in the $\Lambda_c^+ \to \pi^0\phi p$ decay by considering a triangle singularity mechanism. In Ref. [31], Lebed used a $[su]_{\bar{3}}[\bar{s}ud]_3$ diquark-triquark model to investigate the possible existence of P_s pentaquarks and further proposed to create such states

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through the $\Lambda_c^+ \to P_s\pi^0 \to \phi p\pi^0$ decay [32]. Unfortunately, the Belle Collaboration searched for $\Lambda_c^+ \to \phi p\pi^0$ decays in 2017 and found no evidence of the intermediate hidden-strange pentaquark decay $P_s \to \phi p$ [33].

Comparing the hidden-charm P_c states observed in the $J/\psi p$ final state, one possible reason for the absence of P_s pentaquarks may be the limited phase space of the $P_s \to \phi p$ decay. Nevertheless, it is still interesting to investigate the properties of P_s states in theory. In this study, we investigate $\bar{s}suud$ systems in both the $[udu][\bar{s}s]$ and $[uds][\bar{s}u]$ configurations with quantum numbers $J^P = 1/2^-$ using QCD sum rules [34–36].

This paper is organized as follows. In Sec. II, we construct interpolating currents for hidden-strange pentaquarks in both the $[udu][\bar{s}s]$ and $[uds][\bar{s}u]$ configurations with $J^P = 1/2^-$ and then evaluate the correlation functions and spectral densities for these interpolating currents. In Sec. III, we perform QCD sum rule analyses to extract the mass spectra of P_s pentaquarks. The final section contains a brief summary and discussion.

II. QCD SUM RULES FOR HIDDEN-STRANGE PENTAQUARKS

In this section, we introduce QCD sum rules for hidden-strange pentaquarks with a quark content of $qqqs\bar{s}$. The start of QCD sum rules is the two-point correlator

$$\Pi(q) = \mathrm{i}\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}q\cdot x}\langle 0|T\{\eta(x)\bar{\eta}(0)\}|0\rangle,$$
$$\Pi_{\mu\nu}(q) = \mathrm{i}\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}q\cdot x}\left\langle 0\left|T\left\{\eta_\mu(x)\bar{\eta}_\nu(0)\right\}\right|0\right\rangle, \quad (1)$$

where $\eta(x)$ is a hidden-strange pentaquark interpolating current. There are two possible color configurations for such pentaquark operators: $[\epsilon^{abc}q_a q_b q_c][\bar{s}_d s_d]$ and $[\epsilon^{abc}q_a q_b s_c][\bar{s}_d q_d]$, where a,b,c,d are color indices, q denotes a light quark, and s is a strange quark. We use both types of interpolating currents to investigate hidden-strange pentaquark systems. To construct the pentaquark currents, we use the following operators for $L=0$ baryons [36–38]:

$$\begin{aligned}
\eta^p &= \epsilon^{abc}\left[(u_a^T C d_b)\gamma_5 u_c - (u_a^T C\gamma_5 d_b)u_c\right],\\
\eta_\mu^p &= \epsilon^{abc}\left[(u_a^T C d_b)\gamma_\mu u_c - (u_a^T C\gamma_5 d_b)\gamma_\mu\gamma_5 u_c\right],\\
\eta^\Lambda &= \epsilon^{abc}\left[(u_a^T C d_b)\gamma_5 s_c - (u_a^T C\gamma_5 d_b)s_c\right],\\
\eta^\Sigma &= \epsilon^{abc}\left[(u_a^T C\gamma_\mu d_b)\gamma_5\gamma^\mu s_c\right],\\
\eta_\mu^{\Sigma^*} &= \epsilon^{abc}\left[2(u_a^T C\gamma_\mu s_b)u_c + (u_a^T C\gamma_\mu u_b)s_c\right],
\end{aligned} \quad (2)$$

in which T denotes the transposition, and C is the charge conjugation matrix. Together with the operators $\bar{q}_a\gamma_5 q_a(0^-)$and $\bar{q}_a\gamma_\mu q_a(1^-)$ for $L=0$ mesons, we compose the following hidden-strange pentaquark currents with $J^P = 1/2^-$

$$\begin{aligned}
\eta_1 &= \epsilon^{abc}\left[(u_a^T C d_b)\gamma_5 u_c - (u_a^T C\gamma_5 d_b)u_c\right][\bar{s}_d\gamma_5 s_d],\\
\eta_2 &= \epsilon^{abc}\left[(u_a^T C d_b)\gamma_\mu u_c - (u_a^T C\gamma_5 d_b)\gamma_\mu\gamma_5 u_c\right][\bar{s}_d\gamma^\mu s_d],\\
\eta_3 &= \epsilon^{abc}\left[(u_a^T C d_b)\gamma_5 s_c - (u_a^T C\gamma_5 d_b)s_c\right][\bar{s}_d\gamma_5 u_d],\\
\eta_4 &= \epsilon^{abc}\left[(u_a^T C\gamma_\mu d_b)\gamma_5\gamma^\mu s_c\right][\bar{s}_d\gamma_5 u_d],\\
\eta_5 &= \epsilon^{abc}\left[2(u_a^T C\gamma_\mu s_b)u_c + (u_a^T C\gamma_\mu u_b)s_c\right][\bar{s}_d\gamma^\mu d_d],
\end{aligned} \quad (3)$$

which can couple to $p\eta'$, $p\phi$, ΛK, ΣK, and $\Sigma^* K^*$, respectively. We also compose the following hidden-strange pentaquark currents with $J^P = 3/2^-$

$$\begin{aligned}
\eta_{6\mu} &= \epsilon^{abc}\left[(u_a^T C\gamma_\nu d_b)\gamma_5\gamma^\nu s_c\right]\left[\bar{s}_d\gamma_\mu u_d\right],\\
\eta_{7\mu} &= \epsilon^{abc}\left[2(u_a^T C\gamma_\mu s_b)u_c + (u_a^T C\gamma_\mu u_b)s_c\right][\bar{s}_d\gamma_5 d_d],
\end{aligned} \quad (4)$$

which can couple to ΣK^* and $\Sigma^* K$, respectively. In principle, these interpolating currents can couple to pentaquark states with both negative and positive parities.

$$\langle 0|\eta(x)|X^-\rangle = f_{X^-}u(q), \quad (5)$$

$$\langle 0|\eta(x)|X^+\rangle = \mathrm{i}f_{X^+}\gamma_5 u(q), \quad (6)$$

where $u(q)$ is the Dirac spinor, and f_{X^-} and f_{X^+} are coupling constants. Accordingly, the two-point correlators induced by these pentaquark currents can be written as

$$\begin{aligned}
\Pi^-(q) &= \mathrm{i}\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}q\cdot x}\langle 0|T\{\eta(x)\bar{\eta}(0)\}|0\rangle\\
&= \not{q}\Pi_q\left(q^2\right) + \Pi_m\left(q^2\right),
\end{aligned} \quad (7)$$

$$\begin{aligned}
\Pi^+(q) &= \mathrm{i}\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}q\cdot x}\left\langle 0\left|T\left\{\eta'(x)\bar{\eta}'(0)\right\}\right|0\right\rangle\\
&= \not{q}\Pi_q\left(q^2\right) - \Pi_m\left(q^2\right),
\end{aligned} \quad (8)$$

where $\Pi_q(q^2)$ and $\Pi_m(q^2)$ are the invariant functions proportional to $\not{q}$ and 1, respectively. Note that these two invariant functions appear in $\Pi^-(q)$ and $\Pi^+(q)$ simultaneously, implying that they contain hadron information for both negative and positive parity pentaquark states. However, the signs of $\Pi_m(q^2)$ in $\Pi^-(q)$ and $\Pi^+(q)$ are different owing to the coupling definitions in Eqs. (5)–(6). See Refs. [39–42] for detailed discussions.

Moreover, $\eta_{6\mu}(x)$ and $\eta_{7\mu}(x)$ can couple to both $J = 3/2$ and $1/2$ channels

$$\Pi_{\mu\nu}(q)=\left(\frac{q_\mu q_\nu}{q^2}-g_{\mu\nu}\right)(\not{q}+M_X)\Pi^{3/2}(q)+\cdots, \quad (9)$$

where $\cdots$ contains the spin $1/2$ components of $\eta_{6\mu}$ or $\eta_{7\mu}$.

At the hadronic level, the invariant functions can be described by the dispersion relation

$$\Pi\left(q^2\right)=\frac{\left(q^2\right)^N}{\pi}\int_{4m_s^2}^{\infty}\frac{\mathrm{Im}\,\Pi(s)}{s^N(s-q^2-\mathrm{i}\epsilon)}\mathrm{d}s+\sum_{n=0}^{N-1}b_n\left(q^2\right)^n, \quad (10)$$

where b_n are N unknown subtraction constants and will be removed after performing a Borel transform. The imaginary part of the correlation function is usually defined as the spectral function

$$\rho(s)\equiv\frac{1}{\pi}\mathrm{Im}\Pi(s)=\sum_n\delta\left(s-m_n^2\right)\langle0|\eta|n\rangle\langle n|\bar\eta|0\rangle$$
$$=f_X^2\delta(s-m_X^2)+\cdots, \quad (11)$$

in which the “pole plus continuum” parameterization assumption is adopted, and "$\cdots$" contains contributions from the QCD continuum and higher excited states. The intermediate state $|n\rangle$ can be either a positive or negative parity pentaquark. The parameters f_X and m_X are the coupling constant and hadron mass of the lowest-lying state, respectively.

The two-point correlation function and spectral function can be evaluated as the functions of various QCD condensates via operator product expansion (OPE) at the quark-gluonic level. We use coordinate space expressions for the light quark and strange quark propagators [43]

$$\begin{aligned}\mathrm{i}S^{ab}(x)=&\langle T\{q^a(x)\bar q^b(0)\}\rangle=\frac{\mathrm{i}\delta^{ab}}{2\pi^2x^4}+\frac{\mathrm{i}}{32\pi^2}t^n_{ab}g_sG^n_{\mu\nu}\frac{1}{x^2}(\sigma^{\mu\nu}\not{x}+\not{x}\sigma^{\mu\nu})-\frac{\delta^{ab}}{12}\langle\bar qq\rangle+\frac{\delta^{ab}x^2}{192}\langle g_s\bar q\sigma Gq\rangle\\&-\frac{\mathrm{i}g_s^2\langle\bar qq\rangle^2x^2}{2^5\times3^5}\delta^{ab}\not{x}-\frac{g_s^2\langle\bar qq\rangle\langle G^2\rangle x^4}{2^9\times3^3}\delta^{ab}-\frac{m_q\delta^{ab}}{4\pi^2x^2}+\frac{m_q}{16\pi^2}t^n_{ab}g_sG^n_{\mu\nu}\sigma^{\mu\nu}\log(-x^2)-\frac{\delta^{ab}\langle g_s^2G^2\rangle}{2^9\times3\pi^2}m_qx^2\log(-x^2)\\&+\frac{\mathrm{i}\delta^{ab}m_q\langle\bar qq\rangle}{48}\not{x}-\frac{\mathrm{i}m_q\langle g_s\bar q\sigma Gq\rangle\delta^{ab}x^2}{2^7\times3^2}\not{x}-\frac{g_s^2m_q\langle\bar qq\rangle^2}{2^7\times3^5}x^4\delta^{ab}+\cdots\end{aligned} \quad (12)$$

where $q=u,d,s$ quarks, $\not{x}=x^\mu\gamma_\mu$, and $t^n_{ab}=\lambda^n_{ab}/2$. In this study, we evaluate correlation functions and spectral functions up to dimension-11 condensates. As an example, we show the spectral function for the interpolating current $\eta_1(x)$ as

$$\begin{aligned}\rho_1^q(s)=&\frac{s^5}{45875200\pi^7}-\frac{s^3m_s\langle\bar ss\rangle}{40960\pi^5}+\frac{s^3\langle g_s^2G^2\rangle}{1310720\pi^7}+\frac{g_s^2s^2\langle\bar uu\rangle^2}{110592\pi^5}+\frac{g_s^2s^2\langle\bar ss\rangle^2}{110592\pi^5}+\frac{g_s^2s^2\langle\bar dd\rangle^2}{221184\pi^5}+\frac{s^2\langle\bar ss\rangle^2}{2048\pi^3}\\&-\frac{415s^2m_s\langle g_s\bar s\sigma\cdot Gs\rangle}{1572864\pi^5}+\frac{127g_s^3\langle\bar dd\rangle^2\langle g_s^2G^2\rangle}{31850496\pi^5}-\frac{g_s^2m_s\langle\bar dd\rangle^2\langle\bar ss\rangle}{6912\pi^3}-\frac{65m_s\langle g_s^2G^2\rangle\langle g_s\bar s\sigma\cdot Gs\rangle}{2359296\pi^5}\\&-\frac{161sm_s\langle\bar ss\rangle\langle g_s^2G^2\rangle}{2359296\pi^5}+\frac{161\langle\bar ss\rangle^2\langle g_s^2G^2\rangle}{589824\pi^3}+\frac{65g_s^3\langle\bar ss\rangle^2\langle g_s^2G^2\rangle}{31850496\pi^5}+\frac{5g_s^3\langle\bar uu\rangle^2\langle g_s^2G^2\rangle}{663552\pi^5}+\frac{s\langle\bar ss\rangle\langle g_s\bar s\sigma\cdot Gs\rangle}{512\pi^3}\\&+\frac{3\langle g_s\bar s\sigma\cdot Gs\rangle^2}{4096\pi^3}-\frac{g_s^2m_s\langle\bar ss\rangle\langle\bar uu\rangle^2}{3456\pi^3}+\frac{g_s^2m_s\langle\bar ss\rangle^3}{3456\pi^3},\end{aligned} \quad (13)$$

$$\begin{aligned}\rho_1^m(s)=&\frac{s^4\langle\bar dd\rangle}{491520\pi^5}+\frac{s^2\langle\bar dd\rangle\langle g_s^2G^2\rangle}{73728\pi^5}-\frac{s^2m_s\langle\bar dd\rangle\langle\bar ss\rangle}{1536\pi^3}+\frac{s\langle g_s^2G^2\rangle\langle g_s\bar d\sigma\cdot Gd\rangle}{65536\pi^5}+\frac{g_s^2s\langle\bar dd\rangle\langle\bar uu\rangle^2}{10368\pi^3}+\frac{g_s^2s\langle\bar dd\rangle\langle\bar ss\rangle^2}{10368\pi^3}\\&-\frac{sm_s\langle\bar dd\rangle\langle g_s\bar s\sigma\cdot Gs\rangle}{768\pi^3}-\frac{m_s\langle g_s\bar d\sigma\cdot Gd\rangle\langle g_s\bar s\sigma\cdot Gs\rangle}{1024\pi^3}+\frac{\langle\bar dd\rangle\langle\bar ss\rangle\langle g_s\bar s\sigma\cdot Gs\rangle}{192\pi}+\frac{\langle\bar uu\rangle^2\langle g_s\bar d\sigma\cdot Gd\rangle}{192\pi}\\&+\frac{\langle\bar dd\rangle\langle\bar uu\rangle\langle g_s\bar u\sigma\cdot Gu\rangle}{96\pi}+\frac{\langle\bar dd\rangle\langle g_s^2G^2\rangle^2}{1179648\pi^5}+\frac{g_s^2\langle\bar uu\rangle^2\langle g_s\bar d\sigma\cdot Gd\rangle}{41472\pi^3}+\frac{s\langle\bar dd\rangle\langle\bar uu\rangle^2}{96\pi}+\frac{s\langle\bar dd\rangle\langle\bar ss\rangle^2}{192\pi},\end{aligned} \quad (14)$$

where $\rho^q(s)=\mathrm{Im}\Pi_\mathrm{q}(s)/\pi$ and $\rho^m(s)=\mathrm{Im}\Pi_\mathrm{m}(s)/\pi$ are the spectral functions of the invariant structures $\Pi_q(q^2)$ and $\Pi_m(q^2)$ in Eqs. (7)–(8), respectively. Their dimensions are different because $\Pi_q(q^2)$ is proportional to $\not{q}$, whereas $\Pi_m(q^2)$ is proportional to 1. As shown in Eqs. (13)–(14), there are more terms in $\rho^q(s)$ than in $\rho^m(s)$, including the perturbative term. We use $\rho^q(s)$ to perform our numerical analyses in the following section. The expressions for the spectral functions of other interpolating currents are given in Appendix A. Note that all the correlation functions in this study are evaluated at the leading order of α_s. It is known that the α_s corrections of the perturbative terms give important contributions for heavy quarkonium systems [35]; however, we do not consider the α_s corrections here because of the complexity and difficulty of calculations. Nevertheless, the NLO effects in multiquark systems deserve to be studied, and we will attempt to conduct such research in the future.

QCD sum rules can be established by assuming that the two-point correlation functions obtained from the hadronic and quark-gluonic levels are equal to each other. After applying the Borel transform, the correlation function can be written as

$$\Pi(M_{\rm B}^2) \equiv \mathcal{B}_{M_{\rm B}^2}\Pi(q^2) = \int_{4m_s^2}^{\infty} {\rm d}s\, {\rm e}^{-s/M_{\rm B}^2}\rho(s)\,, \tag{15}$$

in which $M_{\rm B}$ is the Borel parameter introduced by the Borel transform. It is clear that the Borel transform suppresses the contributions from the continuum and higher excited states in Eq. (11). The lowest-lying resonance is then chosen as

$$f_X^2 {\rm e}^{-m_X^2/M_{\rm B}^2} = \int_{4m_s^2}^{s_0} {\rm d}s\, {\rm e}^{-s/M_{\rm B}^2}\rho(s)\,, \tag{16}$$

where s_0 is the continuum threshold. The mass of the lowest-lying hadron can thus be extracted as

$$m_X^2(s_0, M_{\rm B}^2) = \frac{\displaystyle\int_{4m_s^2}^{s_0} {\rm d}s\, {\rm e}^{-s/M_{\rm B}^2} s\rho(s)}{\displaystyle\int_{4m_s^2}^{s_0} {\rm d}s\, {\rm e}^{-s/M_{\rm B}^2}\rho(s)}, \tag{17}$$

which is a function of the two parameters $M_{\rm B}^2$ and s_0. We discuss the details in the next section to obtain suitable parameter working regions in the QCD sum rule analysis.

III. NUMERICAL RESULTS

To perform numerical analyses, we adopt parameter values for various QCD condensates and quark masses as follows [43–46]: $\langle\bar{u}u\rangle = \langle\bar{d}d\rangle = \langle\bar{q}q\rangle = -(0.24\pm0.01)^3$ GeV^3, $\langle\bar{q}g_s\sigma Gq\rangle = m_0^2\langle\bar{q}q\rangle$, $m_0^2 = -0.8$ GeV^2, $\langle\bar{s}s\rangle = (0.8\pm0.1)\langle\bar{q}q\rangle$, $\langle\bar{s}g_s\sigma Gs\rangle = m_0^2\langle\bar{s}s\rangle$, $\langle g_s^2GG\rangle = (0.44\pm0.02)$ GeV^4, and the $\overline{\rm MS}$ strange quark mass $m_s = (0.095\pm0.005)$ GeV. We let $m_u = m_d = m_q = 0$, $\langle\bar{u}u\rangle = \langle\bar{d}d\rangle = \langle\bar{q}q\rangle$, and $\langle\bar{u}g_s\sigma\cdot Gu\rangle = \langle\bar{d}g_s\sigma\cdot Gd\rangle = \langle\bar{q}g_s\sigma\cdot Gq\rangle$ for the up and down quarks in the chiral limit. The definition of the coupling constant g_s has a minus sign difference compared to that in Ref. [35].

As shown in Eq. (17), the extracted hadron mass is a function of the Borel mass $M_{\rm B}$ and continuum threshold s_0. To choose suitable working windows for these two parameters, we must study the OPE convergence and pole contribution for the correlation function. On the one hand, the perturbative term should be two times larger than the nonperturbative contribution to ensure good OPE convergence, which will limit the lower bound on the Borel parameter $M_{\rm B}^2$. On the other hand, we require the following pole contribution be larger than 10% to give the upper bound on $M_{\rm B}^2$

$$PC(s_0, M_{\rm B}^2) = \frac{\displaystyle\int_{4m_s^2}^{s_0} {\rm d}s\, {\rm e}^{-s/M_{\rm B}^2}\rho(s)}{\displaystyle\int_{4m_s^2}^{\infty} {\rm d}s\, {\rm e}^{-s/M_{\rm B}^2}\rho(s)}, \tag{18}$$

which is also the function of $M_{\rm B}^2$ and s_0. A proper value of s_0 is required before determining the Borel window.

We take the interpolating current $\eta_4(x)$ as an example to present our numerical analysis of the hidden-strange ΣK pentaquark state. In Fig. 1, we plot the OPE behavior term by term with respect to $M_{\rm B}^2$. It is shown that the most important nonperturbative contribution is from the $\langle\bar{q}q\rangle^2$ term. By requiring the perturbative term to be two times larger than the $\langle\bar{q}q\rangle^2$ term, we find the lower bound on the Borel parameter $M_{\rm min}^2 = 2.32$ GeV^2. To choose an optimum value of s_0, we show the variation in the hadron mass with respect to s_0 at different values of the Borel mass in Fig. 2. The best choice of the threshold parameter is then fixed as $s_0 = 8.35$ GeV^2, around which the hadron mass is very stable against $M_{\rm B}^2$.

To determine the upper limit of $M_{\rm B}^2$, we study the pole contribution for $s_0 = 8.35$ GeV^2 in Fig. 3. We find that the pole contribution is larger than 10% for $M_{\rm B}^2 \leq 2.95$ GeV^2, which is the upper bound on the Borel window. Finally, we can study the behavior of the Borel curves in Fig. 4 using the parameter $s_0 = 8.35$ GeV^2 and the Borel window 2.32 GeV$^2 \leq M_{\rm B}^2 \leq 2.95$ GeV^2. We find

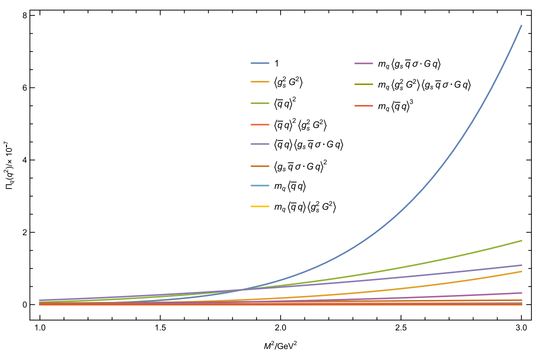

Fig. 1. (color online) Contributions of each term in the OPE series for the interpolating current $\eta_4(x)$ with $J^P = 1/2^-$.

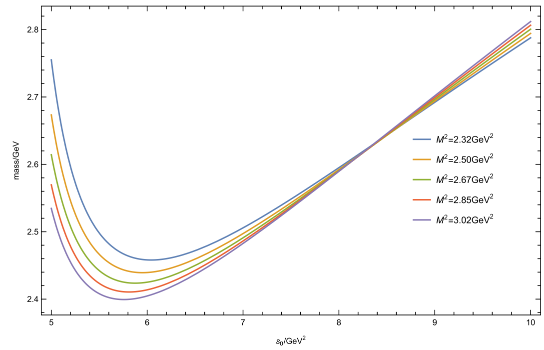

Fig. 2. (color online) Hadron mass with respect to the threshold parameter s_0 with different values of the Borel mass for $\eta_4(x)$.

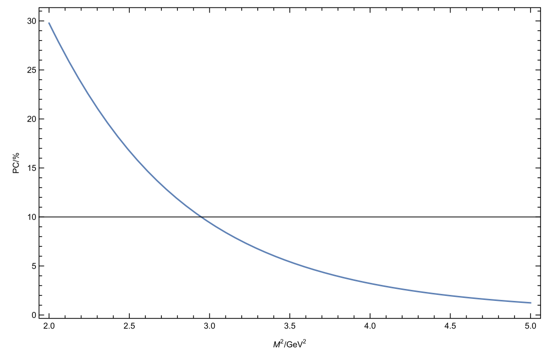

Fig. 3. (color online) Pole contribution behavior of $\eta_4(x)$ with $s_0 = 8.35$ GeV^2.

that the mass curves are very stable with respect to $M_{\rm B}^2$ in the these parameter regions. The hadron mass for such a hidden-strange ΣK pentaquark with $J^P = 1/2^-$ is extracted as

$$m_X = (2.63 \pm 0.14 \pm 0.13)\ \text{GeV}, \tag{19}$$

in which the first error originates from the uncertainties in various input QCD parameters (mainly from the quark condensate), whereas the second error arises from the uncertainty in the threshold value s_0. For all other interpolating currents in Eq. (3), we perform analyses similar to the above procedure and obtain the hadron masses in

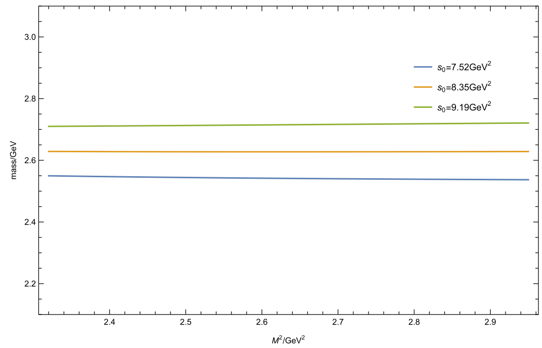

Fig. 4. (color online) Variations in the hadron mass m_X with respect to $M_{\rm B}^2$ for $\eta_4(x)$.

Table 1. We find that the mass of the $p\phi$ pentaquark is several MeV below the $M_{p\phi}$ threshold. However, it is still considerably higher than the mass threshold of $M_{\Sigma K}$, which prevents the existence of such a $p\phi$ pentaquark state with $J^P = 1/2^-$. Moreover, our results do not support the existence of bound states for the $p\eta'$, ΛK, ΣK, and $\Sigma^* K^*$ channels with $J^P = 1/2^-$ and ΣK^* and $\Sigma^* K$ channels with $J^P = 3/2^-$ because the obtained hadron masses are significantly higher than the corresponding two-hadron thresholds.

IV. SUMMARY

Inspired by the observations of hidden-charm P_c states, we investigate hidden-strange pentaquark states in both the $[udu][\bar{s}s]$ and $[uds][\bar{s}u]$ configurations with quantum numbers $J^P = 1/2^-$ and $J^P = 3/2^-$ via the QCD sum rule method. We construct the corresponding interpolating currents using $L = 0$ baryonic and mesonic operators. After calculating two-point correlation functions up to dimension-11, we perform mass sum rule analyses using the invariant structures proportional to $\not{q}$ and extract mass spectra for the $p\eta'$, $p\phi$, ΛK, ΣK, and $\Sigma^* K^*$ pentaquarks with $J^P = 1/2^-$ and ΣK^* and $\Sigma^* K$ with $J^P = 3/2^-$.

There is only a recombination of quarks before and after strong decay, in which all pentaquarks with $J^P = 1/2^-$ can decay into Σ and K. Our calculations show

Table 1. Extracted hadron masses of hidden-strange molecular pentaquark states with $J^P = 1/2^-$ and $J^P = 3/2^-$.

Current	Structure	J^P	m_X/GeV	Threshold s_0/GeV^2	Borel window/GeV^2
$\eta_1(x)$	$[p\eta']$	$1/2^-$	$1.91 \pm 0.06 \pm 0.19$	4.77	2.35–2.48
$\eta_2(x)$	$[p\phi]$	$1/2^-$	$1.95 \pm 0.10 \pm 0.11$	4.62	2.15–2.35
$\eta_3(x)$	$[\Lambda K]$	$1/2^-$	$2.93 \pm 0.16 \pm 0.20$	10.57	2.10–3.51
$\eta_4(x)$	$[\Sigma K]$	$1/2^-$	$2.63 \pm 0.14 \pm 0.13$	8.35	2.32–2.95
$\eta_5(x)$	$[\Sigma^* K^*]$	$1/2^-$	$2.78 \pm 0.08 \pm 0.12$	9.60	2.47–3.32
$\eta_{6\mu}(x)$	$[\Sigma K^*]$	$3/2^-$	$2.39 \pm 0.08 \pm 0.11$	6.95	2.47–3.32
$\eta_{7\mu}(x)$	$[\Sigma^* K]$	$3/2^-$	$2.27 \pm 0.10 \pm 0.10$	5.82	3.06–3.62

that the masses for all the $p\eta'$, $p\phi$, ΛK, ΣK, and $\Sigma^* K^*$ pentaquarks are considerably higher than their two-hadron decay thresholds; hence, there is no bound hidden-strange pentaquark state in these channels.

The states $\Sigma^* K$ and ΣK^* with $J^P = 3/2^-$ can decay into the corresponding components $\Sigma^{(*)} K^{(*)}$. Our calculations show that these states also have masses higher than the mass thresholds of the corresponding components. Therefore, there is a no bound hidden-strange pentaquark state as well in such channels. This result is consistent with the experimental status of hidden-strange P_s states [33].

APPENDIX

In this appendix, we show the spectral densities for the interpolating currents $\eta_2(x)$, $\eta_3(x)$, $\eta_4(x)$, and $\eta_5(x)$, including both $\rho_i^q(s)$ and $\rho_i^m(s)$ structures proportional to $\not{q}$ and 1, respectively.

$$\begin{aligned}\rho_2^q =&\frac{s^5}{17203200\pi^7}+\frac{s^3\langle g_s^2G^2\rangle}{983040\pi^7}+\frac{s^2\langle\bar uu\rangle^2}{1536\pi^3}+\frac{s^2\langle\bar ss\rangle^2}{1536\pi^3}-\frac{s^2m_s\langle g_s\bar s\sigma\cdot Gs\rangle}{12288\pi^5}+\frac{g_s^2s^2\langle\bar uu\rangle^2}{41472\pi^5}+\frac{g_s^2s^2\langle\bar ss\rangle^2}{41472\pi^5}+\frac{g_s^2s^2\langle\bar dd\rangle^2}{82944\pi^5}\\&-\frac{sm_s\langle\bar ss\rangle\langle g_s^2G^2\rangle}{18432\pi^5}+\frac{s\langle\bar ss\rangle\langle g_s\bar s\sigma\cdot Gs\rangle}{384\pi^3}+\frac{s\langle\bar uu\rangle\langle g_s\bar u\sigma\cdot Gu\rangle}{768\pi^3}-\frac{m_s\langle g_s^2G^2\rangle\langle g_s\bar s\sigma\cdot Gs\rangle}{24576\pi^5}+\frac{5\langle\bar ss\rangle^2\langle g_s^2G^2\rangle}{9216\pi^3}\\&+\frac{\langle\bar uu\rangle^2\langle g_s^2G^2\rangle}{4608\pi^3}+\frac{g_s^3\langle\bar ss\rangle^2\langle g_s^2G^2\rangle}{82944\pi^5}+\frac{g_s^3\langle\bar uu\rangle^2\langle g_s^2G^2\rangle}{165888\pi^5}+\frac{g_s^2m_s\langle\bar ss\rangle^3}{1728\pi^3}+\frac{\langle g_s\bar s\sigma\cdot Gs\rangle^2}{1024\pi^3},\end{aligned}\tag{A1}$$

$$\begin{aligned}\rho_3^q =&\frac{11s^5}{825753600\pi^7}+\frac{3s^3\langle g_s^2G^2\rangle}{5242880\pi^7}-\frac{s^3m_s\langle\bar uu\rangle}{73728\pi^5}+\frac{11s^3m_s\langle\bar ss\rangle}{737280\pi^5}-\frac{s^2\langle\bar uu\rangle^2}{36864\pi^3}+\frac{5s^2\langle\bar ss\rangle\langle\bar uu\rangle}{18432\pi^3}-\frac{13s^2m_s\langle g_s\bar s\sigma\cdot Gs\rangle}{1179648\pi^5}\\&-\frac{13s^2m_s\langle g_s\bar u\sigma\cdot Gu\rangle}{196608\pi^5}+\frac{11g_s^2s^2\langle\bar uu\rangle^2}{1990656\pi^5}+\frac{11g_s^2s^2\langle\bar ss\rangle^2}{1990656\pi^5}+\frac{11g_s^2s^2\langle\bar dd\rangle^2}{3981312\pi^5}-\frac{5sm_s\langle\bar uu\rangle\langle g_s^2G^2\rangle}{110592\pi^5}\\&+\frac{35sm_s\langle\bar ss\rangle\langle g_s^2G^2\rangle}{589824\pi^5}+\frac{13s\langle\bar uu\rangle\langle g_s\bar s\sigma\cdot Gs\rangle}{18432\pi^3}+\frac{13s\langle\bar ss\rangle\langle g_s\bar u\sigma\cdot Gu\rangle}{18432\pi^3}+\frac{s\langle\bar uu\rangle\langle g_s\bar u\sigma\cdot Gu\rangle}{18432\pi^3}+\frac{g_s^3\langle\bar dd\rangle^2\langle g_s^2G^2\rangle}{442368\pi^5}\\&-\frac{5g_s^2m_s\langle\bar dd\rangle^2\langle\bar uu\rangle}{62208\pi^3}+\frac{11g_s^2m_s\langle\bar dd\rangle^2\langle\bar ss\rangle}{124416\pi^3}+\frac{35m_s\langle g_s^2G^2\rangle\langle g_s\bar s\sigma\cdot Gs\rangle}{1179648\pi^5}-\frac{5m_s\langle g_s^2G^2\rangle\langle g_s\bar u\sigma\cdot Gu\rangle}{196608\pi^5}\\&+\frac{25\langle\bar ss\rangle\langle\bar uu\rangle\langle g_s^2G^2\rangle}{110592\pi^3}-\frac{5\langle\bar uu\rangle^2\langle g_s^2G^2\rangle}{221184\pi^3}+\frac{35g_s^3\langle\bar ss\rangle^2\langle g_s^2G^2\rangle}{7962624\pi^5}+\frac{7g_s^3\langle\bar uu\rangle^2\langle g_s^2G^2\rangle}{1990656\pi^5}+\frac{\langle g_s\bar s\sigma\cdot Gs\rangle\langle g_s\bar u\sigma\cdot Gu\rangle}{1536\pi^3}\\&+\frac{\langle g_s\bar u\sigma\cdot Gu\rangle^2}{12288\pi^3}-\frac{5g_s^2m_s\langle\bar uu\rangle^3}{62208\pi^3}+\frac{11g_s^2m_s\langle\bar ss\rangle\langle\bar uu\rangle^2}{62208\pi^3}-\frac{5g_s^2m_s\langle\bar ss\rangle^2\langle\bar uu\rangle}{124416\pi^3}+\frac{11g_s^2m_s\langle\bar ss\rangle^3}{124416\pi^3}-\frac{m_s\langle\bar ss\rangle\langle\bar uu\rangle^2}{1152\pi}\\&+\frac{5m_s\langle\bar ss\rangle^2\langle\bar uu\rangle}{1152\pi},\end{aligned}\tag{A2}$$

$$\begin{aligned}\rho_4^q =&\frac{11s^5}{412876800\pi^7}+\frac{7\langle g_s^2G^2\rangle s^3}{5898240\pi^7}+\frac{\langle\bar dd\rangle m_ss^3}{737280\pi^5}+\frac{11\langle\bar ss\rangle m_ss^3}{368640\pi^5}-\frac{\langle\bar uu\rangle m_ss^3}{36864\pi^5}+\frac{11g_s^2\langle\bar dd\rangle^2s^2}{1990656\pi^5}+\frac{11g_s^2\langle\bar ss\rangle^2s^2}{995328\pi^5}\\&-\frac{\langle\bar uu\rangle^2s^2}{18432\pi^3}+\frac{11g_s^2\langle\bar uu\rangle^2s^2}{995328\pi^5}-\frac{\langle\bar dd\rangle\langle\bar ss\rangle s^2}{36864\pi^3}+\frac{5\langle\bar dd\rangle\langle\bar uu\rangle s^2}{18432\pi^3}+\frac{5\langle\bar ss\rangle\langle\bar uu\rangle s^2}{9216\pi^3}+\frac{\langle g_s\bar d\sigma\cdot Gd\rangle m_ss^2}{131072\pi^5}\\&-\frac{89\langle g_s\bar s\sigma\cdot Gs\rangle m_ss^2}{1179648\pi^5}-\frac{25\langle g_s\bar u\sigma\cdot Gu\rangle m_ss^2}{196608\pi^5}-\frac{\langle g_s\bar d\sigma\cdot Gd\rangle\langle\bar ss\rangle s}{12288\pi^3}+\frac{\langle\bar dd\rangle\langle g_s\bar s\sigma\cdot Gs\rangle s}{12288\pi^3}+\frac{\langle g_s\bar d\sigma\cdot Gd\rangle\langle\bar uu\rangle s}{6144\pi^3}\\&+\frac{25\langle g_s\bar s\sigma\cdot Gs\rangle\langle\bar uu\rangle s}{18432\pi^3}+\frac{\langle\bar dd\rangle\langle g_s\bar u\sigma\cdot Gu\rangle s}{2048\pi^3}+\frac{25\langle\bar ss\rangle\langle g_s\bar u\sigma\cdot Gu\rangle s}{18432\pi^3}+\frac{\langle\bar uu\rangle\langle g_s\bar u\sigma\cdot Gu\rangle s}{18432\pi^3}+\frac{13\langle g_s^2G^2\rangle\langle\bar dd\rangle m_ss}{884736\pi^5}\\&+\frac{53\langle g_s^2G^2\rangle\langle\bar ss\rangle m_ss}{589824\pi^5}-\frac{5\langle g_s^2G^2\rangle\langle\bar uu\rangle m_ss}{55296\pi^5}+\frac{5g_s^3\langle g_s^2G^2\rangle\langle\bar dd\rangle^2}{884736\pi^5}+\frac{53g_s^3\langle g_s^2G^2\rangle\langle\bar ss\rangle^2}{7962624\pi^5}-\frac{5\langle g_s^2G^2\rangle\langle\bar uu\rangle^2}{110592\pi^3}\end{aligned}$$

$$+\frac{35g_s^3\langle g_s^2G^2\rangle\langle\bar uu\rangle^2}{3981312\pi^5}+\frac{\langle g_s\bar u\sigma\cdot Gu\rangle^2}{8192\pi^3}-\frac{7\langle g_s^2G^2\rangle\langle\bar dd\rangle\langle\bar ss\rangle}{110592\pi^3}+\frac{\langle g_s\bar d\sigma\cdot Gd\rangle\langle g_s\bar s\sigma\cdot Gs\rangle}{24576\pi^3}+\frac{17\langle g_s^2G^2\rangle\langle\bar dd\rangle\langle\bar uu\rangle}{55296\pi^3}$$
$$+\frac{25\langle g_s^2G^2\rangle\langle\bar ss\rangle\langle\bar uu\rangle}{55296\pi^3}+\frac{\langle g_s\bar d\sigma\cdot Gd\rangle\langle g_s\bar u\sigma\cdot Gu\rangle}{12288\pi^3}+\frac{5\langle g_s\bar s\sigma\cdot Gs\rangle\langle g_s\bar u\sigma\cdot Gu\rangle}{4096\pi^3}+\frac{11g_s^2\langle\bar ss\rangle^3m_s}{62208\pi^3}-\frac{5g_s^2\langle\bar uu\rangle^3m_s}{31104\pi^3}$$
$$-\frac{\langle\bar dd\rangle\langle\bar ss\rangle^2m_s}{2304\pi}+\frac{g_s^2\langle\bar dd\rangle\langle\bar ss\rangle^2m_s}{248832\pi^3}-\frac{11\langle\bar dd\rangle\langle\bar uu\rangle^2m_s}{1152\pi}+\frac{g_s^2\langle\bar dd\rangle\langle\bar uu\rangle^2m_s}{62208\pi^3}-\frac{\langle\bar ss\rangle\langle\bar uu\rangle^2m_s}{576\pi}$$
$$+\frac{11g_s^2\langle\bar ss\rangle\langle\bar uu\rangle^2m_s}{31104\pi^3}+\frac{\langle g_s^2G^2\rangle\langle g_s\bar d\sigma\cdot Gd\rangle m_s}{98304\pi^5}+\frac{11g_s^2\langle\bar dd\rangle^2\langle\bar ss\rangle m_s}{62208\pi^3}+\frac{53\langle g_s^2G^2\rangle\langle g_s\bar s\sigma\cdot Gs\rangle m_s}{1179648\pi^5}$$
$$-\frac{5g_s^2\langle\bar dd\rangle^2\langle\bar uu\rangle m_s}{31104\pi^3}+\frac{5\langle\bar ss\rangle^2\langle\bar uu\rangle m_s}{576\pi}-\frac{5g_s^2\langle\bar ss\rangle^2\langle\bar uu\rangle m_s}{62208\pi^3}+\frac{5\langle\bar dd\rangle\langle\bar ss\rangle\langle\bar uu\rangle m_s}{576\pi}-\frac{5\langle g_s^2G^2\rangle\langle g_s\bar u\sigma\cdot Gu\rangle m_s}{98304\pi^5}, \tag{A3}$$

$$\rho_5^q=\frac{3s^5}{11468800\pi^7}-\frac{\langle g_s^2G^2\rangle s^3}{655360\pi^7}-\frac{3\langle\bar dd\rangle m_ss^3}{20480\pi^5}+\frac{3\langle\bar ss\rangle m_ss^3}{10240\pi^5}-\frac{\langle\bar uu\rangle m_ss^3}{5120\pi^5}+\frac{g_s^2\langle\bar dd\rangle^2s^2}{18432\pi^5}+\frac{g_s^2\langle\bar ss\rangle^2s^2}{9216\pi^5}$$
$$+\frac{\langle\bar uu\rangle^2s^2}{512\pi^3}+\frac{g_s^2\langle\bar uu\rangle^2s^2}{9216\pi^5}+\frac{3\langle\bar dd\rangle\langle\bar ss\rangle s^2}{1024\pi^3}+\frac{\langle\bar ss\rangle\langle\bar uu\rangle s^2}{256\pi^3}-\frac{9\langle g_s\bar d\sigma\cdot Gd\rangle m_ss^2}{16384\pi^5}+\frac{7\langle g_s\bar s\sigma\cdot Gs\rangle m_ss^2}{8192\pi^5}-\frac{7\langle g_s\bar u\sigma\cdot Gu\rangle m_ss^2}{8192\pi^5}$$
$$+\frac{3\langle g_s\bar d\sigma\cdot Gd\rangle\langle\bar ss\rangle s}{512\pi^3}+\frac{3\langle\bar dd\rangle\langle g_s\bar s\sigma\cdot Gs\rangle s}{512\pi^3}+\frac{7\langle g_s\bar s\sigma\cdot Gs\rangle\langle\bar uu\rangle s}{768\pi^3}+\frac{7\langle\bar ss\rangle\langle g_s\bar u\sigma\cdot Gu\rangle s}{768\pi^3}+\frac{7\langle\bar uu\rangle\langle g_s\bar u\sigma\cdot Gu\rangle s}{768\pi^3}$$
$$+\frac{\langle g_s^2G^2\rangle\langle\bar dd\rangle m_ss}{2048\pi^5}-\frac{\langle g_s^2G^2\rangle\langle\bar ss\rangle m_ss}{6144\pi^5}+\frac{\langle g_s^2G^2\rangle\langle\bar uu\rangle m_ss}{2048\pi^5}-\frac{g_s^3\langle g_s^2G^2\rangle\langle\bar dd\rangle^2}{110592\pi^5}-\frac{g_s^3\langle g_s^2G^2\rangle\langle\bar ss\rangle^2}{82944\pi^5}$$
$$-\frac{\langle g_s^2G^2\rangle\langle\bar uu\rangle^2}{1536\pi^3}-\frac{g_s^3\langle g_s^2G^2\rangle\langle\bar uu\rangle^2}{165888\pi^5}+\frac{\langle g_s\bar u\sigma\cdot Gu\rangle^2}{256\pi^3}-\frac{3\langle g_s^2G^2\rangle\langle\bar dd\rangle\langle\bar ss\rangle}{2048\pi^3}+\frac{9\langle g_s\bar d\sigma\cdot Gd\rangle\langle g_s\bar s\sigma\cdot Gs\rangle}{2048\pi^3}$$
$$-\frac{\langle g_s^2G^2\rangle\langle\bar ss\rangle\langle\bar uu\rangle}{768\pi^3}+\frac{\langle g_s\bar s\sigma\cdot Gs\rangle\langle g_s\bar u\sigma\cdot Gu\rangle}{128\pi^3}+\frac{g_s^2\langle\bar ss\rangle^3m_s}{576\pi^3}-\frac{g_s^2\langle\bar uu\rangle^3m_s}{864\pi^3}+\frac{3\langle\bar dd\rangle\langle\bar ss\rangle^2m_s}{64\pi}$$
$$-\frac{g_s^2\langle\bar dd\rangle\langle\bar ss\rangle^2m_s}{2304\pi^3}-\frac{3\langle\bar dd\rangle\langle\bar uu\rangle^2m_s}{32\pi}-\frac{g_s^2\langle\bar dd\rangle\langle\bar uu\rangle^2m_s}{576\pi^3}+\frac{\langle\bar ss\rangle\langle\bar uu\rangle^2m_s}{16\pi}+\frac{g_s^2\langle\bar ss\rangle\langle\bar uu\rangle^2m_s}{288\pi^3}$$
$$+\frac{15\langle g_s^2G^2\rangle\langle g_s\bar d\sigma\cdot Gd\rangle m_s}{32768\pi^5}+\frac{g_s^2\langle\bar dd\rangle^2\langle\bar ss\rangle m_s}{576\pi^3}-\frac{\langle g_s^2G^2\rangle\langle g_s\bar s\sigma\cdot Gs\rangle m_s}{12288\pi^5}-\frac{g_s^2\langle\bar dd\rangle^2\langle\bar uu\rangle m_s}{864\pi^3}+\frac{\langle\bar ss\rangle^2\langle\bar uu\rangle m_s}{16\pi}$$
$$-\frac{g_s^2\langle\bar ss\rangle^2\langle\bar uu\rangle m_s}{1728\pi^3}-\frac{3\langle\bar dd\rangle\langle\bar ss\rangle\langle\bar uu\rangle m_s}{8\pi}+\frac{\langle g_s^2G^2\rangle\langle g_s\bar u\sigma\cdot Gu\rangle m_s}{2048\pi^5}, \tag{A4}$$

$$\rho_6^q=\frac{31s^5}{440401920\pi^7}-\frac{11\langle g_s^2G^2\rangle s^3}{283115520\pi^7}+\frac{\langle\bar dd\rangle m_ss^3}{245760\pi^5}+\frac{37\langle\bar ss\rangle m_ss^3}{491520\pi^5}-\frac{\langle\bar uu\rangle m_ss^3}{12288\pi^5}+\frac{89g_s^2\langle\bar dd\rangle^2s^2}{6635520\pi^5}+\frac{89g_s^2\langle\bar ss\rangle^2s^2}{3317760\pi^5}$$
$$-\frac{3\langle\bar uu\rangle^2s^2}{20480\pi^3}+\frac{89g_s^2\langle\bar uu\rangle^2s^2}{3317760\pi^5}-\frac{\langle\bar dd\rangle\langle\bar ss\rangle s^2}{12288\pi^3}+\frac{\langle\bar dd\rangle\langle\bar uu\rangle s^2}{1536\pi^3}+\frac{5\langle\bar ss\rangle\langle\bar uu\rangle s^2}{3072\pi^3}+\frac{3\langle g_s\bar d\sigma\cdot Gd\rangle m_ss^2}{131072\pi^5}$$
$$+\frac{65\langle g_s\bar s\sigma\cdot Gs\rangle m_ss^2}{393216\pi^5}-\frac{25\langle g_s\bar u\sigma\cdot Gu\rangle m_ss^2}{65536\pi^5}-\frac{\langle g_s\bar d\sigma\cdot Gd\rangle\langle\bar ss\rangle s}{4096\pi^3}-\frac{\langle\bar dd\rangle\langle g_s\bar s\sigma\cdot Gs\rangle s}{4608\pi^3}+\frac{13\langle g_s\bar d\sigma\cdot Gd\rangle\langle\bar uu\rangle s}{24576\pi^3}$$
$$+\frac{113\langle g_s\bar s\sigma\cdot Gs\rangle\langle\bar uu\rangle s}{36864\pi^3}+\frac{17\langle\bar dd\rangle\langle g_s\bar u\sigma\cdot Gu\rangle s}{24576\pi^3}+\frac{25\langle\bar ss\rangle\langle g_s\bar u\sigma\cdot Gu\rangle s}{6144\pi^3}-\frac{25\langle\bar uu\rangle\langle g_s\bar u\sigma\cdot Gu\rangle s}{36864\pi^3}$$
$$+\frac{13\langle g_s^2G^2\rangle\langle\bar dd\rangle m_ss}{294912\pi^5}-\frac{19\langle g_s^2G^2\rangle\langle\bar ss\rangle m_ss}{589824\pi^5}-\frac{5\langle g_s^2G^2\rangle\langle\bar uu\rangle m_ss}{18432\pi^5}-\frac{59g_s^3\langle g_s^2G^2\rangle\langle\bar dd\rangle^2}{11943936\pi^5}-\frac{25g_s^3\langle g_s^2G^2\rangle\langle\bar ss\rangle^2}{7962624\pi^5}$$
$$-\frac{5\langle g_s^2G^2\rangle\langle\bar uu\rangle^2}{73728\pi^3}+\frac{13g_s^3\langle g_s^2G^2\rangle\langle\bar uu\rangle^2}{5971968\pi^5}-\frac{11\langle g_s\bar u\sigma\cdot Gu\rangle^2}{36864\pi^3}-\frac{7\langle g_s^2G^2\rangle\langle\bar dd\rangle\langle\bar ss\rangle}{36864\pi^3}-\frac{5\langle g_s\bar d\sigma\cdot Gd\rangle\langle g_s\bar s\sigma\cdot Gs\rangle}{24576\pi^3}$$

$$-\frac{\langle g_s^2G^2\rangle\langle\bar{d}d\rangle\langle\bar{u}u\rangle}{4608\pi^3}+\frac{25\langle g_s^2G^2\rangle\langle\bar{s}s\rangle\langle\bar{u}u\rangle}{18432\pi^3}-\frac{\langle g_s\bar{d}\sigma\cdot Gd\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle}{36864\pi^3}+\frac{3\langle g_s\bar{s}\sigma\cdot Gs\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle}{1024\pi^3}$$
$$+\frac{5g_s^2\langle\bar{s}s\rangle^3m_s}{13824\pi^3}-\frac{5g_s^2\langle\bar{u}u\rangle^3m_s}{10368\pi^3}-\frac{\langle\bar{d}d\rangle\langle\bar{s}s\rangle^2m_s}{768\pi}+\frac{g_s^2\langle\bar{d}d\rangle\langle\bar{s}s\rangle^2m_s}{82944\pi^3}-\frac{11\langle\bar{d}d\rangle\langle\bar{u}u\rangle^2m_s}{384\pi}+\frac{g_s^2\langle\bar{d}d\rangle\langle\bar{u}u\rangle^2m_s}{20736\pi^3}$$
$$-\frac{5\langle\bar{s}s\rangle\langle\bar{u}u\rangle^2m_s}{1152\pi}+\frac{5g_s^2\langle\bar{s}s\rangle\langle\bar{u}u\rangle^2m_s}{6912\pi^3}+\frac{\langle g_s^2G^2\rangle\langle g_s\bar{d}\sigma\cdot Gd\rangle m_s}{32768\pi^5}+\frac{5g_s^2\langle\bar{d}d\rangle^2\langle\bar{s}s\rangle m_s}{13824\pi^3}$$
$$-\frac{25\langle g_s^2G^2\rangle\langle g_s\bar{s}\sigma\cdot Gs\rangle m_s}{1179648\pi^5}-\frac{5g_s^2\langle\bar{d}d\rangle^2\langle\bar{u}u\rangle m_s}{10368\pi^3}+\frac{5\langle\bar{s}s\rangle^2\langle\bar{u}u\rangle m_s}{192\pi}-\frac{5g_s^2\langle\bar{s}s\rangle^2\langle\bar{u}u\rangle m_s}{20736\pi^3}+\frac{5\langle\bar{d}d\rangle\langle\bar{s}s\rangle\langle\bar{u}u\rangle m_s}{288\pi}$$
$$-\frac{5\langle g_s^2G^2\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle m_s}{32768\pi^5}, \tag{A5}$$

$$\rho_7^q=\frac{3s^5}{18350080\pi^7}+\frac{19\langle g_s^2G^2\rangle s^3}{7864320\pi^7}-\frac{7\langle\bar{d}d\rangle m_s s^3}{40960\pi^5}+\frac{7\langle\bar{s}s\rangle m_s s^3}{40960\pi^5}-\frac{\langle\bar{u}u\rangle m_s s^3}{4096\pi^5}+\frac{11g_s^2\langle\bar{d}d\rangle^2s^2}{368640\pi^5}+\frac{11g_s^2\langle\bar{s}s\rangle^2s^2}{184320\pi^5}$$
$$+\frac{5\langle\bar{u}u\rangle^2s^2}{2048\pi^3}+\frac{11g_s^2\langle\bar{u}u\rangle^2s^2}{184320\pi^5}+\frac{33\langle\bar{d}d\rangle\langle\bar{s}s\rangle s^2}{10240\pi^3}+\frac{5\langle\bar{s}s\rangle\langle\bar{u}u\rangle s^2}{1024\pi^3}-\frac{99\langle g_s\bar{d}\sigma\cdot Gd\rangle m_s s^2}{163840\pi^5}+\frac{3\langle g_s\bar{s}\sigma\cdot Gs\rangle m_s s^2}{81920\pi^5}$$
$$-\frac{35\langle g_s\bar{u}\sigma\cdot Gu\rangle m_s s^2}{32768\pi^5}+\frac{3\langle g_s\bar{d}\sigma\cdot Gd\rangle\langle\bar{s}s\rangle s}{512\pi^3}+\frac{3\langle\bar{d}d\rangle\langle g_s\bar{s}\sigma\cdot Gs\rangle s}{512\pi^3}+\frac{35\langle g_s\bar{s}\sigma\cdot Gs\rangle\langle\bar{u}u\rangle s}{3072\pi^3}$$
$$+\frac{35\langle\bar{s}s\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle s}{3072\pi^3}+\frac{35\langle\bar{u}u\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle s}{3072\pi^3}+\frac{3\langle g_s^2G^2\rangle\langle\bar{d}d\rangle m_s s}{8192\pi^5}+\frac{\langle g_s^2G^2\rangle\langle\bar{s}s\rangle m_s s}{24576\pi^5}-\frac{35\langle g_s^2G^2\rangle\langle\bar{u}u\rangle m_s s}{24576\pi^5}$$
$$-\frac{25g_s^3\langle g_s^2G^2\rangle\langle\bar{d}d\rangle^2}{1327104\pi^5}-\frac{5g_s^3\langle g_s^2G^2\rangle\langle\bar{s}s\rangle^2}{1990656\pi^5}+\frac{5\langle g_s^2G^2\rangle\langle\bar{u}u\rangle^2}{1536\pi^3}+\frac{65g_s^3\langle g_s^2G^2\rangle\langle\bar{u}u\rangle^2}{1990656\pi^5}+\frac{5\langle g_s\bar{u}\sigma\cdot Gu\rangle^2}{1024\pi^3}$$
$$-\frac{5\langle g_s^2G^2\rangle\langle\bar{d}d\rangle\langle\bar{s}s\rangle}{4096\pi^3}+\frac{15\langle g_s\bar{d}\sigma\cdot Gd\rangle\langle g_s\bar{s}\sigma\cdot Gs\rangle}{4096\pi^3}+\frac{5\langle g_s^2G^2\rangle\langle\bar{s}s\rangle\langle\bar{u}u\rangle}{768\pi^3}+\frac{5\langle g_s\bar{s}\sigma\cdot Gs\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle}{512\pi^3}$$
$$+\frac{5g_s^2\langle\bar{s}s\rangle^3m_s}{6912\pi^3}-\frac{5g_s^2\langle\bar{u}u\rangle^3m_s}{3456\pi^3}+\frac{5\langle\bar{d}d\rangle\langle\bar{s}s\rangle^2m_s}{128\pi}-\frac{5g_s^2\langle\bar{d}d\rangle\langle\bar{s}s\rangle^2m_s}{13824\pi^3}$$
$$-\frac{5\langle\bar{d}d\rangle\langle\bar{u}u\rangle^2m_s}{64\pi}-\frac{5g_s^2\langle\bar{d}d\rangle\langle\bar{u}u\rangle^2m_s}{3456\pi^3}+\frac{5\langle\bar{s}s\rangle\langle\bar{u}u\rangle^2m_s}{64\pi}+\frac{5g_s^2\langle\bar{s}s\rangle\langle\bar{u}u\rangle^2m_s}{3456\pi^3}+\frac{25\langle g_s^2G^2\rangle\langle g_s\bar{d}\sigma\cdot Gd\rangle m_s}{65536\pi^5}$$
$$+\frac{5g_s^2\langle\bar{d}d\rangle^2\langle\bar{s}s\rangle m_s}{6912\pi^3}-\frac{5\langle g_s^2G^2\rangle\langle g_s\bar{s}\sigma\cdot Gs\rangle m_s}{294912\pi^5}-\frac{5g_s^2\langle\bar{d}d\rangle^2\langle\bar{u}u\rangle m_s}{3456\pi^3}+\frac{5\langle\bar{s}s\rangle^2\langle\bar{u}u\rangle m_s}{64\pi}-\frac{5g_s^2\langle\bar{s}s\rangle^2\langle\bar{u}u\rangle m_s}{6912\pi^3}$$
$$-\frac{5\langle\bar{d}d\rangle\langle\bar{s}s\rangle\langle\bar{u}u\rangle m_s}{16\pi}-\frac{15\langle g_s^2G^2\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle m_s}{16384\pi^5}. \tag{A6}$$

The spectral densities ρ_i^m are

$$\rho_2^m=\frac{s^4\langle\bar{d}d\rangle}{245760\pi^5}-\frac{s^2m_s\langle\bar{d}d\rangle\langle\bar{s}s\rangle}{256\pi^3}-\frac{s^2\langle\bar{d}d\rangle\langle g_s^2G^2\rangle}{18432\pi^5}+\frac{s\langle\bar{d}d\rangle\langle\bar{u}u\rangle^2}{48\pi}+\frac{s\langle\bar{d}d\rangle\langle\bar{s}s\rangle^2}{48\pi}-\frac{3s\langle g_s^2G^2\rangle\langle g_s\bar{d}\sigma\cdot Gd\rangle}{32768\pi^5}$$
$$-\frac{5sm_s\langle\bar{d}d\rangle\langle g_s\bar{s}\sigma\cdot Gs\rangle}{768\pi^3}+\frac{g_s^2s\langle\bar{d}d\rangle\langle\bar{u}u\rangle^2}{5184\pi^3}+\frac{g_s^2s\langle\bar{d}d\rangle\langle\bar{s}s\rangle^2}{5184\pi^3}+\frac{m_s\langle\bar{d}d\rangle\langle\bar{s}s\rangle\langle g_s^2G^2\rangle}{4608\pi^3}-\frac{\langle\bar{d}d\rangle\langle g_s^2G^2\rangle^2}{196608\pi^5}$$
$$+\frac{g_s^2\langle\bar{u}u\rangle^2\langle g_s\bar{d}\sigma\cdot Gd\rangle}{20736\pi^3}+\frac{m_s\langle g_s\bar{d}\sigma\cdot Gd\rangle\langle g_s\bar{s}\sigma\cdot Gs\rangle}{512\pi^3}+\frac{\langle\bar{d}d\rangle\langle\bar{s}s\rangle\langle g_s\bar{s}\sigma\cdot Gs\rangle}{48\pi}+\frac{\langle\bar{u}u\rangle^2\langle g_s\bar{d}\sigma\cdot Gd\rangle}{96\pi}$$
$$+\frac{\langle\bar{d}d\rangle\langle\bar{u}u\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle}{48\pi}, \tag{A7}$$

$$\rho_3^m = \frac{5s^2 m_s\langle\bar{d}d\rangle\langle\bar{u}u\rangle}{4608\pi^3} - \frac{s^2 m_s\langle\bar{d}d\rangle\langle\bar{s}s\rangle}{4608\pi^3} - \frac{5s\langle\bar{d}d\rangle\langle\bar{s}s\rangle\langle\bar{u}u\rangle}{576\pi} + \frac{s\langle\bar{d}d\rangle\langle\bar{s}s\rangle^2}{1152\pi} \frac{5m_s\langle\bar{d}d\rangle\langle\bar{u}u\rangle\langle g_s^2G^2\rangle}{6912\pi^3}$$
$$- \frac{m_s\langle\bar{d}d\rangle\langle\bar{s}s\rangle\langle g_s^2G^2\rangle}{6912\pi^3} + \frac{m_s\langle g_s\bar{d}\sigma\cdot Gd\rangle\langle g_s\bar{s}\sigma\cdot Gs\rangle}{6144\pi^3} + \frac{sm_s\langle\bar{u}u\rangle\langle g_s\bar{d}\sigma\cdot Gd\rangle}{512\pi^3} + \frac{7sm_s\langle\bar{d}d\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle}{3072\pi^3}$$
$$+ \frac{m_s\langle g_s\bar{d}\sigma\cdot Gd\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle}{768\pi^3} - \frac{\langle\bar{s}s\rangle\langle\bar{u}u\rangle\langle g_s\bar{d}\sigma\cdot Gd\rangle}{192\pi} - \frac{5\langle\bar{d}d\rangle\langle\bar{u}u\rangle\langle g_s\bar{s}\sigma\cdot Gs\rangle}{1152\pi} - \frac{7\langle\bar{d}d\rangle\langle\bar{s}s\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle}{1152\pi}, \tag{A8}$$

$$\rho_4^m = \frac{11m_s s^5}{117964800\pi^7} - \frac{11\langle\bar{s}s\rangle s^4}{2949120\pi^5} - \frac{\langle g_s\bar{s}\sigma\cdot Gs\rangle s^3}{589824\pi^5} + \frac{\langle g_s^2G^2\rangle m_s s^3}{1048576\pi^7} - \frac{19\langle g_s^2G^2\rangle\langle\bar{s}s\rangle s^2}{884736\pi^5}$$
$$+ \frac{11g_s^2\langle\bar{d}d\rangle^2 m_s s^2}{995328\pi^5} - \frac{11\langle\bar{s}s\rangle^2 m_s s^2}{18432\pi^3} + \frac{11g_s^2\langle\bar{s}s\rangle^2 m_s s^2}{1990656\pi^5} - \frac{\langle\bar{u}u\rangle^2 m_s s^2}{9216\pi^3} + \frac{11g_s^2\langle\bar{u}u\rangle^2 m_s s^2}{497664\pi^5} - \frac{\langle\bar{d}d\rangle\langle\bar{s}s\rangle m_s s^2}{2304\pi^3}$$
$$+ \frac{5\langle\bar{d}d\rangle\langle\bar{u}u\rangle m_s s^2}{2304\pi^3} + \frac{5\langle\bar{s}s\rangle\langle\bar{u}u\rangle m_s s^2}{2304\pi^3} - \frac{11g_s^2\langle\bar{s}s\rangle^3 s}{124416\pi^3} + \frac{\langle\bar{d}d\rangle\langle\bar{s}s\rangle^2 s}{576\pi} + \frac{\langle\bar{s}s\rangle\langle\bar{u}u\rangle^2 s}{1152\pi} - \frac{11g_s^2\langle\bar{s}s\rangle\langle\bar{u}u\rangle^2 s}{62208\pi^3}$$
$$- \frac{11g_s^2\langle\bar{d}d\rangle^2\langle\bar{s}s\rangle s}{124416\pi^3} - \frac{3\langle g_s^2G^2\rangle\langle g_s\bar{s}\sigma\cdot Gs\rangle s}{131072\pi^5} - \frac{5\langle\bar{s}s\rangle^2\langle\bar{u}u\rangle s}{576\pi} - \frac{5\langle\bar{d}d\rangle\langle\bar{s}s\rangle\langle\bar{u}u\rangle s}{288\pi} - \frac{\langle g_s\bar{d}\sigma\cdot Gd\rangle\langle\bar{s}s\rangle m_s s}{1536\pi^3}$$
$$+ \frac{\langle\bar{d}d\rangle\langle g_s\bar{s}\sigma\cdot Gs\rangle m_s s}{6144\pi^3} - \frac{47\langle\bar{s}s\rangle\langle g_s\bar{s}\sigma\cdot Gs\rangle m_s s}{73728\pi^3} + \frac{5\langle g_s\bar{d}\sigma\cdot Gd\rangle\langle\bar{u}u\rangle m_s s}{1536\pi^3} + \frac{11\langle g_s\bar{s}\sigma\cdot Gs\rangle\langle\bar{u}u\rangle m_s s}{6144\pi^3}$$
$$+ \frac{7\langle\bar{d}d\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle m_s s}{1536\pi^3} + \frac{11\langle\bar{s}s\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle m_s s}{3072\pi^3} - \frac{\langle\bar{u}u\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle m_s s}{6144\pi^3} + \frac{\langle g_s\bar{d}\sigma\cdot Gd\rangle\langle\bar{s}s\rangle^2}{1152\pi}$$
$$- \frac{7g_s^2\langle g_s\bar{s}\sigma\cdot Gs\rangle\langle\bar{u}u\rangle^2}{248832\pi^3} - \frac{\langle g_s^2G^2\rangle^2\langle\bar{s}s\rangle}{786432\pi^5} - \frac{13g_s^2\langle\bar{d}d\rangle^2\langle g_s\bar{s}\sigma\cdot Gs\rangle}{497664\pi^3} - \frac{g_s^2\langle\bar{s}s\rangle^2\langle g_s\bar{s}\sigma\cdot Gs\rangle}{497664\pi^3} - \frac{\langle\bar{d}d\rangle\langle\bar{s}s\rangle\langle g_s\bar{s}\sigma\cdot Gs\rangle}{2304\pi}$$
$$- \frac{5\langle g_s\bar{d}\sigma\cdot Gd\rangle\langle\bar{s}s\rangle\langle\bar{u}u\rangle}{576\pi} - \frac{11\langle\bar{d}d\rangle\langle g_s\bar{s}\sigma\cdot Gs\rangle\langle\bar{u}u\rangle}{1152\pi} - \frac{11\langle\bar{s}s\rangle\langle g_s\bar{s}\sigma\cdot Gs\rangle\langle\bar{u}u\rangle}{2304\pi} - \frac{11\langle\bar{s}s\rangle^2\langle g_s\bar{u}\sigma\cdot Gu\rangle}{2304\pi}$$
$$- \frac{7\langle\bar{d}d\rangle\langle\bar{s}s\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle}{576\pi} + \frac{\langle\bar{s}s\rangle\langle\bar{u}u\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle}{2304\pi} + \frac{g_s^3\langle g_s^2G^2\rangle\langle\bar{d}d\rangle^2 m_s}{221184\pi^5} + \frac{25\langle g_s^2G^2\rangle\langle\bar{s}s\rangle^2 m_s}{221184\pi^3}$$
$$- \frac{11g_s^3\langle g_s^2G^2\rangle\langle\bar{s}s\rangle^2 m_s}{2654208\pi^5} + \frac{31\langle g_s\bar{s}\sigma\cdot Gs\rangle^2 m_s}{36864\pi^3} + \frac{\langle g_s^2G^2\rangle\langle\bar{u}u\rangle^2 m_s}{110592\pi^3} + \frac{g_s^3\langle g_s^2G^2\rangle\langle\bar{u}u\rangle^2 m_s}{331776\pi^5} - \frac{\langle g_s^2G^2\rangle\langle\bar{d}d\rangle\langle\bar{s}s\rangle m_s}{3456\pi^3}$$
$$+ \frac{\langle g_s\bar{d}\sigma\cdot Gd\rangle\langle g_s\bar{s}\sigma\cdot Gs\rangle m_s}{12288\pi^3} + \frac{5\langle g_s^2G^2\rangle\langle\bar{d}d\rangle\langle\bar{u}u\rangle m_s}{3456\pi^3} - \frac{5\langle g_s^2G^2\rangle\langle\bar{s}s\rangle\langle\bar{u}u\rangle m_s}{27648\pi^3} + \frac{7\langle g_s\bar{d}\sigma\cdot Gd\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle m_s}{3072\pi^3}$$
$$+ \frac{13\langle g_s\bar{s}\sigma\cdot Gs\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle m_s}{12288\pi^3}, \tag{A9}$$

$$\rho_5^m = \frac{3m_s s^5}{3276800\pi^7} - \frac{3\langle\bar{s}s\rangle s^4}{81920\pi^5} - \frac{3\langle\bar{u}u\rangle s^4}{40960\pi^5} - \frac{\langle g_s\bar{s}\sigma\cdot Gs\rangle s^3}{4096\pi^5} - \frac{\langle g_s\bar{u}\sigma\cdot Gu\rangle s^3}{2048\pi^5} - \frac{\langle g_s^2G^2\rangle m_s s^3}{262144\pi^7} + \frac{g_s^2\langle\bar{d}d\rangle^2 m_s s^2}{9216\pi^5}$$
$$- \frac{3\langle\bar{s}s\rangle^2 m_s s^2}{512\pi^3} + \frac{g_s^2\langle\bar{s}s\rangle^2 m_s s^2}{18432\pi^5} + \frac{3\langle\bar{u}u\rangle^2 m_s s^2}{256\pi^3} + \frac{g_s^2\langle\bar{u}u\rangle^2 m_s s^2}{4608\pi^5} + \frac{\langle\bar{d}d\rangle\langle\bar{s}s\rangle m_s s^2}{64\pi^3} + \frac{\langle\bar{d}d\rangle\langle\bar{u}u\rangle m_s s^2}{64\pi^3}$$
$$- \frac{3\langle\bar{s}s\rangle\langle\bar{u}u\rangle m_s s^2}{128\pi^3} - \frac{g_s^2\langle\bar{s}s\rangle^3 s}{1152\pi^3} - \frac{g_s^2\langle\bar{u}u\rangle^3 s}{576\pi^3} - \frac{\langle\bar{d}d\rangle\langle\bar{s}s\rangle^2 s}{16\pi} - \frac{3\langle\bar{s}s\rangle\langle\bar{u}u\rangle^2 s}{32\pi} - \frac{g_s^2\langle\bar{s}s\rangle\langle\bar{u}u\rangle^2 s}{576\pi^3} - \frac{g_s^2\langle\bar{d}d\rangle^2\langle\bar{s}s\rangle s}{1152\pi^3}$$
$$+ \frac{3\langle g_s^2G^2\rangle\langle g_s\bar{s}\sigma\cdot Gs\rangle s}{32768\pi^5} - \frac{g_s^2\langle\bar{d}d\rangle^2\langle\bar{u}u\rangle s}{576\pi^3} - \frac{g_s^2\langle\bar{s}s\rangle^2\langle\bar{u}u\rangle s}{288\pi^3} - \frac{\langle\bar{d}d\rangle\langle\bar{s}s\rangle\langle\bar{u}u\rangle s}{8\pi} + \frac{3\langle g_s^2G^2\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle s}{16384\pi^5}$$
$$+ \frac{3\langle g_s\bar{d}\sigma\cdot Gd\rangle\langle\bar{s}s\rangle m_s s}{128\pi^3} + \frac{7\langle\bar{d}d\rangle\langle g_s\bar{s}\sigma\cdot Gs\rangle m_s s}{256\pi^3} - \frac{9\langle\bar{s}s\rangle\langle g_s\bar{s}\sigma\cdot Gs\rangle m_s s}{512\pi^3} + \frac{3\langle g_s\bar{d}\sigma\cdot Gd\rangle\langle\bar{u}u\rangle m_s s}{128\pi^3}$$

$$
-\frac{3\langle g_s\bar{s}\sigma\cdot Gs\rangle\langle\bar{u}u\rangle m_s s}{128\pi^3}+\frac{\langle\bar{d}d\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle m_s s}{32\pi^3}-\frac{45\langle\bar{s}s\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle m_s s}{1024\pi^3}+\frac{9\langle\bar{u}u\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle m_s s}{256\pi^3}
$$
$$
-\frac{\langle g_s\bar{d}\sigma\cdot Gd\rangle\langle\bar{s}s\rangle^2}{32\pi}-\frac{3\langle g_s\bar{s}\sigma\cdot Gs\rangle\langle\bar{u}u\rangle^2}{64\pi}-\frac{7g_s^2\langle g_s\bar{s}\sigma\cdot Gs\rangle\langle\bar{u}u\rangle^2}{6912\pi^3}+\frac{\langle g_s^2G^2\rangle^2\langle\bar{s}s\rangle}{196608\pi^5}-\frac{g_s^2\langle\bar{d}d\rangle^2\langle g_s\bar{s}\sigma\cdot Gs\rangle}{1728\pi^3}
$$
$$
-\frac{g_s^2\langle\bar{s}s\rangle^2\langle g_s\bar{s}\sigma\cdot Gs\rangle}{1728\pi^3}-\frac{7\langle\bar{d}d\rangle\langle\bar{s}s\rangle\langle g_s\bar{s}\sigma\cdot Gs\rangle}{96\pi}+\frac{\langle g_s^2G^2\rangle^2\langle\bar{u}u\rangle}{98304\pi^5}-\frac{\langle g_s\bar{d}\sigma\cdot Gd\rangle\langle\bar{s}s\rangle\langle\bar{u}u\rangle}{16\pi}
$$
$$
-\frac{\langle\bar{d}d\rangle\langle g_s\bar{s}\sigma\cdot Gs\rangle\langle\bar{u}u\rangle}{16\pi}-\frac{g_s^2\langle\bar{d}d\rangle^2\langle g_s\bar{u}\sigma\cdot Gu\rangle}{864\pi^3}-\frac{5g_s^2\langle\bar{s}s\rangle^2\langle g_s\bar{u}\sigma\cdot Gu\rangle}{2304\pi^3}-\frac{7g_s^2\langle\bar{u}u\rangle^2\langle g_s\bar{u}\sigma\cdot Gu\rangle}{6912\pi^3}
$$
$$
-\frac{\langle\bar{d}d\rangle\langle\bar{s}s\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle}{12\pi}-\frac{3\langle\bar{s}s\rangle\langle\bar{u}u\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle}{32\pi}-\frac{g_s^3\langle g_s^2G^2\rangle\langle\bar{d}d\rangle^2m_s}{331776\pi^5}-\frac{\langle g_s^2G^2\rangle\langle\bar{s}s\rangle^2m_s}{3072\pi^3}
$$
$$
+\frac{g_s^3\langle g_s^2G^2\rangle\langle\bar{s}s\rangle^2m_s}{663552\pi^5}-\frac{5\langle g_s\bar{s}\sigma\cdot Gs\rangle^2m_s}{1024\pi^3}-\frac{\langle g_s^2G^2\rangle\langle\bar{u}u\rangle^2m_s}{256\pi^3}-\frac{g_s^3\langle g_s^2G^2\rangle\langle\bar{u}u\rangle^2m_s}{82944\pi^5}+\frac{9\langle g_s\bar{u}\sigma\cdot Gu\rangle^2m_s}{1024\pi^3}
$$
$$
-\frac{\langle g_s^2G^2\rangle\langle\bar{d}d\rangle\langle\bar{s}s\rangle m_s}{768\pi^3}+\frac{7\langle g_s\bar{d}\sigma\cdot Gd\rangle\langle g_s\bar{s}\sigma\cdot Gs\rangle m_s}{512\pi^3}-\frac{\langle g_s^2G^2\rangle\langle\bar{d}d\rangle\langle\bar{u}u\rangle m_s}{768\pi^3}-\frac{\langle g_s^2G^2\rangle\langle\bar{s}s\rangle\langle\bar{u}u\rangle m_s}{1024\pi^3}
$$
$$
+\frac{\langle g_s\bar{d}\sigma\cdot Gd\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle m_s}{64\pi^3}-\frac{9\langle g_s\bar{s}\sigma\cdot Gs\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle m_s}{512\pi^3}. \tag{A10}
$$

$$
\rho_6^m=\frac{11m_s s^5}{45875200\pi^7}-\frac{11\langle\bar{s}s\rangle s^4}{1179648\pi^5}-\frac{\langle g_s\bar{s}\sigma\cdot Gs\rangle s^3}{983040\pi^5}-\frac{3\langle g_s^2G^2\rangle m_s s^3}{524288\pi^7}+\frac{5\langle g_s^2G^2\rangle\langle\bar{s}s\rangle s^2}{65536\pi^5}+\frac{11g_s^2\langle\bar{d}d\rangle^2m_s s^2}{442368\pi^5}
$$
$$
-\frac{11\langle\bar{s}s\rangle^2m_s s^2}{8192\pi^3}+\frac{11g_s^2\langle\bar{s}s\rangle^2m_s s^2}{884736\pi^5}-\frac{\langle\bar{u}u\rangle^2m_s s^2}{4096\pi^3}+\frac{11g_s^2\langle\bar{u}u\rangle^2m_s s^2}{221184\pi^5}-\frac{\langle\bar{d}d\rangle\langle\bar{s}s\rangle m_s s^2}{768\pi^3}+\frac{5\langle\bar{d}d\rangle\langle\bar{u}u\rangle m_s s^2}{1024\pi^3}
$$
$$
+\frac{5\langle\bar{s}s\rangle\langle\bar{u}u\rangle m_s s^2}{768\pi^3}-\frac{11g_s^2\langle\bar{s}s\rangle^3 s}{62208\pi^3}+\frac{\langle\bar{d}d\rangle\langle\bar{s}s\rangle^2 s}{192\pi}+\frac{\langle\bar{s}s\rangle\langle\bar{u}u\rangle^2 s}{576\pi}-\frac{11g_s^2\langle\bar{s}s\rangle\langle\bar{u}u\rangle^2 s}{31104\pi^3}-\frac{11g_s^2\langle\bar{d}d\rangle^2\langle\bar{s}s\rangle s}{62208\pi^3}
$$
$$
+\frac{13\langle g_s^2G^2\rangle\langle g_s\bar{s}\sigma\cdot Gs\rangle s}{98304\pi^5}-\frac{5\langle\bar{s}s\rangle^2\langle\bar{u}u\rangle s}{192\pi}-\frac{5\langle\bar{d}d\rangle\langle\bar{s}s\rangle\langle\bar{u}u\rangle s}{144\pi}-\frac{\langle g_s\bar{d}\sigma\cdot Gd\rangle\langle\bar{s}s\rangle m_s s}{512\pi^3}
$$
$$
-\frac{31\langle\bar{d}d\rangle\langle g_s\bar{s}\sigma\cdot Gs\rangle m_s s}{18432\pi^3}-\frac{47\langle\bar{s}s\rangle\langle g_s\bar{s}\sigma\cdot Gs\rangle m_s s}{36864\pi^3}+\frac{5\langle g_s\bar{d}\sigma\cdot Gd\rangle\langle\bar{u}u\rangle m_s s}{768\pi^3}+\frac{89\langle g_s\bar{s}\sigma\cdot Gs\rangle\langle\bar{u}u\rangle m_s s}{18432\pi^3}
$$
$$
+\frac{\langle\bar{d}d\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle m_s s}{144\pi^3}+\frac{11\langle\bar{s}s\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle m_s s}{1024\pi^3}-\frac{13\langle\bar{u}u\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle m_s s}{9216\pi^3}+\frac{\langle g_s\bar{d}\sigma\cdot Gd\rangle\langle\bar{s}s\rangle^2}{384\pi}
$$
$$
+\frac{\langle g_s\bar{s}\sigma\cdot Gs\rangle\langle\bar{u}u\rangle^2}{1536\pi}-\frac{5g_s^2\langle g_s\bar{s}\sigma\cdot Gs\rangle\langle\bar{u}u\rangle^2}{165888\pi^3}+\frac{11\langle g_s^2G^2\rangle^2\langle\bar{s}s\rangle}{1572864\pi^5}-\frac{11g_s^2\langle\bar{d}d\rangle^2\langle g_s\bar{s}\sigma\cdot Gs\rangle}{331776\pi^3}-\frac{g_s^2\langle\bar{s}s\rangle^2\langle g_s\bar{s}\sigma\cdot Gs\rangle}{331776\pi^3}
$$
$$
+\frac{\langle\bar{d}d\rangle\langle\bar{s}s\rangle\langle g_s\bar{s}\sigma\cdot Gs\rangle}{256\pi}-\frac{5\langle g_s\bar{d}\sigma\cdot Gd\rangle\langle\bar{s}s\rangle\langle\bar{u}u\rangle}{384\pi}-\frac{5\langle\bar{d}d\rangle\langle g_s\bar{s}\sigma\cdot Gs\rangle\langle\bar{u}u\rangle}{384\pi}-\frac{5\langle\bar{s}s\rangle\langle g_s\bar{s}\sigma\cdot Gs\rangle\langle\bar{u}u\rangle}{384\pi}
$$
$$
-\frac{11\langle\bar{s}s\rangle^2\langle g_s\bar{u}\sigma\cdot Gu\rangle}{768\pi}-\frac{5\langle\bar{d}d\rangle\langle\bar{s}s\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle}{384\pi}+\frac{5\langle\bar{s}s\rangle\langle\bar{u}u\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle}{1536\pi}-\frac{11g_s^3\langle g_s^2G^2\rangle\langle\bar{d}d\rangle^2m_s}{1327104\pi^5}
$$
$$
+\frac{25\langle g_s^2G^2\rangle\langle\bar{s}s\rangle^2m_s}{147456\pi^3}+\frac{g_s^3\langle g_s^2G^2\rangle\langle\bar{s}s\rangle^2m_s}{589824\pi^5}-\frac{\langle g_s\bar{s}\sigma\cdot Gs\rangle^2m_s}{768\pi^3}-\frac{5\langle g_s^2G^2\rangle\langle\bar{u}u\rangle^2m_s}{73728\pi^3}-\frac{g_s^3\langle g_s^2G^2\rangle\langle\bar{u}u\rangle^2m_s}{82944\pi^5}
$$
$$
-\frac{\langle g_s\bar{u}\sigma\cdot Gu\rangle^2m_s}{2048\pi^3}-\frac{\langle g_s^2G^2\rangle\langle\bar{d}d\rangle\langle\bar{s}s\rangle m_s}{1152\pi^3}-\frac{3\langle g_s\bar{d}\sigma\cdot Gd\rangle\langle g_s\bar{s}\sigma\cdot Gs\rangle m_s}{4096\pi^3}-\frac{5\langle g_s^2G^2\rangle\langle\bar{d}d\rangle\langle\bar{u}u\rangle m_s}{4608\pi^3}
$$
$$
-\frac{5\langle g_s^2G^2\rangle\langle\bar{s}s\rangle\langle\bar{u}u\rangle m_s}{9216\pi^3}+\frac{5\langle g_s\bar{d}\sigma\cdot Gd\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle m_s}{2048\pi^3}+\frac{3\langle g_s\bar{s}\sigma\cdot Gs\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle m_s}{1024\pi^3}, \tag{A11}
$$

$$
\begin{aligned}
\rho_7^m = & \frac{31m_s s^5}{45875200\pi^7} - \frac{13\langle\bar{s}s\rangle s^4}{491520\pi^5} - \frac{13\langle\bar{u}u\rangle s^4}{245760\pi^5} - \frac{7\langle g_s\bar{s}\sigma\cdot Gs\rangle s^3}{40960\pi^5} - \frac{7\langle g_s\bar{u}\sigma\cdot Gu\rangle s^3}{20480\pi^5} + \frac{29\langle g_s^2G^2\rangle m_s s^3}{2621440\pi^7} \\
& - \frac{5\langle g_s^2G^2\rangle\langle\bar{s}s\rangle s^2}{24576\pi^5} - \frac{5\langle g_s^2G^2\rangle\langle\bar{u}u\rangle s^2}{12288\pi^5} + \frac{g_s^2\langle\bar{d}d\rangle^2 m_s s^2}{13824\pi^5} - \frac{\langle\bar{s}s\rangle^2 m_s s^2}{256\pi^3} + \frac{g_s^2\langle\bar{s}s\rangle^2 m_s s^2}{27648\pi^5} + \frac{9\langle\bar{u}u\rangle^2 m_s s^2}{512\pi^3} \\
& + \frac{g_s^2\langle\bar{u}u\rangle^2 m_s s^2}{6912\pi^5} + \frac{\langle\bar{d}d\rangle\langle\bar{s}s\rangle m_s s^2}{64\pi^3} + \frac{\langle\bar{d}d\rangle\langle\bar{u}u\rangle m_s s^2}{64\pi^3} - \frac{\langle\bar{s}s\rangle\langle\bar{u}u\rangle m_s s^2}{64\pi^3} - \frac{11g_s^2\langle\bar{s}s\rangle^3 s}{20736\pi^3} - \frac{11g_s^2\langle\bar{u}u\rangle^3 s}{10368\pi^3} \\
& - \frac{11\langle\bar{d}d\rangle\langle\bar{s}s\rangle^2 s}{192\pi} - \frac{9\langle\bar{s}s\rangle\langle\bar{u}u\rangle^2 s}{64\pi} - \frac{11g_s^2\langle\bar{s}s\rangle\langle\bar{u}u\rangle^2 s}{10368\pi^3} - \frac{11g_s^2\langle\bar{d}d\rangle^2\langle\bar{s}s\rangle s}{20736\pi^3} - \frac{41\langle g_s^2G^2\rangle\langle g_s\bar{s}\sigma\cdot Gs\rangle s}{196608\pi^5} \\
& - \frac{11g_s^2\langle\bar{d}d\rangle^2\langle\bar{u}u\rangle s}{10368\pi^3} - \frac{11g_s^2\langle\bar{s}s\rangle^2\langle\bar{u}u\rangle s}{5184\pi^3} - \frac{11\langle\bar{d}d\rangle\langle\bar{s}s\rangle\langle\bar{u}u\rangle s}{96\pi} - \frac{41\langle g_s^2G^2\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle s}{98304\pi^5} \\
& + \frac{11\langle g_s\bar{d}\sigma\cdot Gd\rangle\langle\bar{s}s\rangle m_s s}{512\pi^3} + \frac{77\langle\bar{d}d\rangle\langle g_s\bar{s}\sigma\cdot Gs\rangle m_s s}{3072\pi^3} - \frac{11\langle\bar{s}s\rangle\langle g_s\bar{s}\sigma\cdot Gs\rangle m_s s}{1024\pi^3} + \frac{11\langle g_s\bar{d}\sigma\cdot Gd\rangle\langle\bar{u}u\rangle m_s s}{512\pi^3} \\
& - \frac{11\langle g_s\bar{s}\sigma\cdot Gs\rangle\langle\bar{u}u\rangle m_s s}{768\pi^3} + \frac{11\langle\bar{d}d\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle m_s s}{384\pi^3} - \frac{55\langle\bar{s}s\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle m_s s}{2048\pi^3} + \frac{27\langle\bar{u}u\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle m_s s}{512\pi^3} \\
& - \frac{3\langle g_s\bar{d}\sigma\cdot Gd\rangle\langle\bar{s}s\rangle^2}{128\pi} - \frac{9\langle g_s\bar{s}\sigma\cdot Gs\rangle\langle\bar{u}u\rangle^2}{128\pi} - \frac{7g_s^2\langle g_s\bar{s}\sigma\cdot Gs\rangle\langle\bar{u}u\rangle^2}{13824\pi^3} - \frac{\langle g_s^2G^2\rangle^2\langle\bar{s}s\rangle}{131072\pi^5} - \frac{g_s^2\langle\bar{d}d\rangle^2\langle g_s\bar{s}\sigma\cdot Gs\rangle}{3456\pi^3} \\
& - \frac{g_s^2\langle\bar{s}s\rangle^2\langle g_s\bar{s}\sigma\cdot Gs\rangle}{3456\pi^3} - \frac{7\langle\bar{d}d\rangle\langle\bar{s}s\rangle\langle g_s\bar{s}\sigma\cdot Gs\rangle}{128\pi} - \frac{\langle g_s^2G^2\rangle^2\langle\bar{u}u\rangle}{65536\pi^5} - \frac{3\langle g_s\bar{d}\sigma\cdot Gd\rangle\langle\bar{s}s\rangle\langle\bar{u}u\rangle}{64\pi} \\
& - \frac{3\langle\bar{d}d\rangle\langle g_s\bar{s}\sigma\cdot Gs\rangle\langle\bar{u}u\rangle}{64\pi} - \frac{g_s^2\langle\bar{d}d\rangle^2\langle g_s\bar{u}\sigma\cdot Gu\rangle}{1728\pi^3} - \frac{5g_s^2\langle\bar{s}s\rangle^2\langle g_s\bar{u}\sigma\cdot Gu\rangle}{4608\pi^3} - \frac{7g_s^2\langle\bar{u}u\rangle^2\langle g_s\bar{u}\sigma\cdot Gu\rangle}{13824\pi^3} \\
& - \frac{\langle\bar{d}d\rangle\langle\bar{s}s\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle}{16\pi} - \frac{9\langle\bar{s}s\rangle\langle\bar{u}u\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle}{64\pi} - \frac{g_s^3\langle g_s^2G^2\rangle\langle\bar{d}d\rangle^2 m_s}{73728\pi^5} + \frac{\langle g_s^2G^2\rangle\langle\bar{s}s\rangle^2 m_s}{2048\pi^3} \\
& - \frac{g_s^3\langle g_s^2G^2\rangle\langle\bar{s}s\rangle^2 m_s}{49152\pi^5} + \frac{5\langle g_s\bar{s}\sigma\cdot Gs\rangle^2 m_s}{2048\pi^3} + \frac{3\langle g_s^2G^2\rangle\langle\bar{u}u\rangle^2 m_s}{256\pi^3} + \frac{g_s^3\langle g_s^2G^2\rangle\langle\bar{u}u\rangle^2 m_s}{18432\pi^5} + \frac{27\langle g_s\bar{u}\sigma\cdot Gu\rangle^2 m_s}{2048\pi^3} \\
& - \frac{\langle g_s^2G^2\rangle\langle\bar{d}d\rangle\langle\bar{s}s\rangle m_s}{1024\pi^3} + \frac{21\langle g_s\bar{d}\sigma\cdot Gd\rangle\langle g_s\bar{s}\sigma\cdot Gs\rangle m_s}{2048\pi^3} - \frac{\langle g_s^2G^2\rangle\langle\bar{d}d\rangle\langle\bar{u}u\rangle m_s}{1024\pi^3} - \frac{5\langle g_s^2G^2\rangle\langle\bar{s}s\rangle\langle\bar{u}u\rangle m_s}{2048\pi^3} \\
& + \frac{3\langle g_s\bar{d}\sigma\cdot Gd\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle m_s}{256\pi^3} - \frac{3\langle g_s\bar{s}\sigma\cdot Gs\rangle\langle g_s\bar{u}\sigma\cdot Gu\rangle m_s}{1024\pi^3}. \qquad \text{(A12)}
\end{aligned}
$$

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