Leptogenesis via a varying Weinberg operator: a semi-classical approach*

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Abstract: In this paper, we introduce leptogenesis via a varying Weinberg operator from a semi-classical perspective. This mechanism is motivated by the breaking of an underlying symmetry which triggers a phase transition that causes the coupling of the Weinberg operator to become dynamical. Consequently, a lepton anti-lepton asymmetry arises from the interference of the Weinberg operator at two different spacetime points. Using the semi-classical approach, we treat the Higgs as a background field and show that a reflection asymmetry between leptons and anti-leptons is generated in the vicinity of the bubble wall. We solve the equations of motion of the lepton and anti-lepton quasi-particles to obtain the final lepton asymmetry.

Keywords: leptogenesis, Weinberg operator, phase transition

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1 Introduction

The origin of tiny neutrino masses and the asymmetry between baryons and anti-baryons in the Universe are two fundamental and open questions in particle physics. An important theoretical development linking both is baryogenesis via leptogenesis [1], which applies the new physics, motivated by tiny neutrino masses, to generate an asymmetry between leptons and anti-leptons. This lepton asymmetry is later converted into baryon asymmetry via weak sphaleron processes.

Recently, we proposed a new mechanism to generate the lepton asymmetry via the Weinberg operator [2] (see also [3, 4]). This operator is given by

$$\mathcal{L}_{\rm W} = -\frac{\lambda_{\alpha\beta}}{\Lambda}\ell^i_{\alpha L}\varepsilon^{ij}H^j C\ell^k_{\beta L}\varepsilon^{kl}H^l + {\rm h.c}, \tag{1}$$

where $\ell_L = (\nu_L, l_L)^T$ in the $SU(2)_{\rm L}$ gauge space, $\lambda_{\alpha\beta} = \lambda_{\beta\alpha}$ are the effective Yukawa couplings with flavour indices $\alpha,\beta = e,\mu,\tau$, and C is the charge conjugation matrix. We demonstrated that the dimension-five Weinberg operator can play a crucial role in leptogenesis without the need to specify the completion of this operator. It provides two ingredients for the leptogenesis recipe:

● The Weinberg operator violates the lepton number by two units and triggers lepton-number-violating (LNV) processes, including

$$\begin{aligned}&H^*H^* \leftrightarrow \ell\ell, \quad \bar{\ell}H^* \leftrightarrow \ell H, \quad \bar{\ell}H^*H^* \leftrightarrow \ell,\\ &\bar{\ell} \leftrightarrow \ell HH, \quad H^* \leftrightarrow \ell\ell H, \quad 0 \leftrightarrow \ell\ell HH,\end{aligned} \tag{2}$$

and their CP conjugate processes, where ℓ and H are the left-handed leptonic and Higgs doublets of the Standard Model, respectively. The CP violating phase transition occurs at much higher temperatures than the electroweak (EW) scale, and, therefore, the Higgs has not acquired a non-zero vacuum expectation value (VEV) and it is almost in thermal equilibrium.

● After electroweak symmetry breaking (EWSB), the Higgs acquires a VEV $\langle H\rangle = (0, v_H/\sqrt{2})^T$ and the neutrino mass matrix is given by

$$(m_\nu)_{\alpha\beta} = \frac{\lambda_{\alpha\beta}}{\Lambda}v_H^2. \tag{3}$$

This operator violates the lepton number and generates Majorana masses for neutrinos. As the primary motivation for the Weinberg operator is the generation of tiny

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neutrino masses, all processes triggered by this operator are very weak [5]. The rate of these LNV processes is approximately

$$\Gamma_{\rm W} \sim \frac{3}{4\pi^3}\frac{m_\nu^2}{v_H^4}T^3, \tag{4}$$

where $v_H = 246$ GeV is the Higgs VEV and $m_\nu \lesssim 0.1$ eV is the neutrino mass. For temperatures $T < 10^{13}$ GeV, as $\Gamma_{\rm W}$ is smaller than the Hubble expansion rate, $H \sim O(10)\frac{T^2}{m_{\rm Pl}}$, the LNV processes generated by the Weinberg operator are out of thermal equilibrium. Moreover, because of the smallness of the LNV rates, the washout mediated by the dimension-five operator is highly suppressed and can be safely ignored.

In our mechanism, CP violation is provided by a CP-violating phase transition (CPPT) in the very early Universe. This phase transition causes the coefficient of the Weinberg operator to be dynamically realised and to contain irremovable complex phases. Such a phase transition is strongly motivated by a variety of new symmetries such as $B-L$ and flavour symmetries. In order to generate sufficient baryon asymmetry, we found the temperature of the phase transition to be approximately 10^{11} GeV. We discussed this mechanism in [2] and calculated the lepton asymmetry using non-equilibrium field theory methods. Moreover, in our twin paper [6] , we provide some additional discussion of the influence of the phase transition dynamics, and how the particle thermal properties contribute to the mechanism.

In this paper, we present a simplified and intuitive description of this mechanism based on the semi-classical approximation. In order to do so, we follow the method introduced in [7] , where the transition between left-and right-handed fermions is calculated via a varying mass during the electroweak phase transition (EWPT)[1]. The techniques applied in [7] are particularly amenable as the baryon asymmetry is calculated by solving the equations of motion of the Green's functions of the left- and right-handed quasiparticles, where the asymmetry itself manifests by the CP violating reflections of particles off the bubble wall. The calculation is rather transparent and some of the simplifying assumptions that are made, such as a thin and fast moving bubble wall, parallel our own.

We emphasise that the CPPT mechanism works only if the UV-completion scale, Λ, is higher than the temperature of the phase transition T. If $\Lambda \lesssim T$, new lepton-number-violating particles, for example, right-handed neutrinos needed for the type-I seesaw mechanism, may be produced in the thermal bath during the phase transition. Subsequently, the phase transition may influence the leptogenesis via the decays of these particles, as is studied in [10].

We organise the remainder of this paper as follows: we first review the mechanism in Section 2; we then state the main assumptions of the semi-classical description in Section 3. Finally, we present the calculation of lepton asymmetry in Section 4 and make concluding remarks in Section 5.

2 The CPPT mechanism

The Weinberg operator of Eq. (1) is the simplest higher-dimensional operator needed to explain tiny neutrino masses. As discussed in Refs. [2, 6], in many models, the coupling of the Weinberg operator can be functionally dependent on a SM-singlet scalar, ϕ, such that $\lambda_{\alpha\beta} = \lambda^0_{\alpha\beta} + \lambda^1_{\alpha\beta}\langle\phi\rangle/v_\phi$. Associated to ϕ is a finite temperature scalar potential, which is symmetric under a $U(1)_{B-L}$ or flavour symmetry at sufficiently high temperatures. As the temperature of the Universe lowers, the minimum at the origin of this potential becomes metastable and a phase transition occurs. As a result, the minimum changes from the vacuum at the origin to a deeper, true vacuum which is stable and non-zero, $\langle\phi\rangle$, and activates the CP violating coupling coefficient, $\lambda_{\alpha\beta}$. The ensemble expectation value (EEV) of ϕ spontaneously breaks the high-scale symmetry and, if it is a flavour symmetry, results in the observed pattern of leptonic masses and mixing.

Assuming a first order phase transition, bubbles of the leptonically CP-violating broken phase nucleate. We denote the bubble wall width and bubble wall velocity as L_w and v_w, respectively. In the following calculation, we work within the bubble wall rest frame, where the bubble wall is stationary and the thermal plasma moves against the wall with a velocity $-v_w$. Inside the bubble wall, the EEV is spacetime-dependent, and, therefore, the coupling of the Weinberg operator, $\lambda_{\alpha\beta}$, must also vary with spacetime. This has as an effect that the interference of the Weinberg operator at different times produces a lepton asymmetry.

Before CPPT is triggered, there are equal amounts of leptons and anti-leptons in the thermal plasma and they are thermally distributed. Once CPPT begins, a bubble nucleates with the bubble wall separating the symmetric and broken phases, which are denoted in Fig. 1 as Phase I and II, respectively. The majority of leptons, anti-leptons and the Higgs pass through the bubble wall; however, there will be some of these particle species which reflect off the wall. As the bubble wall causes the coupling of

1) This work, along with several others [8, 9], demonstrated that the amount of CP violation within the Standard Model (SM) is not sufficient to produce the observed baryon asymmetry of the Universe (BAU).

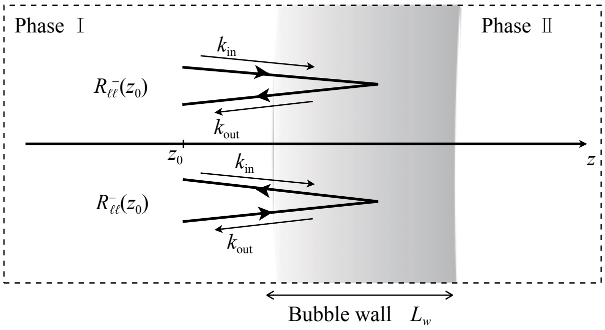

Fig. 1. Lepton and antilepton reflection off the bubble wall during a phase transition from Phase I ($\langle\phi\rangle = 0$) to Phase II ($\langle\phi\rangle = v_\phi$), in the bubble wall rest frame. We set the bubble wall perpendicular to the z direction. $R_{\ell\bar{\ell}}(z_0)$ and $R_{\bar{\ell}\ell}(z_0)$ represent the z-dependent transition amplitudes for lepton to anti-lepton and anti-lepton to lepton, respectively, at $z = z_0$.

the Weinberg operator to be CP violating, the transition from leptons to anti-leptons and from anti-leptons to leptons will be different in the presence of the bubble wall. Therefore, different amounts of anti-leptons and leptons will be produced after these scatterings. As discussed, the interactions mediated by the Weinberg operator are out of thermal equilibrium, and, therefore, LNV processes do occur but are rather rare.

We note that the coefficient of the Weinberg operator varies only along the z direction in the wall, as shown in Fig. 1. Since the PT scale is much higher than the electroweak scale, all particles are massless, and the lepton ℓ and anti-lepton $\bar{\ell}$ have helicity -1 and $+1$, respectively. To further elaborate, we consider a group of leptons, ℓ, propagating to the wall from the left hand side (Phase I). While most of the particles move freely through the wall to the right hand side (Phase II) without reflecting off the wall, a small proportion of the leptons will hit the wall and subsequently convert to anti-leptons via the Weinberg operator. Since leptons and anti-leptons have opposite helicities, $\bar{\ell}$ should move backwards to the Phase I zone. This process leads to the non-conservation of the momentum in the z direction.

We denote the amplitude for a transition from lepton to anti-lepton at $z = z_0$ as $R_{\ell\bar{\ell}}(z_0)$. Likewise, for anti-lepton to lepton at $z = z_0$, the amplitude is $R_{\bar{\ell}\ell}(z_0)$. These transitions originate from the varying Weinberg operator, and the CP asymmetry between these two processes is given by

$$\Delta_{\rm CP}(z_0) \equiv |R_{\bar{\ell}\ell}(z_0)|^2 - |R_{\ell\bar{\ell}}(z_0)|^2. \tag{5}$$

The interference of the Weinberg operators at different z can lead to non-zero CP violating effects in the thermal plasma. In this case, given an equivalent amount of initial leptons and anti-leptons propagating from the right hand side, a different amount of anti-leptons and leptons can be generated via the reflection. This asymmetry finally diffuses to Phase II and will be preserved. The number density asymmetry of leptons and antileptons is given by

$$\Delta n_\ell = \int \frac{{\rm d}^3 k}{(2\pi)^3}[f_\ell(k) - f_{\bar{\ell}}(k)] = \int \frac{{\rm d}^3 k}{2\pi} f_{\rm th}(k)\Delta_{\rm CP}(z_0), \tag{6}$$

where $f_{\rm th}(k) = \left[\exp\left(\beta\dfrac{\omega_k - v_w k_z}{\sqrt{1-v_w^2}}\right)+1\right]^{-1}$ is the Fermi-Dirac thermal distribution boosted to the wall frame.

In the following, we carry out the semi-classical approximation to relate $\Delta_{\rm CP}(z_0)$ with the varying Weinberg operator.

The CPPT mechanism shares a common feature with EWBG, namely that a phase transition is necessary to drive the generation of a baryon asymmetry. However, the two mechanisms differ markedly and it is worthwhile to remark on the features which distinguish them. First, in EWBG, the baryon number violation is provided by sphaleron transitions in the symmetric phase. Both the out-of-equilibrium condition and the C/CP violations are induced by the EW phase transition. Therefore, in EWBG, the phase transition is key to the generation of the non-equilibrium evolution. In order to achieve this, rapidly expanding bubble walls are required, such that the back-reactions are not efficient in washing out the generated baryon asymmetry. In the CPPT mechanism, the $B-L$ number violation and departure from thermodynamic equilibrium are directly provided by the very weakly coupled Weinberg operator. The PT is only necessary to provide a source of C/CP violation and is not needed for the efficiency of reactions in the system. Consequently, successful leptogenesis in this setup does not necessarily require a first-order PT, and it is possible that a CP-violating second-order PT would also generate lepton asymmetry. The purpose of assuming the first-order phase transition in this work is to simplify the discussion.

3 The semi-classical approximations

In this section, we introduce the semi-classical approximations we use for the lepton asymmetry calculation. Firstly, we introduce the equations of motion (EOM) for the leptonic doublets and the effective mass-like matrix which parametrises the lepton anti-lepton transitions. Secondly, we review our treatment of the Higgs as a background field.

3.1 Equation of motion for leptonic quasiparticles

We begin from the well-known equation of motion for Majorana neutrinos at low energy. It is expressed as

$$\begin{pmatrix} {\rm i}\sigma^\mu\partial_\mu & m_\nu \\ m_\nu^\dagger & {\rm i}\bar{\sigma}^\mu\partial_\mu \end{pmatrix}\begin{pmatrix}\chi_\nu \\ \chi_{\bar{\nu}}\end{pmatrix} = 0, \tag{7}$$

where $\nu^c_L \equiv C\overline{\nu_L}^T = (\nu^c)_R$. The Majorana mass matrix, m_ν, results in the neutrino anti-neutrino transitions and oscillations (see, e.g., [11]).

In the early Universe, when the Higgs is in its symmetric phase, the Higgs field may fluctuate. Such fluctuations can be enhanced by temperature, and influence the behaviour of neutrinos as well as of the charged leptons. For this reason, we treat the Higgs as a background field. Taking into account the $SU(2)_L$ symmetry, the effective EOM for the leptonic doublet quasiparticles is directly obtained from Eq. (1) as

$$\begin{pmatrix} i\sigma^\mu\partial_\mu & M_\ell(x) \\ M^\dagger_\ell(x) & i\bar{\sigma}^\mu\partial_\mu \end{pmatrix}\begin{pmatrix}\chi_\ell(x)\\ \chi_{\bar{\ell}}(x)\end{pmatrix} = 0. \tag{8}$$

In the $SU(2)_L$ gauge space the wave functions and mass-like matrix are given by

$$\chi_\ell(x) = \begin{pmatrix}\chi_\nu(x)\\ \chi_l(x)\end{pmatrix}, \quad \chi_{\bar{\ell}}(x) = \begin{pmatrix}-\chi_{\bar{\nu}}(x)\\ \chi_{\bar{l}}(x)\end{pmatrix}, \tag{9}$$

$$M^\dagger_\ell(x) = \frac{\lambda(x)}{\Lambda}\begin{pmatrix} 2\left[H^0(x)\right]^2 & -2H^0(x)H^+(x) \\ -2H^0(x)H^+(x) & 2\left[H^+(x)\right]^2 \end{pmatrix}, \tag{10}$$

where we have made the x^μ-dependence explicit to emphasise the spacetime-dependence of M_ℓ. Note that the effective Majorana mass-like matrix, $M_\ell(x)$, originates from the Weinberg operator and leads to the transition between a lepton and anti-lepton, which will be of importance for the lepton asymmetry generation.

3.2 The Higgs as a background field

As the Majorana mass-like matrix, $M_\ell(x)$, derives from the Higgs field, the thermal properties of this scalar field are of fundamental importance to the semi-classical treatment we detail in this paper. Above the EWSB scale, the mean value of the Higgs field may be zero at finite temperatures, $\langle H\rangle = 0$. However, the mean value of $\langle H^\dagger H\rangle$ is non-zero and such fluctuations correspond to particle excitations and annihilations in the thermal plasma.

As a complex field, the mean value is given by

$$\langle H^{0*}H^0\rangle = \langle H^{+*}H^+\rangle = \frac{1}{2}\langle H^\dagger H\rangle = 2\int\frac{d^3k}{(2\pi)^3}\frac{1}{2\omega}\frac{1}{e^{\beta\omega}-1} = \frac{T^2}{12}, \tag{11}$$

where we have ignored the effective thermal masses and chemical potential of the Higgs. It is worth noting that the mean values of $\langle (H^0)^2\rangle$, $\langle (H^+)^2\rangle$ and $\langle H^0H^+\rangle$ should be zero. As we shall see later, this property will be important for the enhancement of the lepton asymmetry production at high temperatures.

Another interesting property is that the mean value $\langle (H^\dagger(x)H(x))^2\rangle$ is correlated with $\langle H^\dagger(x)H(x)\rangle$[1)] by

$$\langle (H^{0*}(x)H^0(x))^2\rangle = \langle (H^{+*}(x)H^+(x))^2\rangle = \frac{1}{3}\langle (H^\dagger(x)H(x))^2\rangle = \frac{T^4}{72}. \tag{12}$$

The expectation values for H and $H^\dagger$ at different spacetimes give the Wightman propagators, e.g.,

$$\langle H^{0*}(x_2)H^0(x_1)\rangle = S^<_{H^0}(x_1,x_2),$$
$$\langle H^0(x_1)H^{0*}(x_2)\rangle = S^>_{H^0}(x_1,x_2). \tag{13}$$

For the detailed discussion of correlations between lepton asymmetry and Wightman propagators, please see Ref. [6]. In this paper, we ignore the spacetime difference between H and $H^\dagger$. This treatment simplifies the discussion and is sufficiently good to derive the CP asymmetry qualitatively.

4 Lepton asymmetry in the semi-classical approximation

The concept of quasiparticles has been known for many decades [13, 14] and manifests as particle properties become modified in a medium; for example, particles may acquire a different mass from that in vacuum as a result of their interactions in plasma. In general, such properties can be described by collective excitations, or using a quasiparticle description. These quasiparticles are characterised by their dispersion relation, which gives their energy (ω) as a function of their momentum ($\boldsymbol{k}$). Moreover, a stable particle in vacuum may have a finite lifetime in a medium and this corresponds to the quasiparticle damping rate, γ. The damping characterises the degree of decoherence of particles, and, therefore, gives a measure of the spread in the particle energy due to their interactions in the medium. We define the decoherence length, L, similarly to [7]

$$L = \frac{v_g}{2\gamma} = \frac{1}{6\gamma}, \tag{14}$$

where v_g is the group velocity of the quasiparticle. As the quasiparticles of interest in our mechanism are leptons, the decoherence results mainly from the electroweak gauge interaction. In this case, as the quasiparticles have homogeneous distributions parallel to the wall, it is reasonable to restrict our attention to quasiparticles with momenta perpendicular to the bubble wall [7]. We move to the wall rest frame and expand ℓ_L and $\overline{\ell_L}$ into positive and negative frequencies in the spinor space, respectively. As left-handed particles, they can be parametrised as

1) It is proved in the following. For a real scalar φ_i, $\langle\varphi_i^2\rangle = T^2/12$, $\langle\varphi_i^{2n}\rangle = (2n-1)!!\langle\varphi_i^2\rangle^n$ [12]. For a complex scalar $\Phi = (\varphi_1 + i\varphi_2)/\sqrt{2}$, $\langle\Phi^*\Phi\rangle = \frac{1}{2}\langle\varphi_1^2+\varphi_2^2\rangle = T^2/12$, $\langle(\Phi^*\Phi)^2\rangle = \frac{1}{4}\langle(\varphi_1^2+\phi_2^2)^2\rangle = \frac{1}{4}\langle\varphi_1^4+\varphi_2^4+2\varphi_1^2\phi_2^2\rangle = \frac{1}{4}(3\langle\varphi_1^2\rangle^2+3\langle\varphi_2^2\rangle^2+2\langle\varphi_1^2\rangle\langle\varphi_1^2\rangle) = 2\langle\Phi^*\Phi\rangle^2$.

$$\ell_L = \begin{pmatrix} \exp[-\mathrm{i}(\omega t - k_{\rm in}z)]\chi_{1\ell}(z) \\ \exp[-\mathrm{i}(\omega t + k_{\rm out}z)]\chi_{2\ell}(z) \\ 0 \\ 0 \end{pmatrix},$$

$$\overline{\ell_L}^T = \begin{pmatrix} 0 \\ 0 \\ \exp[+\mathrm{i}(\omega t - k_{\rm in}z)]\chi_{1\bar{\ell}}(z) \\ \exp[+\mathrm{i}(\omega t + k_{\rm out}z)]\chi_{2\bar{\ell}}(z) \end{pmatrix}. \quad (15)$$

Here, we have required $\chi_{1\ell}$ and $\chi_{1\bar{\ell}}$ to be incoming quasiparticles moving in the $+z$ direction, and $\chi_{2\ell}$ and $\chi_{2\bar{\ell}}$ to be outgoing quasiparticles moving in the $-z$ direction (i.e. the quasiparticles in the upper component of the spinor are moving *into* the bubble, and the lower component are reflected back to Phase I by the wall). $\chi_{1\ell}(z)$ and $\chi_{2\bar{\ell}}(z)$ have spin $j_z = -\frac{1}{2}$, while $\chi_{1\bar{\ell}}(z)$ and $\chi_{2\ell}(z)$ have spin $j_z = +\frac{1}{2}$. The coherence of these states may be included using the following replacement

$$k_{\rm in} \to K_{\rm in} = k_{\rm in} + \frac{i}{2L},$$
$$k_{\rm out} \to K_{\rm out} = k_{\rm out} - \frac{i}{2L}, \quad (16)$$

with $\gamma_w = \gamma\sqrt{1-v_w^2}$ being the boosted damping rate. As $M_\ell(z)$ does not change the energy in the wall rest frame, we do not distinguish between the energy, ω, of the leptons and anti-leptons. The EOM is decomposed into two uncoupled equations, one for $j_z = -\frac{1}{2}$ and the other for $j_z = +\frac{1}{2}$ quasiparticles. They are expressed respectively as

$$\left[(-\mathrm{i}\partial_z + \omega)\,\mathbb{1}_2 - \begin{pmatrix} -K_{\rm in} & M_\ell^\dagger(z) \\ -M_\ell(z) & -K_{\rm out}\end{pmatrix}\right]\begin{pmatrix}\chi_{1\ell}(z) \\ \chi_{2\bar{\ell}}(z)\end{pmatrix} = 0, \quad (17)$$

$$\left[(-\mathrm{i}\partial_z - \omega)\,\mathbb{1}_2 - \begin{pmatrix} K_{\rm in} & -M_\ell(z) \\ M_\ell^\dagger(z) & K_{\rm out}\end{pmatrix}\right]\begin{pmatrix}\chi_{1\bar{\ell}}(z) \\ \chi_{2\ell}(z)\end{pmatrix} = 0, \quad (18)$$

The energy-dependent term does not contribute to the CP violation in the wall rest frame, and thus we do not include it in the following discussion.

The calculation of lepton asymmetry generated by CPPT follows from solving the EOM for the leptonic doublet quasiparticles.

We now consider the amplitude of $\chi_{1\ell}$ transition to $\chi_{2\bar{\ell}}$ and use the techniques developed in [7] for electroweak baryogenesis (EWBG). The transition from left-handed fermion to right-handed fermion via a spacetime-varying mass is similar to our case of the transition from left-handed lepton to right-handed anti-lepton via the time-varying Weinberg operator.

The first step is to consider the propagation of quasiparticles in Phase I, where we restrict our discussion to the $j_z = -1/2$ quasiparticles $\chi_{1\ell}$ and $\chi_{2\bar{\ell}}$. The relevant Green functions are

$$\left(-\mathrm{i}\partial_z + K^{\rm in(out)}\right)G_{\ell(\bar{\ell})}(z-z_0) = \mathbb{1}\delta(z-z_0). \quad (19)$$

In order that there are no sources of quasiparticles at spatial infinity, the boundary conditions

$$G_\ell(-\infty) = G_{\bar{\ell}}(+\infty) = 0, \quad (20)$$

are necessary.

The solution of the Green functions with these boundary conditions is given by

$$G_\ell(z-z_0) = \mathrm{i}\theta(z-z_0)\mathrm{e}^{-\mathrm{i}K_{\rm in}(z-z_0)} = \mathrm{i}\theta(z-z_0)\mathrm{e}^{-(z-z_0)/(2L)}\mathrm{e}^{-\mathrm{i}k_{\rm in}(z-z_0)},$$

$$G_{\bar{\ell}}(z-z_0) = -\mathrm{i}\theta(z_0-z)\mathrm{e}^{-\mathrm{i}K_{\rm out}(z-z_0)} = -\mathrm{i}\theta(z_0-z)\mathrm{e}^{-(z_0-z)/(2L)}\mathrm{e}^{-\mathrm{i}k_{\rm out}(z-z_0)}. \quad (21)$$

The lepton quasiparticle will propagate from Phase I into Phase II. For this purpose, we may consider leptons with a δ-function source at $z = z_0$ propagating into the bubble wall. The influence of the wall leads to an effective "mass" term, $M_\ell(z)$, as explained above, and the evolution of quasiparticles is described by Eq. (17). Taking advantage of the Green function method, we obtain

$$\chi_{1\ell}(z) = -\mathrm{i}G_\ell(z-z_0)\chi_{1\ell}(z_0) + \int \mathrm{d}z_1 G_\ell(z-z_1)M_\ell^\dagger(z_1)\chi_{2\bar{\ell}}(z_1),$$

$$\chi_{2\bar{\ell}}(z) = \int \mathrm{d}z_1 G_{\bar{\ell}}(z-z_1)[-M_\ell(z_1)]\chi_{1\ell}(z_1). \quad (22)$$

Since the Weinberg operator is relatively weakly coupled to the thermal plasma, we ignore all corrections $\lesssim O(M_\ell^2)$. Therefore, the amplitude for $\chi_{1\ell}(z_0)\to\chi_{2\bar{\ell}}(z_0)$, $R_{\ell\bar{\ell}}(z_0)$, corresponding to the reflection matrix $R_{\rm LR}$ in [7], is given by

$$R_{\ell\bar{\ell}}(z_0) = \mathrm{i}\int \mathrm{d}z_1 G_{\bar{\ell}}(z_0-z_1)M_\ell(z_1)G_\ell(z_1-z_0) = \mathrm{i}\int_0^{+\infty}\mathrm{d}z_1\mathrm{e}^{-z_1/L}\mathrm{e}^{\mathrm{i}k_{\rm out}z_1}M_\ell(z_0+z_1)\mathrm{e}^{-\mathrm{i}k_{\rm in}z_1}. \quad (23)$$

We can calculate the amplitude for $\chi_{1\bar{\ell}}(z_0)\to\chi_{2\ell}(z_0)$ by assuming a similar treatment of a δ-function source at z_0 and moving in the $+z$ direction. The resultant reflection matrix $R_{\bar{\ell}\ell}$, corresponding to the reflection matrix $\bar{R}_{LR}$ in [7], is

$$R_{\bar{\ell}\ell}(z_0) = \mathrm{i}\int_0^{+\infty}\mathrm{d}z_1\mathrm{e}^{-z_1/L}\mathrm{e}^{\mathrm{i}k_{\rm out}z_1}M_\ell^\dagger(z_0+z_1)\mathrm{e}^{-\mathrm{i}k_{\rm in}z_1}. \quad (24)$$

Finally, we obtain the CP asymmetry of the amplitude (defined in Eq. (5)) as

$$\Delta_{\rm CP}(z_0) = \int_0^{+\infty}\mathrm{d}z_1\mathrm{d}z_2\mathrm{e}^{-(z_1+z_2)/L}\mathrm{e}^{\mathrm{i}(k_{\rm out}-k_{\rm in})(z_1-z_2)} \times[M_\ell^\dagger(z_0+z_1)M_\ell(z_0+z_2) - M_\ell(z_0+z_1)M_\ell^\dagger(z_0+z_2)]$$
$$= 2\int_0^{+\infty}\mathrm{d}z_1\mathrm{d}z_2\mathrm{e}^{-(z_1+z_2)/L}\sin[(k_{\rm out}-k_{\rm in})(z_1-z_2)] \times\mathrm{Im}[M_\ell(z_0+z_1)M_\ell^\dagger(z_0+z_2)]. \quad (25)$$

This quantity is determined by: 1) the momentum change $k_{\rm out}-k_{\rm in}$ due to the pressure from the wall; and 2) the imaginary part of the interference of two varying Majorana mass-like matrices ${\rm Im}[M_\ell^\dagger(z_0+z_1)M_\ell(z_0+z_2)]$. The criteria $\Delta_{\rm CP}\neq 0$ at order $O(M_\ell^2)$ can only be fulfilled if these two conditions are satisfied. As discussed in Section 2, the z-varying Weinberg operator can lead to momentum non-conservation in the z direction. Ignoring the momentum exchange with the Higgs boson, the momentum non-conservation is explicitly written as

$$k_{\rm out}\neq k_{\rm in}. \tag{26}$$

The momentum difference $k_{\rm out}-k_{\rm in}$ represents the impulse of the wall acting on leptons and anti-leptons. A similar problem is encountered in EWPT studies and the on-shell condition is usually assumed, where the momentum difference is correlated with the mass varying along the z direction. The on-shell condition is relaxed once transition radiations are included, and the latter is more important if the bubble wall moves very fast [15]. In our case, applying the on-shell condition can only give a very small momentum change, because M_ℓ is very small. A large momentum change can be obtained through interactions of the scalar excitation with leptons, anti-leptons and the Higgs. Such processes appear as there is an energy gradient within the bubble wall and the scalar excitation can be produced off-shell and interact with leptons, anti-leptons and the Higgs, thereby causing perturbations in their distribution functions from equilibrium. To simplify the problem, we make a reasonable assumption that the maximum value of the momentum transfer is of the order of the plasma temperature [6].

We now discuss in detail the term ${\rm Im}[M_\ell^\dagger(z_0+z_1)M_\ell(z_0+z_2)]$ in Eq. (25). We can rewrite this term as AB/Λ^2, where A and B specify the flavour and gauge component contributions, respectively. For CP violation between $\bar\ell_\alpha\to\ell_\beta$ and its conjugate process to occur, we have

$$A_{\alpha\beta}={\rm Im}\{\lambda_{\alpha\beta}(z_0+z_1)\lambda^*_{\alpha\beta}(z_0+z_2)\}. \tag{27}$$

The total contribution with all flavour summed together is given by

$$A\equiv\sum_{\alpha\beta}A_{\alpha\beta}={\rm Im}\{{\rm tr}[\lambda^*(z_0+z_1)\lambda(z_0+z_2)]\}. \tag{28}$$

For CP asymmetry between $\bar\nu-\nu$, $\bar l-\nu$, $\bar\nu-l$, $\bar l-l$ transitions, B is given respectively by

$$\begin{aligned}B_{\bar\nu\nu}&=4(H^{0*}H^0)^2,\\ B_{\bar l\nu}&=4(H^{0*}H^0)(H^{+*}H^+),\\ B_{\bar\nu l}&=4(H^{0*}H^0)^2,\\ B_{\bar ll}&=4(H^{0*}H^0)(H^{+*}H^+).\end{aligned} \tag{29}$$

Ignoring the energy-momentum exchange between leptons and the Higgs, and taking mean values on the right-hand-side and using (11) and (12), we obtain

$$B_{\bar\nu\nu}=B_{\bar ll}=\frac{T^4}{18},\quad B_{\bar l\nu}=B_{\bar\nu l}=\frac{T^4}{36}. \tag{30}$$

The average among gauge components is given by

$$B\equiv\frac{1}{2}(B_{\bar\nu\nu}+B_{\bar l\nu}+B_{\bar\nu l}+B_{\bar ll})=\frac{T^4}{12}. \tag{31}$$

Taking into account the results for A and B as given above, we can perform the integration of Eq. (25). We find that it depends on three terms: the interference of the coefficient term A, the damping term ${\rm e}^{-(z_1+z_2)/L}$, and the oscillation term $\sin[(k_{\rm out}-k_{\rm in})(z_1-z_2)]$. In general, the wall length and decoherence length are inversely proportional to the temperature, and the momentum transfer is proportional to the temperature. Therefore, the CP asymmetry $\Delta_{\rm CP}(z_0)$ in Eq. (25) is proportional to ${\rm Im}\{{\rm tr}[\lambda^0\lambda^{1*}]\}T^2/\Lambda^2$, with the coefficient depending on the competition of the three terms, where ${\rm Im}\{{\rm tr}[\lambda^0\lambda^{1*}]\}/\Lambda^2={\rm Im}\{{\rm tr}[m_\nu^0m_\nu^*]\}/v_H^4$. The final baryon asymmetry is given by

$$\eta_B\sim\frac{\Delta n_\ell}{n_\gamma}\sim{\rm Im}\{{\rm tr}[m_\nu^0m_\nu^*]\}\frac{T^2}{v_H^4}, \tag{32}$$

which is qualitatively the same as our previous result [2]. Through this simplified treatment, we recover the combination ${\rm Im}\{{\rm tr}[m_\nu^0m_\nu^*]\}$ and the temperature-dependent contribution $\propto T^2$ to the number density asymmetry between lepton number and anti-lepton number.

5 Conclusion

In this paper, we apply the semi-classical approximation to calculate the lepton asymmetry generated by a varying Weinberg operator. Firstly, we approximate the Higgs field as a background field. Following this treatment, we can effectively regard the Weinberg operator as an effective "Majorana mass term" for the leptonic doublet. Then, we write out the EOM for both lepton and anti-lepton quasiparticles, in which the "Majorana mass term" results in lepton anti-lepton transition. During the CP-violating phase transition, the "Majorana mass term" varies with spacetime, and the transitions from lepton to anti-lepton and from anti-lepton to lepton are not equal. This treatment is analogous to the approximation used in EWBG, where the varying fermion mass results in asymmetric transitions between left-handed and right-handed components.

In the semi-classical approximation, we do not try to provide quantitatively precise results for lepton asymmetry as the energy-momentum transfer with the Higgs has been ignored. However, this simplified treatment allows to present the mechanism more intuitively. Moreover, one of the main results of this paper is that, in the single scalar case, the number density asymmetry between leptons and anti-leptons $\Delta n_\ell \propto \mathrm{Im}\{\mathrm{tr}[m_\nu^0 m_\nu^*]\}T^2/v_H^4$ agrees with the result obtained using the non-equilibrium QFT approach.

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