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Diagonal reflection symmetries and universal four-zero texture*

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Abstract: In this paper, we consider a set of new symmetry in the SM, *diagonal reflection* symmetries $R m_{u,\nu}^* R = m_{u,\nu}, m_{d,e}^* = m_{d,e}$ with $R =$ diag $(-1,1,1)$. These generalized CP symmetries predict the Majorana phases to be $\alpha_{2,3}/2 = 0$ or $\pi/2$. A realization of diagonal reflection symmetries implies a broken chiral $U(1)_{\rm PQ}$ symmetry only for the first generations. The axion scale is suggested to be $\langle \theta_{u,d} \rangle \sim \Lambda_{\rm GUT} \sqrt{m_{u,d} m_{c,s}}/v \sim 10^{12}$ [GeV]. By combining the symmetries with the four-zero texture, the mass eigenvalues and mixing matrices of quarks and leptons are well reproduced. This scheme predicts the normal hierarchy, the Dirac phase $\delta_{CP} \simeq 203°$, and $|m_1| \simeq 2.5$ or 6.2 [meV]. In this scheme, the type-I seesaw mechanism and a given neutrino Yukawa matrix Y_ν completely determine the structure of right-handed neutrino mass M_R. An $u-\nu$ unification predicts mass eigenvalues to be $(M_{R1}, M_{R2}, M_{R3}) = (O(10^5), O(10^9), O(10^{14}))$ [GeV].

Keywords: $\mu-\tau$ reflection symmetry, four-zero texture, generalized CP symmetry

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I. INTRODUCTION

The discovery of the neutrino oscillation [1, 2] proved the finite mass and mixing of neutrinos. In order to explain the peculiar mixing pattern, a lot of flavor structures based on some symmetry such as four-zero texture [3-13], democratic texture [14-33], $\mu-\tau$ symmetry [34-55], and $\mu-\tau$ reflection symmetry [56–78] have been studied. However, these symmetries often have large corrections of symmetry breaking on the order of $\sim O(0.1)$. Among them, $\mu-\tau$ reflection symmetries for quarks and leptons are recently discussed [79].

In this paper, we consider a set of new symmetry with the accuracy of $\simeq O(2,3\ \%)$ in the Standard Model (SM), diagonal reflection symmetries for quarks and leptons. The previous study of $\mu-\tau$ reflection symmetries are translated to forms $R m_{u,\nu}^* R = m_{u,\nu}$, $m_{d,e}^* = m_{d,e}$ with $R =$ diag $(-1,1,1)$ by a redefinition of fermion fields. We call such a symmetry *diagonal reflection* because it is a diagonal remnant of $\mu-\tau$ reflection symmetry after deduction of $\mu-\tau$ symmetry. Each of them is just a generalized CP (GCP) symmetry [80-99] and no longer a $\mu-\tau$ reflection.

The form of the symmetries suggests that the flavored CP violation only comes from a chiral symmetry breaking of the first generations. As a justification of diagonal reflection symmetries and a zero texture $(m_f)_{11} = 0$, simultaneous breaking of a chiral $U(1)_{\rm PQ}$ [100] and a generalized CP symmetry is discussed in a specific two Higgs doublet model (2HDM). As a result, an invisible (flavored) axion [101-108] (a *flaxion* [109] or *axiflavon* [110]) appears in conjunction with solving the strong CP problem [111]. The axion scale is suggested to be $\langle \theta_{u,d} \rangle \sim \Lambda_{\rm GUT} \sqrt{m_{u,d} m_{c,s}}/v \sim 10^{12}$ [GeV]. This value can produce the dark matter abundance $\Omega_a h^2 \sim 0.2$ and very intriguing. It is also applicable to a solution of the strong CP problem by the discrete symmetry P [112, 113] or CP [114], because the diagonal reflection symmetries can reconcile the CKM phase $\delta_{\rm CKM}$ and $\theta_{\rm QFD}^{\rm tree} = {\rm Arg\,Det}[m_u m_d] = 0$ without Hermiticity or mirror fermions [115].

An additional assumption $(m_\nu)_{13} = 0$ (that can be justified by Eq. (38) in the left-right symmetric models [116-118]) realizes diagonal reflection with universal four-zero texture, which restrict fermion mass matrices to have only four parameters. This scheme provides proper masses, mixing, and CP phases of quarks and leptons. It predicts the Dirac phase $\delta_{CP} \simeq 203°$, the normal mass hierarchy, and the lightest neutrino mass $|m_1| \simeq 2.5$ or 6.2 [meV].

The main purpose of this paper is to constrain the mass matrix of the right-handed neutrinos M_R by the diagonal reflection symmetries, the four-zero texture, and the type-I seesaw mechanism [119-122]. The matrix M_R also exhibits the diagonal reflection symmetry with a four-zero texture because four-zero textures are type-I

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seesaw invariant [4, 6]. For a given neutrino Yukawa matrix Y_ν, the texture of M_R is completely determined by the seesaw mechanism in this scheme. A $u-\nu$ unification predicts mass eigenvalues as $(M_{R1}, M_{R2}, M_{R3}) = (O(10^5), O(10^9), O(10^{14}))$ [GeV].

Quantum corrections almost do not break these symmetries because couplings of the first generations are very tiny. A qualitative analysis shows that the symmetries are retained as approximate ones under the renormalization group equations of SM.

This paper is organized as follows. The next section gives definition of diagonal reflection symmetries. Sec. III discusses a realization of diagonal reflection symmetries and implications to the strong *CP* problem. Sec. IV is an analysis of physical parameters and universal four-zero texture. In Sec. V, we discuss stability under quantum corrections. The final section is devoted to summary.

II. DIAGONAL REFLECTION SYMMETRIES

In the beginning, we show a new set of symmetry. The mass matrices of the SM fermions $f = u,d,e$, and neutrinos ν_L are defined by

$$\mathcal{L} \ni \sum_f -\bar f_{Li} m^{BM}_{fij} f_{Rj} - \bar\nu_{Li} m^{BM}_{\nu ij} \nu^c_{Lj} + \text{h.c.}. \tag{1}$$

Here, we assume Hermitian m_f^{BM} and complex-symmetric m_ν^{BM} which can produce successful mass eigenvalues and mixing matrices $V_{\rm CKM}$ and $U_{\rm MNS}$ [79];

$$m_u^{BM} = \begin{pmatrix} 0 & -\frac{C_u}{\sqrt2} & -\frac{C_u}{\sqrt2} \\ -\frac{C_u}{\sqrt2} & \frac{\tilde B_u}{2} + \frac{A_u}{2} & \frac{\tilde B_u}{2} - \frac{A_u}{2} - iB_u \\ -\frac{C_u}{\sqrt2} & \frac{\tilde B_u}{2} - \frac{A_u}{2} + iB_u & \frac{\tilde B_u}{2} + \frac{A_u}{2} \end{pmatrix}, \tag{2}$$

$$m_d^{BM} = \begin{pmatrix} 0 & \frac{iC_d}{\sqrt2} & \frac{iC_d}{\sqrt2} \\ -\frac{iC_d}{\sqrt2} & \frac{\tilde B_d}{2} + \frac{A_d}{2} & \frac{\tilde B_d}{2} - \frac{A_d}{2} - iB_d \\ -\frac{iC_d}{\sqrt2} & \frac{\tilde B_d}{2} - \frac{A_d}{2} + iB_d & \frac{\tilde B_d}{2} + \frac{A_d}{2} \end{pmatrix}, \tag{3}$$

and

$$m_\nu^{BM} = \begin{pmatrix} -a_\nu & \frac{1}{\sqrt2}(b_\nu - ic_\nu) & \frac{1}{\sqrt2}(b_\nu + ic_\nu) \\ \frac{1}{\sqrt2}(b_\nu - ic_\nu) & \frac{f_\nu}{2} - \frac{d_\nu}{2} + ie_\nu & -\frac{f_\nu}{2} - \frac{d_\nu}{2} \\ \frac{1}{\sqrt2}(b_\nu + ic_\nu) & -\frac{f_\nu}{2} - \frac{d_\nu}{2} & \frac{f_\nu}{2} - \frac{d_\nu}{2} - ie_\nu \end{pmatrix}, \tag{4}$$

$$m_e^{BM} = \begin{pmatrix} 0 & \frac{iC_e}{\sqrt2} & \frac{iC_e}{\sqrt2} \\ -\frac{iC_e}{\sqrt2} & \frac{\tilde B_e}{2} + \frac{A_e}{2} & \frac{\tilde B_e}{2} - \frac{A_e}{2} - iB_e \\ -\frac{iC_e}{\sqrt2} & \frac{\tilde B_e}{2} - \frac{A_e}{2} + iB_e & \frac{\tilde B_e}{2} + \frac{A_e}{2} \end{pmatrix}. \tag{5}$$

Hermiticity of Yukawa matrices are justified by the parity symmetry in the left-right symmetric models [116-118]. These matrices (2)-(5) separately satisfy $\mu-\tau$ reflection symmetries [56, 57]:

$$T_u \left(m^{BM}_{u,\nu}\right)^* T_u = m^{BM}_{u,\nu}, \quad T_d \left(m^{BM}_{d,e}\right)^* T_d = m^{BM}_{d,e}, \tag{6}$$

where

$$T_u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad T_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}. \tag{7}$$

In general, a Hermitian or complex-symmetric matrix with a $\mu-\tau$ reflection symmetry has six parameters. Eq. (4) is a general complex-symmetric matrix which satisfies Eq. (6). Eq. (2), Eq. (3), and Eq. (5) have four parameters with two additional constraints, $(m_f)_{11} = 0$ and $(m_f)_{12} = (m_f)_{13}$.

A simultaneous redefinition of all fermion fields $f' = U_{BM} f$ and $\nu' = U_{BM}\nu$ by the following bi-maximal transformation U_{BM},

$$m_f \equiv U_{BM} m_f^{BM} U_{BM}^\dagger, \quad m_\nu \equiv U_{BM} m_\nu^{BM} U_{BM}^T,$$
$$U_{BM} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{i}{\sqrt2} & \frac{i}{\sqrt2} \\ 0 & -\frac{1}{\sqrt2} & \frac{1}{\sqrt2} \end{pmatrix}, \tag{8}$$

leads to Hermitian four-zero textures [3] and a symmetric neutrino mass;

$$m_u = \begin{pmatrix} i & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & C_u & 0 \\ C_u & \tilde B_u & B_u \\ 0 & B_u & A_u \end{pmatrix} \begin{pmatrix} -i & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad m_d = \begin{pmatrix} 0 & C_d & 0 \\ C_d & \tilde B_d & B_d \\ 0 & B_d & A_d \end{pmatrix}, \tag{9}$$

$$m_\nu = \begin{pmatrix} -i & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a_\nu & b_\nu & c_\nu \\ b_\nu & d_\nu & e_\nu \\ c_\nu & e_\nu & f_\nu \end{pmatrix} \begin{pmatrix} -i & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad m_e = \begin{pmatrix} 0 & C_e & 0 \\ C_e & \tilde{B}_e & B_e \\ 0 & B_e & A_e \end{pmatrix}. \tag{10}$$

Here, $a_\nu \sim f_\nu$ and $A_f \sim C_f$ are real parameters which satisfy $A_f > \tilde{B}_f > B_f \gg C_f$. In this basis, the assumptions are deformed to be $(Y_f)_{11}, (Y_f)_{13}, (Y_f)_{31} = 0$ for $f = u,d,e$. (The sentence that was written here is deleted.) We will partially discuss a justification of the texture later. Note that a $\mu-\tau$ reflection symmetry is not imposed on m_ν (10).

In this basis of the four-zero texture, the $\mu-\tau$ reflection symmetries (6) are rewritten as

$$U_{BM} T_{u,d} U_{BM}^T m_{u,d}^* U_{BM}^* T_{u,d} U_{BM}^\dagger = m_{u,d}. \tag{11}$$

Surprisingly,

$$-U_{BM}^* T_u U_{BM}^\dagger = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \equiv R, \tag{12}$$

$$U_{BM}^* T_d U_{BM}^\dagger = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = 1_3. \tag{13}$$

Then, the $\mu-\tau$ reflection symmetries in the four-zero basis are transformed into

$$R m_{u,\nu}^* R = m_{u,\nu}, \quad m_{d,e}^* = m_{d,e}. \tag{14}$$

Hermitian or symmetric mass matrices which satisfy Eq. (14) are given by

$$m_u = \begin{pmatrix} a_u & ib_u & ic_u \\ -ib_u & d_u & e_u \\ -ic_u & e_u & f_u \end{pmatrix}, \; m_\nu = \begin{pmatrix} a_\nu & ib_\nu & ic_\nu \\ ib_\nu & d_\nu & e_\nu \\ ic_\nu & e_\nu & f_\nu \end{pmatrix},$$
$$m_{d,e} = \begin{pmatrix} a_{d,e} & b_{d,e} & c_{d,e} \\ b_{d,e} & d_{d,e} & e_{d,e} \\ c_{d,e} & e_{d,e} & f_{d,e} \end{pmatrix}, \tag{15}$$

with real parameters $a_f \sim f_f$. The mass matrices (9)-(10) certainly satisfy these conditions. We call such a symmetry *diagonal reflection* because it is a diagonal remnant of $\mu-\tau$ reflection symmetry after deduction of $\mu-\tau$ symmetry. Each of them is just a generalized *CP* symmetry [81, 83-85, 87] and no longer a $\mu-\tau$ reflection. The textures (9) are discussed for quarks and CKM matrices in many studies ([9] and references therein). However, we can not found a paper that indicates the existence of GCP symmetries.

The latest calculation shows an example of Yukawa matrices compatible with all the flavor data of quarks [13]:

$$Y_u^0 \simeq \frac{0.9 m_t \sqrt{2}}{v} \begin{pmatrix} 0 & 0.0002\,i & 0 \\ -0.0002\,i & 0.10 & 0.31\,e^{\pm 0.02\pi} \\ 0 & 0.31\,e^{\mp 0.02\pi} & 1 \end{pmatrix}, \tag{16}$$

$$Y_d^0 \simeq \frac{0.9 m_b \sqrt{2}}{v} \begin{pmatrix} 0 & 0.005 & 0 \\ 0.005 & 0.13 & 0.31\,e^{\mp 0.02\pi} \\ 0 & 0.31\,e^{\pm 0.02\pi} & 1 \end{pmatrix}, \tag{17}$$

where $v = 246\,[\mathrm{GeV}]$ is the vacuum expectation value (vev) of the SM Higgs field. The textures (9) agree with (16) and (17) in the accuracy of $O(2{,}3\,\%)$. Breaking effects come from phases of the 23 element $B_{u,d}\, e^{i\varphi_{u,d}}$, where $\varphi_{u,d} \sim \pm 0.02\pi$.

Since the conditions (14) depend on a basis, they are changed by further redefinitions of fermion fields (the weak basis transformations [123, 124]). For example, rephasing of quark fields $Q = q,u,d$

$$Q' = P_Q^\dagger Q, \quad P_Q = \mathrm{diag}\left(e^{i\phi_Q}, 1, 1\right), \tag{18}$$

leads to *CP*-violating quark masses $\tilde{m}_{u,d}$;

$$\tilde{m}_u = P_q^\dagger m_u P_u = \begin{pmatrix} a_u & ie^{-i\phi_q} b_u & ie^{-i\phi_q} c_u \\ -ie^{i\phi_u} b_u & d_u & e_u \\ -ie^{i\phi_u} c_u & e_u & f_u \end{pmatrix}, \tag{19}$$

$$\tilde{m}_d = P_q^\dagger m_d P_d = \begin{pmatrix} a_d & e^{-i\phi_q} b_d & e^{-i\phi_q} c_d \\ e^{i\phi_d} b_d & d_d & e_d \\ e^{i\phi_d} c_d & e_d & f_d \end{pmatrix}. \tag{20}$$

In this case, by the following equivalent transformation

$$R_{q,u} \equiv P_{q,u} R P_{q,u} = \begin{pmatrix} -e^{2i\phi_{q,u}} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$
$$\tilde{R}_{q,d} \equiv P_{q,d} 1_3 P_{q,d} = \begin{pmatrix} +e^{2i\phi_{q,d}} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{21}$$

deforms the diagonal reflection symmetries (14) as

$$R_q^\dagger \tilde{m}_u^* R_u = \tilde{m}_u, \quad \tilde{R}_q^\dagger \tilde{m}_d^* \tilde{R}_d = \tilde{m}_d. \tag{22}$$

In this basis, the Hermiticity of the quark masses is lost, as shown in Eqs. (19) and (20). The symmetries Eq. (6), Eq. (14), and Eq. (22) are all equivalent under redefinitions of fermion fields.

III. REALIZATION OF THE SYMMETRIES

The $\mu-\tau$ reflection symmetry is often realized as a remnant of a larger flavor symmetry, such as $A_4, Z_2\times Z_2, U(1)_{L_\mu-L_\tau}$, and so on [56-78]. The origin of four-zero texture is also discussed in $S_{3L}\times S_{3R}$ model [125-128]. Then, in this section, we concentrate on a realization of the diagonal reflection symmetries. Since Eq. (6) or Eq. (14) imposes two independent GCP, underlying CP should be broken separately in the up- and down-sector [88].

To this end, the following $U(1)_{\rm PQ}\times Z_2$ flavor symmetry and a GCP symmetry are imposed on the 2HDM. A similar model-building and its UV completion can be found in [129-131].

- $Z_2^{\rm NFC}$: It realizes the natural flavor conservation (NFC) [132] and prohibits flavor changing neutral currents (FCNCs) by two Higgs doublets.
- $U(1)_{\rm PQ}$: A chiral (PQ) symmetry [100] that prohibits the mass of the first generations[1]. It is a kind of flavored PQ symmetry [105-108].
- CP : A generalized CP symmetry that restricts phases of Yukawa couplings. As an alternative way, the driving field method [133] is utilized to generate the relative phases.

Two SM singlet flavon fields $\theta_{u,d}$ are introduced to the 2HDM. These flavons have nontrivial charges under the $U(1)_{\rm PQ}$ and CP symmetry. Simultaneous breaking of these symmetries by vevs of $\theta_{u,d}$ provokes CPV only for the first generations. The charge assignment of fields is presented in Table 1.

Under the $U(1)_{\rm PQ}$ symmetry, only the first-generations have nontrivial charges as

$$q_{1L}\to e^{-i\alpha} q_{1L}, \quad u_{1R}\to e^{i\alpha} u_{1R}, \quad d_{1R}\to e^{i\alpha} d_{1R}, \tag{23}$$

$$l_{1L}\to e^{-i\alpha} l_{1L}, \quad \nu_{1R}\to e^{i\alpha} \nu_{1R}, \quad e_{1R}\to e^{i\alpha} e_{1R}. \tag{24}$$

The bilinear terms $\bar{q}_{Li}u_{Rj}, \bar{q}_{Li}d_{Rj}, \bar{l}_{Li}\nu_{Rj}$ and, $\bar{l}_{Li}e_{Rj}$ (associated with Yukawa interactions) are transformed under $U(1)_{\rm PQ}$ as

1) A discrete symmetry larger than Z_3 is also a possible choice.

Table 1. The charge assignments of the SM fermions and scalar fields under the gauge and the flavor symmetries.

	$SU(2)_L$	$U(1)_Y$	$Z_2^{\rm NFC}$	$U(1)_{\rm PQ}$	CP
q_{Li}	**2**	1/6	1	−1,0,0	1
u_{Ri}	**1**	2/3	1	1,0,0	1
d_{Ri}	**1**	−1/3	−1	1,0,0	1
l_{Li}	**2**	−1/2	1	−1,0,0	1
ν_{Ri}	**1**	0	1	1,0,0	1
e_{Ri}	**1**	−1	−1	1,0,0	1
H_u	**2**	−1/2	1	0	1
H_d	**2**	1/2	−1	0	1
θ_u	**1**	1	1	−1	$+i$
θ_d	**1**	1	−1	−1	$-i$

$$\begin{pmatrix} e^{2i\alpha} & \vline & e^{i\alpha} & e^{i\alpha} \\ \hline e^{i\alpha} & \vline & 1 & 1 \\ e^{i\alpha} & \vline & 1 & 1 \end{pmatrix}. \tag{25}$$

Under these discrete symmetries, the most general Yukawa interactions are written by

$$-\mathcal{L} \ni \bar{q}_L \left(\tilde{Y}_u^0 + \frac{\theta_u}{\Lambda}\tilde{Y}_u^1 + \frac{\theta_u^2}{\Lambda^2}\tilde{Y}_u^2 + \frac{\theta_d^2}{\Lambda^2}\tilde{Y}_u'^2 \right) u_R H_u \tag{26}$$

$$+\bar{q}_L \left(\tilde{Y}_d^0 + \frac{\theta_d}{\Lambda}\tilde{Y}_d^1 + \frac{\theta_u\theta_d}{\Lambda^2}\tilde{Y}_d^2 \right) d_R H_d + \text{h.c.}, \tag{27}$$

where Λ is a cut-off scale. Analogous formula holds in the lepton sector. The Yukawa matrices are parameterized as

$$\tilde{Y}_{u,d}^0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \tilde{d}_{u,d} & \tilde{c}_{u,d} \\ 0 & \tilde{b}_{u,d} & \tilde{a}_{u,d} \end{pmatrix}, \quad \tilde{Y}_{u,d}^1 = \begin{pmatrix} 0 & \tilde{e}_{u,d} & \tilde{f}_{u,d} \\ \tilde{g}_{u,d} & 0 & 0 \\ \tilde{h}_{u,d} & 0 & 0 \end{pmatrix}, \tag{28}$$

and $\tilde{Y}_f^2$ have only the 11 matrix element, which has a small influence. These Yukawa matrices satisfy a condition

$$(\tilde{Y}_{u,d}^0)_{ij}(\tilde{Y}_{u,d}^1)_{ij} = 0 \ \ (\text{no sum}), \tag{29}$$

similar to consistency conditions of general parity (or CP) and a flavor symmetry [80, 81].

The generalized CP invariance

$$\theta_u^* = +i\theta_u, \quad \theta_d^* = -i\theta_d, \quad \phi^* = \phi \ \text{ for other fields} \tag{30}$$

restricts relative complex phases of the matrix elements as

$$\left(\tilde{Y}^{0}_{u,d}\right)^{*} = \tilde{Y}^{0}_{u,d}, \quad \tilde{Y}^{1}_{u} = e^{i\pi/4}|\tilde{Y}^{1}_{u}|, \quad \tilde{Y}^{1}_{d} = e^{-i\pi/4}|\tilde{Y}^{1}_{d}|. \tag{31}$$

Next, we investigate transformation properties of the Higgs potential. The potential can be written as

$$V = V^{1}(H_u, H_d) + V^{2}(H_{u,d}, \theta_{u,d}) + V^{3}(\theta_u, \theta_d). \tag{32}$$

V^1 is obviously real because the GCP is the canonical *CP* for the Higgs doublets $H_{u,d}$. Among bi-linear terms made from θ_u and θ_d, only $\theta_u^*\theta_u$ and $\theta_d^*\theta_d$ are invariant under $U(1)_{\rm PQ}\times Z_2^{\rm NFC}$ (Both of $\theta_u^*\theta_d$ and its complex conjugate $\theta_d^*\theta_u$ has charge -1 under $Z_2^{\rm NFC}$ and -1 under *CP*). Then V_2 has only real terms because $\theta_u^*\theta_u$ and $\theta_d^*\theta_d$ have trivial *CP* charges. Finally, quartic terms made from the flavons should be a combination between $\{|\theta_u|^2, |\theta_d^2|\}$ or $\{\theta_u^*\theta_d, \theta_d^*\theta_u\}$, such as $|\theta_u|^2|\theta_d^2|$ or $\theta_u^*\theta_d\theta_u^*\theta_d$. Since these terms have trivial charges under *CP*, V_3 is a GCP invariant and then the whole Higgs potential V is invariant under *CP*. Therefore, in this basis, CP phases are localized only in the first generations of Yukawa matrices. Real vevs of the flavon fields $\langle\theta_{u,d}\rangle$ provokes a spontaneous symmetry breaking (SSB) of $U(1)_{\rm PQ}, Z_2^{\rm NFC}$, and *CP*.

As a result, the vevs $\langle\theta_{u,d}\rangle$ produce the following textures

$$Y_{u,d} = \left(\tilde{Y}^{0}_{u,d} + \frac{\langle\theta_{u,d}\rangle}{\Lambda}\tilde{Y}^{1}_{u,d} + \frac{\langle\theta_{u,d}\rangle^2}{\Lambda^2}\tilde{Y}^{2}_{u,d}\right) = \begin{pmatrix} O(\frac{\langle\theta_{u,d}\rangle^2}{\Lambda^2}) & \tilde{e}\frac{\langle\theta_{u,d}\rangle}{\Lambda}e^{i\varphi_{u,d}} & \tilde{f}\frac{\langle\theta_{u,d}\rangle}{\Lambda}e^{i\varphi_{u,d}} \\ \tilde{g}\frac{\langle\theta_{u,d}\rangle}{\Lambda}e^{i\varphi_{u,d}} & \tilde{d}_{u,d} & \tilde{c}_{u,d} \\ \tilde{h}\frac{\langle\theta_{u,d}\rangle}{\Lambda}e^{i\varphi_{u,d}} & \tilde{b}_{u,d} & \tilde{a}_{u,d} \end{pmatrix}, \tag{33}$$

where

$$\varphi_u = +\pi/4, \quad \varphi_d = -\pi/4. \tag{34}$$

These vevs can be estimated from the best fit values for $Y_{u,d}$ (16) and (17) as

$$\frac{\langle\theta_u\rangle}{\Lambda}|\tilde{Y}^1_u| \simeq \frac{\sqrt{2m_u m_c}}{v\sin\beta} \simeq \frac{3\times10^{-4}}{\sin\beta}, \tag{35}$$

$$\frac{\langle\theta_d\rangle}{\Lambda}|\tilde{Y}^1_d| \simeq \frac{\sqrt{2m_d m_s}}{v\cos\beta} \simeq \frac{1\times10^{-4}}{\cos\beta}, \tag{36}$$

where $\langle H_u^0\rangle \equiv v\sin\beta/\sqrt{2}, \langle H_d^0\rangle \equiv v\cos\beta/\sqrt{2}$ with $\langle H_u^0\rangle^2 + \langle H_d^0\rangle^2 = v^2/2$. The small 11 matrix elements in Eq. (33) are generated from $\tilde{Y}^2_f$. In many cases, they are negligible compared to Yukawa eigenvalues of the first generations:

$$\frac{\langle\theta_{u,d}\rangle^2}{\Lambda^2} \simeq \frac{10^{-8}(\times\tan^2\beta)}{|\tilde{Y}^1_{u,d}|^2} \lesssim (y_u, y_d) \simeq (\frac{m_u}{v\sin\beta}, \frac{m_d}{v\cos\beta}) \simeq (10^{-5}, 10^{-5}\tan\beta). \tag{37}$$

Therefore, Eq. (33) and (34) satisfy the diagonal reflection symmetries (22) with $\phi_u = 3\pi/4$, $\phi_q = -\phi_d = \pi/4$ and $(m_f)_{11} \simeq 0$.

In this construction, Eqs. (16) and (17) stand for $\tilde{Y}^0_u \simeq \tilde{Y}^0_d$ and $\tilde{Y}^1_u \sim \tilde{Y}^1_d$. It indicates an existence of $u-d$ unification, such as the left-right symmetric model. Moreover, with a $u-d$ unified relation $\tilde{Y}^1_u = \tilde{Y}^1_d$ (in the other basis of *CP* phases), simultaneous rotation of 2-3 generations by a real orthogonal matrix O_{23} can realize zero textures

$$(Y_u)_{13} = (Y_d)_{13} = (Y_u)_{31} = (Y_d)_{31} = 0. \tag{38}$$

Then the four-zero textures with the diagonal reflection symmetries appear. Note that O_{23} is commutative with the diagonal reflection symmetries, because it satisfies $RO_{23}^*R = O_{23}$.

Realization of four-zero texture in the left-right symmetric model, such as a model in [13], seems to lead a more concise model. We leave it for future work.

A. Implications to the strong CP problem

As a related issue, the strong *CP* problem is considered [111]. This is a fine-tuning problem of $\bar{\theta} = \theta_{\rm QCD} + \theta_{\rm QFD}$, a sum of the QCD θ-term $\theta_{\rm QCD}$ and its fermionic contribution $\theta_{\rm QFD} = {\rm Arg\,Det}[m_u m_d]$ [134].

Although $Y_{u,d}$ in Eq. (33) are not Hermitian matrices, $\theta^{\rm tree}_{\rm QFD} = 0$ holds because they satisfy

$$\phi_u + \phi_d - 2\phi_q = 0. \tag{39}$$

Under the condition (39), mass matrices generally have two more free parameters (for example, ϕ_q and $\phi_u + \phi_d$). Then, the diagonal reflection symmetries can have a similar feature (for $\theta_{\rm QFD}$) to the discrete symmetry P [112, 113] or *CP* [114] in a solution of the strong CP problem.

Moreover, $\bar{\theta}$ is dynamically retained to zero by a flavored axion [105-110] (*flaxion* [109] or *axiflavon* [110]) associates with the SSB of $U(1)_{\rm PQ}$. If the cut-off scale Λ is taken to be the GUT scale $\Lambda_{\rm GUT} \simeq 10^{16}$ [GeV], Eqs. (35) and (36) suggests that

$$\langle \theta_{u,d} \rangle \sim \Lambda_{\rm GUT} \frac{\sqrt{m_{u,d} m_{c,s}}}{v} \sim 10^{12} \, [\text{GeV}]. \tag{40}$$

This is consistent with phenomenological constraints [109] and predicts the axion mass $m_a \simeq 10^{-6}$ [eV], the dark matter abundance $\Omega_a h^2 \sim 0.2$. These chiral and GCP symmetry may shed light on the Strong CP problem and the origin of CP violation.

IV. PHYSICAL PARAMETERS

Next, let us consider predictions of mass eigenvalues and mixings. Derivation of these physical parameters is done in the previous study [79]. It is well known that the four-zero texture can reproduce quark masses and the CKM matrix. Then, we focus on the lepton sector.

Diagonalizing the mass matrices $m_f^{\rm diag} = U_{Lf}^\dagger m_f U_{Rf}$, one obtains the MNS matrix

$$U_{\rm MNS} = U_{Le}^\dagger U_{L\nu}. \tag{41}$$

An approximate form of the MNS matrix is found to be

$$U_{\rm MNS} = V_e^T \begin{pmatrix} -i & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} V_\nu P_M, \tag{42}$$

where

$$V_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{43}$$

$$V_e \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{r_e} & \sqrt{1-r_e} \\ 0 & -\sqrt{1-r_e} & \sqrt{r_e} \end{pmatrix} \begin{pmatrix} 1 & -\sqrt{\frac{m_e}{m_\mu}} & 0 \\ \sqrt{\frac{m_e}{m_\mu}} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{44}$$

with $r_e \equiv A_e/m_\tau$. $P_M \equiv {\rm diag}(1, e^{i\alpha_2/2}, e^{i\alpha_3/2})$ is the Majorana phases.

The mixing angles and mass differences of the latest global fit [135]

$$\theta_{23}^{PDG} = 49.7^\circ, \quad \theta_{12}^{PDG} = 33.82^\circ, \quad \theta_{13}^{PDG} = 8.61^\circ, \tag{45}$$

$$\Delta m_{21}^2 = 73.9 \left[\text{meV}^2\right], \quad \Delta m_{31}^2 = 2525 \left[\text{meV}^2\right], \tag{46}$$

determines the Dirac phase in the PDG parameterization δ_{CP} as

$$\sin\delta_{CP} = -0.390 \simeq \sqrt{\frac{m_e}{m_\mu}} \frac{c_{13} s_{23}}{s_{13}}, \quad \delta_{CP} \simeq 203^\circ. \tag{47}$$

It is very close to the best fit for the normal hierarchy (NH) $\delta_{CP}/^\circ = 217^{+40}_{-28}$ [135].

Including the Majorana phases, one can reconstruct the neutrino mass matrix m_ν as

$$m_\nu = V_e U_{\rm MNS} \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U_{\rm MNS}^T V_e^T. \tag{48}$$

The $\mu-\tau$ reflection symmetries (6) restrict the Majorana phases to be $\alpha_{2,3}/2 = n\pi/2$ $(n = 0, 1)$ [73]. The nontrivial phase $\pi/2$ comes from a negative mass eigenvalues [73, 75]. Moreover, if universal texture $(m_f)_{11} = 0$ for $f = u, d, \nu, e$ [38] and small 2-3 mixing of V_e is assumed, we can determine the lightest neutrino mass m_1 from the condition of the texture

$$m_1 = \frac{-e^{i\alpha_2} m_2 s_{12}^2 - e^{i\alpha_3} m_3 t_{13}^2}{c_{12}^2}, \tag{49}$$

where $t_{13} \equiv s_{13}/c_{13}$. The numerical values of the mass are found to be

$$|m_1| = 6.20 \, [\text{meV}] \ \text{ for } \ (\alpha_2, \alpha_3) = (0,0) \text{ or } (\pi,\pi), \tag{50}$$

$$= 2.54 \, [\text{meV}] \ \text{ for } \ (\alpha_2, \alpha_3) = (0,\pi) \text{ or } (\pi,0), \tag{51}$$

for the normal hierarchy case. For the inverted mass hierarchy, the solutions do not have real values and then contradict the diagonal reflection.

In the previous study [79], the effective mass m_{ee} of the double beta decay is also evaluated as

$$|m_{ee}| = \left| \sum_{i=1}^{3} m_i U_{ei}^2 \right| \tag{52}$$

$$= 0.17 \, [\text{meV}] \ \text{ for } \ (\alpha_2, \alpha_3) = (0,0) \ \textbf{or} \ (\pi,\pi), \tag{53}$$

$$= 1.24 \, [\text{meV}] \ \text{ for } \ (\alpha_2, \alpha_3) = (0,\pi) \ \textbf{or} \ (\pi,0). \tag{54}$$

A. Universal four-zero texture

Here, we show a universal four-zero texture compatible with neutrino mixing parameters. An additional assumption in this paper is $(m_\nu)_{13} = 0$. This assumption can be justified like Eq. (38) in the left-right symmetric models. This constraint realizes the universal four-zero texture and determines the mixing parameter $r_e = A_e/m_\tau$ in Eq. (44).

The mass matrix m_ν (48) is a matrix function of α_2, α_3, m_1, and r_e. Solving an equation $(m_\nu)_{13} = 0$, we found two solutions of universal four-zero texture. The first solution with a large $r_e \simeq 0.996$ and its mass eigenvalues are found to be

$$m_{\nu 0} \simeq \begin{pmatrix} 0 & -8.86i & 0 \\ -8.86i & 29.3 & 26.4 \\ 0 & 26.4 & 14.6 \end{pmatrix} [\text{meV}] \ \text{ for } (\alpha_2, \alpha_3) = (\pi, 0), \tag{55}$$

$$(m_1, m_2, m_3) = (2.54, -8.96, 50.3)\,[\text{meV}]. \tag{56}$$

Indeed the Majorana phases $\alpha_2 = \pi, \alpha_3 = 0$ are realized. In this basis, the charged lepton mass matrix also shows the four-zero texture

$$m_e \simeq \begin{pmatrix} 0 & -7.058 & 0 \\ -7.058 & 107.873 & 96.12 \\ 0 & 96.12 & 1740 \end{pmatrix} [\text{MeV}] \ \text{ for } (m_e^{\text{diag}})_{11} < 0, (m_e^{\text{diag}})_{22} > 0, \tag{57}$$

$$\simeq \begin{pmatrix} 0. & 7.058 & 0 \\ 7.058 & -95.898 & 108.1 \\ 0 & 108.1 & 1740 \end{pmatrix} [\text{MeV}] \ \text{ for } (m_e^{\text{diag}})_{11} > 0, (m_e^{\text{diag}})_{22} < 0). \tag{58}$$

The second solution has a small $r_e \simeq 0.0024$;

$$\tilde{m}_{\nu 0} = \begin{pmatrix} 0 & 10.5i & 0 \\ 10.5i & 24.9 & -22.0 \\ 0 & -22.0 & 30.1 \end{pmatrix} [\text{meV}] \ \text{ for } (\alpha_2, \alpha_3) = (0, 0), \tag{59}$$

$$(m_1, m_2, m_3) = (-6.20, 10.6, 50.6)\,[\text{meV}]. \tag{60}$$

This solution results in $(m_e)_{22} \simeq m_\tau$ and seems to be somewhat unnatural. However, perhaps it relates large 22 and 23 elements of quarks Eq. (16) and (17) by a grand unified theory (GUT).

The right-handed neutrino mass matrix M_R can be reconstructed from the type-I seesaw mechanism [119-122] with some GUT relations. A $u-\nu$ unification such as in the Pati–Salam GUT [116] can determine Y_ν from Eq. (16) as

$$Y_\nu = Y_u \simeq \frac{0.9 m_t \sqrt{2}}{v} \begin{pmatrix} 0 & 0.0002i & 0 \\ -0.0002i & 0.10 & 0.31 \\ 0 & 0.31 & 1 \end{pmatrix}. \tag{61}$$

From Eq. (55) and (61), M_R also displays a four-zero texture because the four-zero texture is seesaw invariant [4, 6],

$$M_R = \frac{v^2}{2} Y_\nu m_{\nu 0}^{-1} Y_\nu^T \tag{62}$$

$$= \begin{pmatrix} 0 & -1.08i \times 10^8 & 0 \\ -1.08i \times 10^8 & 1.26 \times 10^{14} & 4.07 \times 10^{14} \\ 0 & 4.07 \times 10^{14} & 1.32 \times 10^{15} \end{pmatrix} [\text{GeV}]. \tag{63}$$

Evidently M_R also satisfies the diagonal reflection symmetry (14),

$$R M_R^* R = M_R. \tag{64}$$

Therefore, all the fermion mass respects the diagonal reflection symmetry with a four-zero texture.

The eigenvalues of M_R are found to be

$$(M_{R1}, M_{R2}, M_{R3}) = (2.86 \times 10^6, 3.73 \times 10^9, 1.44 \times 10^{15})\,[\text{GeV}]. \tag{65}$$

The Yukawa matrices Y_ν (61) is evaluated at m_Z scale. Other renormalized values of quark masses will lead to smaller eigenvalues of M_R. For example, Y_ν is determined in other Pati–Salam GUT

$$Y_\nu = \begin{pmatrix} i & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & C_\nu & 0 \\ C_\nu & \tilde{B}_\nu & B_\nu \\ 0 & B_\nu & A_\nu \end{pmatrix} \begin{pmatrix} -i & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{66}$$

with $A_\nu = A_u, C_\nu = C_u$ and the Georgi–Jarlskog relation $B_\nu = -3B_u, \tilde{B}_\nu = -3\tilde{B}_u$ [136]. Quark masses at the GUT scale $\Lambda_{\text{GUT}} = 2 \times 10^{16}$ [GeV] [137]

$$m_u = 0.48\,[\text{MeV}],\ m_c = 0.235\,[\text{GeV}],\ m_t = 74\,[\text{GeV}], \tag{67}$$

lead to smaller eigenvalues

$$(M_{R1}, M_{R2}, M_{R3}) = (9.18\times10^4, 1.77\times10^9, 3.02\times10^{14})\,[\text{GeV}]. \quad (68)$$

The precise eigenvalues will be obtained by solving renormalization group equations.

The mass matrix M_R is constrained by the diagonal reflection symmetries, the universal four-zero texture, and the type-I seesaw mechanism. This scheme enhances the predictivity of the leptogenesis [138]. Large CP violation in M_R (and m_ν) is desirable.

Since the mass matrix M_R has strong hierarchy $M_R \sim Y_u^T Y_u$, the lightest mass eigenvalue M_{R1} is too small [139, 140] for the naive thermal leptogenesis. However, leptogenesis may be achieved by the decay of the second lightest neutrino ν_{R2} [141] with the maximal Majorana phase $\alpha_2/2 = \pi/2$.

V. QUANTUM CORRECTIONS

Here we show the stability of the symmetries against quantum corrections. Since quantum corrections are very tiny for the first generations, the symmetries (14) are retained as approximate ones.

The diagonal reflection symmetries are not invariant under the renormalization group equations (RGEs) of the SM. RGEs of quarks at one-loop order are given by [142],

$$16\pi^2 \frac{dY_u}{dt} = \left[\alpha_u + C_u^u\left(Y_u Y_u^\dagger\right) + C_u^d\left(Y_d Y_d^\dagger\right)\right] Y_u, \quad (69)$$

$$16\pi^2 \frac{dY_d}{dt} = \left[\alpha_d + C_d^u\left(Y_u Y_u^\dagger\right) + C_d^d\left(Y_d Y_d^\dagger\right)\right] Y_d, \quad (70)$$

where $t = \ln(\mu)/m_Z$, μ is an arbitrary renormalization scale, α_f are flavor independent contributions from the gauge and Higgs bosons. The coefficients $C_f^{f'}$ are given by

$$C_u^d = C_d^u = -3/2, \quad C_u^u + C_d^d = 3/2. \quad (71)$$

Similar equations hold in the lepton sector.

It has been pointed out that the four-zero texture and its CKM phase are approximately RGE invariant [13, 143]. The same statement holds for the diagonal reflection. One of the best fit values (16) and (17) are roughly written by

$$Y_u \simeq \frac{\sqrt{2}}{v}\begin{pmatrix} 0 & i\sqrt{m_u m_c} & 0 \\ -i\sqrt{m_u m_c} & O(m_t) & O(m_t) \\ 0 & O(m_t) & O(m_t)\end{pmatrix}, \quad (72)$$

$$Y_d \simeq \frac{\sqrt{2}}{v}\begin{pmatrix} 0 & \sqrt{m_d m_s} & 0 \\ \sqrt{m_d m_s} & O(m_b) & O(m_b) \\ 0 & O(m_b) & O(m_b)\end{pmatrix}. \quad (73)$$

A term in Eq. (70) can be reconstructed as

$$Y_u Y_u^\dagger Y_d = \begin{pmatrix} 1.17\times10^{-9} i & 2.34\times10^{-12} + 2.56\times10^{-7} i & 7.99\times10^{-7} i \\ 6.22\times10^{-6} & 0.00140 - 1.17\times10^{-9} i & 0.00438 \\ 2.00\times10^{-5} & 0.00450 - 3.63\times10^{-9} i & 0.0141 \end{pmatrix} \quad (74)$$

$$\simeq \begin{pmatrix} iC_u\tilde{B}_u C_d & iC_u(B_u B_d + \tilde{B}_u\tilde{B}_d) & iC_u(B_u A_d + \tilde{B}_u B_d) \\ (B_u B_u + \tilde{B}_u\tilde{B}_u)C_d & O(B_u A_u B_d) - i\tilde{B}_u C_u C_d & O(B_u A_u A_d) \\ (A_u B_u + B_u\tilde{B}_u)C_d & O(A_u A_u B_d) - iB_u C_u C_d & O(A_u A_u A_d)\end{pmatrix}. \quad (75)$$

In Eq. (75), several terms at the leading order are represented. Matrix elements of the first row and column (specifically, $(1,i)$ and $(j,1)$ elements) of the term $Y_u Y_u^\dagger Y_d$ are insignificant. This is due to the smallness of $|(m_{u,d})_{12}| = |C_{u,d}| \simeq \sqrt{m_{u,d} m_{c,s}}$ (or the chiral symmetry of the first generations $U(1)_{\rm PQ}$). Furthermore, influence to complex phases of $(2,2),(2,3),(3,2)$ and $(3,3)$ elements are also negligible because they are the second-order corrections of the small parameters $C_{u,d}$.

Since the flavor depending terms in Eqs. (69) and (70) have a similar structure, flavor dependent contributions almost do not change the couplings of the first generations. This statement holds without the four-zero texture as long as couplings in the first row and column of the Yukawa matrices are sufficiently small. Therefore, the diagonal reflection symmetries with these properties are approximately RGE invariant and then they inherit flavor structures at a high energy scale.

VI. SUMMARY

In this paper, we considered a set of new symmetry in the SM, *diagonal reflection* symmetries. $\mu-\tau$ reflection symmetries of the previous study are deformed to $R m_{u,\nu}^* R = m_{u,\nu}$, $m_{d,e}^* = m_{d,e}$ with $R = \text{diag}\,(-1,1,1)$ by a redefinition of fermion fields. They can constrain the Majorana phases to be $\alpha_{2,3}/2 = 0$ or $\pi/2$ and then enhance the predictivity of the leptogenesis.

The form of the symmetries suggests that the flavored *CP* violation only comes from a chiral symmetry breaking of the first generation. As a justification of diagonal reflection symmetries and a zero texture $(m_f)_{11} = 0$, simultaneous breaking of a chiral $U(1)_{\rm PQ}$ and a generalized *CP* symmetry is discussed in a specific 2HDM. As a result, a flavored axion appears in conjunction with solving the strong CP problem. The axion scale is suggested to be $\langle \theta_{u,d} \rangle \sim \Lambda_{\rm GUT} \sqrt{m_{u,d} m_{c,s}}/v \sim 10^{12}$ [GeV]. This value can produce the dark matter abundance $\Omega_a h^2 \sim 0.2$ and very intriguing. They can be also applicable to a solution of the Strong CP problem by discrete symmetry P or CP, because the symmetries can reconcile the CKM phase $\delta_{\rm CKM}$ and $\theta^{\rm tree}_{\rm QFD} = {\rm Arg\,Det}[m_u m_d] = 0$ without Hermiticity or mirror fermions.

By combining the symmetries with the four-zero texture, the mass eigenvalues and mixing matrices of quarks and leptons are well reproduced. This scheme predicts the normal hierarchy, the Dirac phase $\delta_{CP} \simeq 203°$, and $|m_1| \simeq 2.5$ or $6.2\,[\rm meV]$.

The type-I seesaw mechanism results in the mass matrix of the right-handed neutrinos M_R which exhibits diagonal reflection symmetries with a four-zero texture. The matrix M_R is completely determined by a given Y_ν and the type-I seesaw mechanism. A $u-\nu$ unification predicts that the mass matrix M_R has a strong hierarchy $M_R \sim Y_u^T Y_u$.

The symmetries are approximately stable under the renormalization of SM. This statement holds without the four-zero texture as long as couplings in the first row and column of the Yukawa matrices are sufficiently small. Then, they can possess information on a high energy scale.

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