Effects of an odd particle on shape phase transitions in odd-even systems

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Abstract: A scheme of solving the Hamiltonian in the interacting boson-fermion model in terms of the SU(3) coupling basis is introduced, by which the effects of an odd particle on shape phase transitions (SPTs) in odd-A nuclei are examined through comparing the critical behaviors of some selected quantities in between the odd-even and even-even systems. The results indicate that the spherical to prolate (U(5)-SU(3)) SPT and the spherical to γ-soft (U(5)-O(6)) SPT may clearly occur in the odd-even system with the SPT signatures revealed from various quantities including the excitation energies, energy ratio, B(E2) ratio, quadrupole moments and one-particle-transfer spectroscopic intensities. In particular, it is shown that the spherical to prolate SPT in the odd-even system can be even strengthened by the effects of the odd particle with the large fluctuations of the quadrupole deformations appearing near the critical point.

Keywords: The interacting boson-fermion model, The SU(3) basis, Shape phase transition, Effects of the odd particle

I. INTRODUCTION

Quantum phase transitions (QPTs) in nuclei have attracted a lot of attention in the past two decades [1–14]. QPTs in nuclei are not of the usual thermodynamic type but related to changes in the ground state shapes of nuclei, hence the name shape phase transitions (SPTs) given to them. In theory, the interacting boson model (IBM) [15] may be the mostly often used framework to study the SPTs in even-even nuclei [2]. In recent years, considerable attentions were given to the SPTs in odd-A nuclei [16–34, 36–39]. A theoretical tool of describing odd-A nuclei is the interacting boson-fermion model (IBFM) [40], in which an odd-A nucleus can be approximately considered as the odd-even system with the even-core (bosons) plus the unpaired particles (fermions). The SPTs in odd-even system can be explored as the QPTs between two different dynamical symmetry limits of the boson core since the ground state shape of the system is assumed to be mainly determined by the core deformation. There are two ways of addressing SPTs in the framework of IBFM, the analysis of the ground state potential surfaces and the direct quantum computation of order parameters [14]. A classical analysis of the ground deformations in the spherical to prolate and spherical to γ-soft SPTs was recently carried [34] in the frame of the IBFM using the coherent state method [40] and the main conclusion is that the single particle (odd particle) can influence different types of SPTs in different ways [24–26]. However, the ground state deformation cannot be directly observed in experiments. A more practical way of studying SPT is to perform a quantal analysis of the observables that are sensitive to the ground state deformations. Such kinds of quantities can be accordingly taken as the effective order parameters to identify the SPTs in experiments [14]. To calculate observables, one has to numerically solve the IBFM Hamiltonian in a transitional situation. The often used IBFM code is “ODDA” developed by Scholten [41] with the wave functions expanded in terms of the weak-coupling U(5) basis [40]. Since the IBFM [40] as the standard model for odd-A nuclei can provide a very convenient frame to study SPTs, developing an alternative scheme to solve the model Hamiltonian would be interesting and also expected.

There are two folds in this work. First we would like to introduce the diagonalization scheme of the IBFM Hamiltonian in terms of the weak-coupling SU(3) basis with the SU(3) part constructed by the Draayer-Akiyama algorithm [42, 43]. Second we will study the effects of an odd particle on the SPTs in odd-even systems by using the proposed diagonalization scheme. Two types of SPT will be emphasized in this work, i.e. the spherical to pro-
late SPT and the spherical to $\gamma$-soft SPT. The article is arranged as follows. In Sec.II, the IBFM Hamiltonian together with the diagonalization scheme is introduced, by which the roles of different boson-fermion interactions in the IBFM are examined. Section III is devoted to studying the effects of an odd particle on two types of SPTs. Finally, a summary is given in Sec.IV.

**II. MODEL**

**A. The IBFM Hamiltonian**

The IBFM Hamiltonian can be generally written as [40]

$$\hat{H} = \hat{H}_B + \hat{H}_F + \hat{V}_{BF},$$  \hspace{1cm} (1)

where $\hat{H}_B$ represents the IBM Hamiltonian describing the boson core, $\hat{H}_F$ is the single particle Hamiltonian describing the unpaired fermions (odd particles), and $\hat{V}_{BF}$ represents the boson-fermion interaction. If only the mean-field part is taken into account, the single particle Hamiltonian can be written as

$$\hat{H}_F = \sum_{j} \varepsilon_j \hat{n}_j,$$  \hspace{1cm} (2)

where $\varepsilon_j$ represents the single-particle energies of the spherical orbit $j$ and

$$\hat{n}_j = -\sqrt{2j+1}(a_j^{\dagger} \times a_j)^{(0)},$$  \hspace{1cm} (3)

with $a_{jm} = (-1)^{m-m_j} a_{j-m}$ is the fermion number operator. In this work, only the situation with one unpaired fermion confined in a single-$j$ orbit is taken into account for simplicity. It means that $\hat{H}_F$ will only contribute a constant for the excitation energies. For the IBM Hamiltonian, we take the consistent-$Q$ form [44]

$$\hat{H}_B = \varepsilon_d \hat{n}_d + \kappa \hat{Q}_B^x \cdot \hat{Q}_B^y,$$  \hspace{1cm} (4)

where the the $d$-boson number operator is defined as

$$\hat{n}_d = \sqrt{5}(d^{\dagger} \times d)^{(0)}$$  \hspace{1cm} (5)

with $d_a = (-1)^{\mu} d_{\mu-a}$ and the quadrupole operator is defined as

$$\hat{Q}_B^x = (d^{\dagger} s + s^\dagger d)^{(2)} + \chi (d^{\dagger} \times d)^{(2)}$$  \hspace{1cm} (6)

with $\chi \in [-\sqrt{7}/2, 0]$. There are three typical dynamical symmetries (DSs) included in the IBM, namely $U(5)$, $O(6)$ and $SU(3)$. One can prove that the Hamiltonian $\hat{H}_B$ is in the $U(5)$ DS when $\kappa = 0$; it is in the $O(6)$ DS when $\varepsilon_d = 0$ and $\chi = 0$; it is in the $SU(3)$ DS when $\varepsilon_d = 0$ and $\chi = -\sqrt{7}/2$. The three DSs in the IBM corresponding to three typical collective modes (or collective shapes) including the spherical vibrator ($U(5)$), axial rotor ($SU(3)$) and $\gamma$-soft rotor ($O(6)$). The often used boson-fermion interaction can be written as [45]

$$\hat{V}_{BF} = \hat{V}_{MON} + \hat{V}_{QUAD} + \hat{V}_{EXC},$$  \hspace{1cm} (7)

which contains the monopole term

$$\hat{V}_{BF}^{MON} = \Lambda \hat{n}_d \hat{\theta}_j,$$  \hspace{1cm} (8)

the quadrupole term

$$\hat{V}_{BF}^{QUAD} = \Gamma \hat{Q}_B^x \cdot \hat{\theta}_F$$  \hspace{1cm} (9)

with

$$\hat{\theta}_F = (\hat{a}_j^{\dagger} \times \hat{a}_j)^{(2)}$$  \hspace{1cm} (10)

and the exchange term

$$\hat{V}_{BF}^{EXC} = \Lambda \sqrt{2j+1} \{[(d^{\dagger} \times \hat{a}_j)^{(0)}] \times (d^{\dagger} \times a_j^{\dagger})^{(0)} \}.$$  \hspace{1cm} (11)

where $\{ \ldots \}$ denotes normal ordering [40]. The interaction strengths $\Lambda_j$, $\Gamma$ and $\Lambda$ in the boson-fermion interaction can in principle be calculated from the fermion-fermion dynamics [45] and semi-microscopically connected with the BCS occupation probabilities [46], but here they are treated as the adjustable parameters.

**B. Diagonalization scheme**

To get the eigenvalues and eigenfunctions, we diagonalize the IBFM Hamiltonian in terms of the weak coupling $SU(3)$ basis

$$|N(\lambda, \mu)\chi(L)JM_j)\rangle = \sum_{M_L, m_L} \langle LM_L, jm_L|N(\lambda, \mu)\chi(LM_L)|jm_j\rangle,$$  \hspace{1cm} (12)

where $N$ is the total boson number, $(\lambda, \mu)$ characterizes the SU(3) irreducible representation and $\chi$ denotes the additional quantum number to distinguish from the different states with the same $(\lambda, \mu)$ and $L$. In Eq. (12), $L, j, J$ represent the angular momentum for the boson core, the odd particle and the whole system with the corresponding third components denoted by $M_L, m_L$ and $M_j$, respectively. The SU(3) coupling basis can be characterized by
the group chain

\[
\left\{ \begin{array}{c}
(U(6) \supset SU(3) \supset SO(3)) \otimes SU(J(2)) \supset SU(J(2)) \supset SO(J(2)) \\
N(\lambda, \mu) \chi L j J M J
\end{array} \right.
\]

(13)

where the additional quantum number \( \tilde{\chi} \) also labels the multiplicity of \( L \) in an \( SU(3) \) representation \((\lambda, \mu)\). It should be mentioned that the complete group symmetry for the fermion part with single \( j \) should replace \( SU(J(2)) \) with

\[
U(2j + 1) \supset SU(2j + 1) \supset SP(2j + 1) \supset SU(j(2))
\]

(14)
especially for the multi-fermion situation because the \( n^2 \) operators \((a^j \times a^j)^k\) with \( k = 0, 1, \ldots, n = 2j \) can generate the maximal group symmetry \( U(2j + 1) \) \[40\]. If only one fermion is taken into account as in the present case, the nontrivial sub-symmetry is just \( SU(j(2)) \).

In the diagonalization, the matrix elements of each term involved in the Hamiltonian (1) can be derived using the \( SU(3) \) algebraic technique. Here, we take the core-particle coupling term

\[
\hat{M}_c = (d^j \times d^j)^2 \cdot (a^j \times a^j)^2
\]

(15)

which corresponds to part of the quadrupole boson-fermion interaction in Eq. (9), as an example to show how to derive the Hamiltonian matrix under the \( SU(3) \) coupling basis. Using the Wigner-Eckart theorem, the matrix element can be derived as

\[
\langle a' L' j' M' j' J M' J | \hat{M}_c | a L j J M J \rangle = \delta_{J J'} \delta_{M M'} (-1)^{\lambda+J+J'} \langle a' L' \| d^j \times d^j \| a L \rangle \times \left\{ \begin{array}{c}
L' j J \\
J L 2
\end{array} \right\} \langle j \| a^j \times a^j \| j \rangle
\]

\[
= -5 \delta_{J J'} \delta_{M M'} (-1)^{\lambda+J+J'} \sum_{\alpha L} \left\{ \begin{array}{c}
2 2 2
\end{array} \right\} \langle L' j J \| d^j \| a' \| \alpha' M' \rangle \langle a' L' \| d^j \| a L \rangle,
\]

(16)

where the abbreviation \( \alpha \equiv N(\lambda, \mu)\tilde{\chi} \) is used. Clearly, the final results will be determined by the reduced elements of the boson operator under the \( SU(3) \) basis. The \( d\)-boson operators can be further expressed as the \( SU(3) \) irreducible tensors, \( T^{(1j\mu)}_{\chi,LM} \). Specifically, it is given \[47\]

\[
d^j_a = A^{(2)}_{1,2a}, \quad \tilde{d}_a = B^{(0)}_{1,2a}.
\]

(17)

Then, the double-barred reduced matrix elements contained in Eq. (16) can be further expanded as

\[
\langle a' L' \| d^j \| a'' L'' \rangle = \sqrt{2L'+1} \langle N|a' L'\| A^{(2)}_{1,0} \| [N-1]|a'' L''\rangle \times \langle (a', \mu') \tilde{\chi}', L'; (2,0) 1, 2 \| (a'', \mu'') \tilde{\chi}'', L'' \rangle
\]

(18)

and

\[
\langle a'' L'' \| d^j \| a L \rangle = \sqrt{2L''+1} \langle (N-1)|a'' L''\| B^{(0,2)}_{1,0} \| [N]|a L\rangle \times \langle (a'', \mu'') \tilde{\chi}'', L'' \| (0, 2) 1, 2 \| (\lambda, \mu) \tilde{\chi}, L \rangle,
\]

(19)

in which the triple-barred matrix elements have been analytically obtained in \[47\] and the isoscalar \( SU(3) \) wigner coefficients \( \langle :, : \rangle \) can be calculated using the algorithm given in \[42, 43\]. The similar derivations can be applied to all the other terms in the IBFM Hamiltonian. Accordingly, the eigenstates of the Hamiltonian can be expanded in terms of the \( SU(3) \) coupling basis as

\[
|N, \xi, JM J \rangle = \sum_{\lambda \mu \tilde{\chi}(L)} C_{\xi, J M J}^{(1, \mu)} N(\lambda, \mu) \tilde{\chi}(L) j J M J \rangle,
\]

(20)

where the expansion coefficients \( C_{\xi, J M J}^{(1, \mu)} \) with \( \xi \) indicating the \( \xi \)th level for a given \( J \) can be obtained through diagonalizing the Hamiltonian.

C. Influences of the boson-fermion interactions

To check the different boson-fermion interactions using the new diagonalization scheme, we make a comparison of the lowest-lying levels in between the IBM and IBFM for the three symmetry limits. In the IBM calculations, the parameters (in MeV) involved in the consistent-Q Hamiltonian (4) are taken as \( (\epsilon_d = 1.0, \kappa = 0) \) for the U(5) limit, \( (\epsilon_d = 0, \kappa = -1/4, \chi = -\sqrt{7}/2) \) for the SU(3) limit and \( (\epsilon_d = 0, \kappa = -1/4, \chi = 0) \) for the O(6) limit. In the IBFM calculations, the three-boson-fermion interaction terms defined in Eq. (8)-(11) are individually added to the IBM Hamiltonian (4) with the adopted parameters given in Fig. 1, where the level patterns calculated for \( j = 9/2 \) and \( N = 10 \) are shown for different cases. In addition, the parameter \( \chi \) involved in the quadrupole term (9) will be set as \( \chi = -\sqrt{7}/2 \) for SU(3) and \( \chi = 0 \) for both O(6) and U(5) in order to be consistent with the consistent-Q Hamiltonian. As seen from Fig. 1(a1)-(a3), the results indicate that the level degeneracies may exactly occur for the states with \( j = 1 \) \( \leq \) \( J \leq j + 1 \) in the IBF if only the monopole term is considered and the associated level pattern in each case is very similar to the corresponding IBM one. Undoubtedly, if no boson-fermion interactions are involved, the exact degeneracies...
will also occur for the states with \( |j - L| \leq J \leq j + L \) and meanwhile the level energies will be exactly equivalent to the corresponding values in the IBM. It means that the monopole term may only cause a renormalization of the boson level energies in each symmetry limit [40]. In contrast, if only the quadrupole term is involved as seen in Fig. 1(b1)-(b3), the levels with different \( J \) are not degenerated anymore. Notably, the quadrupole term at a given strength can cause the level energy splitting in the SU(3) limit much larger than in the U(5) or O(6) limit. As further seen in Fig. 1(c1)-(c3), the exchange term can also break the level degeneracies but with the level order in each case different from the one caused by the quadrupole term. Generally, the three types of boson-fermion interactions are all needed to be taken into account in order to have a quantitatively good descriptions of the experimental data [45, 48].

### III. EFFECTS OF THE ODD PARTICLE ON SPTs

#### A. Shape phase diagram

The SPTs in even-even systems can be illustrated as the QPTs in the IBM. The mean-field analysis indicates that the emergence of an additional odd particle in odd-even systems can cause alternative effects on the SPTs [23–25, 34]. To examine the effects of the odd particle, it is convenient to take the consistent Q IBFM Hamiltonian

\[
\hat{H}_{BF} = \varepsilon \left[ (1 + \eta) b^d_{j+1} - \frac{\eta}{4N} \hat{Q}_{BF} \cdot \hat{Q}_{BF} \right],
\]

where

\[
\hat{Q}_{BF} = \hat{Q}^d_{1} + \hat{Q}^f_{1}
\]

is the quadrupole operator. Compared with Eq. (9), one can derive that the strength of the quadrupole boson-fermion interaction in the Hamiltonian (21) is given as

\[
\Gamma = \frac{\varepsilon \eta}{2N}.
\]

To calculate the \( B(E2) \) transitional rates and quadrupole moments, the transitional operator can be chosen as the quadrupole operator defined in (22). For simplicity, we only take the boson part and the transitional operator is then expressed as

\[
\hat{T}^{E2} = e \hat{Q}^d_{B}
\]

with \( e \) representing the effective charge. In fact, such an approximation agree well with the analysis of some deformed odd-mass nuclei using the Microscopic core-quasiparticle coupling model [35]. Accordingly, the \( B(E2) \) transitional rates and quadrupole moments can be calculated via the formulas

\[
B(E2; J_i \rightarrow J_f) = \frac{|\langle J_f | \hat{T}^{E2} \parallel J_i \rangle|^2}{2J_i + 1}
\]

with \( j \) representing the effective charge. In fact, such an approximation agree well with the analysis of some deformed odd-mass nuclei using the Microscopic core-quasiparticle coupling model [35]. Accordingly, the \( B(E2) \) transitional rates and quadrupole moments can be calculated via the formulas

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\[
B(E2; J_i \rightarrow J_f) = \frac{|\langle J_f | \hat{T}^{E2} \parallel J_i \rangle|^2}{2J_i + 1}
\]
\[
Q(J) = \langle J M_J = J | \sqrt{\frac{16\pi}{5}} T E_2 | J M_J = J \rangle.
\] (26)

In fact, the consistent-\(Q\) Hamiltonian shown above is as same as that adopted in the classical analysis given in [34], where the quadrupole boson-fermion interaction is mainly concerned. It means that some predictions of the classical analysis based on the same Hamiltonian [34] can be checked in the present quantal analysis.

At first, we focus on the boson core dynamics by removing the fermion term \(\hat{q}_F\) in the quadrupole operator (22). The IBFM consistent-\(Q\) Hamiltonian (21) is then reduced back to the IBM consistent-\(Q\) Hamiltonian
\[
\hat{H}_B = \varepsilon \left( 1 - \eta \right) \hat{d}_d - \frac{\eta}{4N} \hat{Q}_B \cdot \hat{Q}_B,
\] (27)
which is as same as the one in (4) but with the parameters rewritten as
\[
\varepsilon_d = \varepsilon (1 - \eta), \quad \kappa = -\frac{\eta}{4N}.
\] (28)

In the discussions, the scale parameter is usually taken as \(\varepsilon = 1.0\). The IBM phase diagram in terms of the parameters in Eq. (27) can be mapped onto the so-called Casten triangle as seen in Fig. 2. It is shown that each vertex of the triangle represents a given DS, i.e. U(5) at \((\eta, \chi) = (0, 0)\), O(6) at \((\eta, \chi) = (1, 0)\) and SU(3) at \((\eta, \chi) = (1, -\frac{\sqrt{10}}{2})\). As mentioned above, these DSs are alternatively associated with different collective modes or collective shapes (deformations). Accordingly, the transitions between different collective shapes are mapped into the QPTs between different DSs and vice versa. It should be noted that the single group G in the IBM phase diagram shown in Fig. 2 should be replaced with the direct product group \(G \otimes \text{U}(j + 1)\) for the IBFM [40], but we keep to use the symbol G to indicate the related situations in both IBM and IBFM for convenience.

Based on the mean-field analysis [14], one can prove that the system in the large-\(N\) limit experiences a 1st-order QPT at \(\eta_c = 8/17 \approx 0.5\) on the U(5)-SU(3) leg and a 2nd-order QPT at \(\eta_c = 0.5\) on the U(5)-O(6) leg. The U(5)-SU(3) QPT may correspond to the spherical to prolate (or the vibrator to axial rotor) SPT in the collective model terminology while the U(5)-O(6) QPT corresponds to the spherical to \(\gamma\)-soft (or the vibrator to \(\gamma\)-soft rotor) SPT. In this work, the two models’ terminology on QPTs will be mutually used without distinction. More generally, the 1st-order spherical to deformed SPTs (the U(5)-SU(3) QPT-like) may widely occur inside the triangle phase diagram with the critical points given as
\[
\eta_c = \frac{14}{28 + \chi^2}.
\] (29)

In the following, we take the cases with \(j = 9/2\) and \(N = 10\) to discuss how an odd particle can affect the U(5)-SU(3) and U(5)-O(6) SPTs in the finite systems. Concretely, we will compare the results solved from the IBFM Hamiltonian (21) to those obtained from the IBM Hamiltonian (27). It should be mentioned that the influences of different boson-fermion interactions on the spectra in the two types of SPTs have been previously investigated in the IBFM both classically and quantum mechanically [26]. In the following, we will focus on revealing the similarities and differences of the critical behaviors in between the odd-even and even-even systems.

**B. Finite-N critical features**

The lowest-lying levels are firstly worked out and the results evolving as functions of \(\eta\) in both the U(5)-SU(3) and U(5)-O(6) transitional regions are given in Fig. 3. As shown in Fig. 3(a), the states with different \(J\) in the odd-even systems are approximately degenerate and divided into groups with the level energies being close to those with \(L = 0, 2, 4\) in the even-even system until \(\eta \sim 0.4\), then the degeneracies are rapidly broken in the range of \(\eta \sim 0.4 - 0.6\) with the levels reorganized in a way of spread. This is actually the finite precursor of the U(5)-SU(3) SPT in odd-even systems. It should be noted that SPT in a finite system may occur in a parameter region rather than at a point due to the finite-\(N\) effect, which also makes the transitional features not as sharp as in the large-\(N\) limit [14], where the concept of QPT is rigorously defined. As further seen in Fig. 3(b), the level degeneracies in the odd-even systems clearly break down in

![Fig. 2. (color online) Shape phase diagram in the IBM described by the Hamiltonian (27). The dashed line denoting the 1st-order transitional points described by (29) cuts the triangle phase diagram into the spherical and deformed regions.](image)
the critical region $\eta \sim 0.4 \sim 0.6$, which confirms that the U(5)-O(6) SPT occurs in the odd-even systems too. In theory, the breaking of level degeneracies can be regarded as a signal for U(5)-O(6) SPT. Meanwhile, the results implies that the transitional features of the 2nd-order QPT (U(5)-O(6) SPT) in a finite-$N$ system may be much weaker than those of the 1st-order QPT (U(5)-SU(3) SPT).

To further identify the critical features in a finite-$N$ situation, the energy ratio $R_{4/2}$ and the $B(E2)$ ratio $B_{4/2}$ are calculated for the two types of SPTs with the corresponding results as a function of $\eta$ shown in Fig. 4 and Fig. 5, respectively. As seen in Fig. 4(a), a sudden increase in $R_{4/2}$ can be observed in the critical region of the U(5)-SU(3) SPT, which confirms again that this type of SPT indeed occurs in the odd-even system as in the adjacent even-even system. It is more interesting to find that the transitional feature in $R_{4/2}$ seems to be more or less enhanced in the odd-even system. In contrast, the results shown in Fig. 4(b) manifest that the finite-$N$ precursor of U(5)-O(6) SPT can be also identified from the evolutions of $R_{4/2}$ but with the transitional amplitude in the odd-even system ($R_{4/2} \sim 2.0 \sim 2.3$) more depressed than in the adjacent even-even system ($R_{4/2} \sim 2.0 \sim 2.5$). It means that the U(5)-O(6) SPT may be smoother in the odd-even nuclei due to the effects of the odd particle, which actually agrees with the classical analysis given in [34]. As further seen in Fig. 5, the results for the $B(E2)$ ratio further confirm the finite-$N$ precursors of the SPTs in the odd-even and even-even systems. It is shown that the U(5)-SU(3) transitional features are indeed strengthened by the effects of the odd particle with the transitional amplitude of $B_{4/2}$ in the odd-even system relatively larger than in the even-even system. In contrast, the U(5)-O(6) SPT features in the odd-even system become relatively weaker with a smaller amplitude of $B_{4/2}$ than in the adjacent even-even systems. It was pointed out [49] that the ratio $B_{4/2}$ in the even-even system can be taken as the effective order parameter to distinguish the 1st-order SPT
(U(5)-SU(3)) from the 2nd-order SPT (U(5)-O(6)) by its different evolitional characteristics in the two types of SPTs. One can find in Fig. 5 that $B_{4/2}$ in the odd-even system can play the same role, as the differences in between the 1st-order and 2nd-order SPTs become even larger due to the effects of the odd particle.

To check the deformations of the finite systems in the SPTs, the quadrupole moments of the even-even system, $Q(J_1)$, and those of the odd-even system, $Q(J)$, have been calculated and the results as a function of $\eta$ are shown in Fig. 6. As clearly seen in Fig. 6(a), the quadrupole moments of the selected states in the U(5)-SU(3) transition may all decrease from the nearly zero values down to the negative values with the fastest change appearing in $\eta = 0.4 \sim 0.6$ as expected. A noticeable case is that $Q(7/2)$ as a function of $\eta$ may slowly increase till $\eta \sim 0.4$ before turning to decrease. It is more interesting to find from the sub-panel that the odd-even differences $\Delta Q$ for the different states nearly all reach their maxima in the critical region, which indicates that the odd particle can induce a larger fluctuation of the quadrupole deformation in the critical systems. This implies that different deformations (phases) may have more chances to coexist in the low-lying structures of the odd-even systems undergoing the U(5)-SU(3) SPT [34]. As to the U(5)-O(6) SPT, one can find from Fig. 6(b) that the quadrupole moments of the even-even system, $Q(2_1)$, remains as zero in the entire transitional process. It is easy to understand this feature from the selection rule as the transitional operator adopted in the calculation for the U(5)-O(6) SPT requires $\Delta L = \pm 2$ for the yrast states [15]. In contrast, the quadrupole moments of the odd-even system, $Q(J)$, may all monotonically decrease from zero to the negative values, which in turn suggests that the largest odd-even difference of quadrupole deformation in the U(5)-O(6) SPT should appear in the O(6) limit. In addition, it is shown that the results in Fig. 6(a) are not exactly equivalent to zero in the case of the U(5) limit. This is mainly because of a different transitional operator $\hat{Q}_0^{2\rightarrow 2} - \hat{Q}_0$ being chosen for the U(5)-SU(3) SPT in order to be consistent with the
In summary, a scheme of diagonalizing the IBFM Hamiltonian in terms of the SU(3) coupling basis has been introduced, by which a quantal analysis of the effect of the odd particle on two types of SPTs is performed in the IBFM through comparing the critical behaviors of some select observables. Like in the even-even systems, it is shown that the U(5)-SU(3) and U(5)-O(6) SPTs can also occur in the odd-even system. More importantly, the results indicate that the effects of the odd particle may further strengthen the U(5)-SU(3) transitional features but weaken the U(5)-O(6) ones. This point agrees well with the previously classical analysis [24, 25, 34] of the two types of SPTs, thus adding observable proofs of the mean-field predictions. It is further revealed that the fluctuations of the quadrupole deformations in the odd-even systems become larger when approaching the critical point of the U(5)-SU(3) SPT, which in turn implies the potential for phase coexistence in the critical odd-A nuclei. The present study is a schematic illustration of the actual situations for odd-A nuclei as the discussions have been confined to the cases with the odd particle assumed to move in a single \( j \) shell. Multi-particle (-hole) in multi-\( j \) cases may need to be taken into account for a more general situation especially for searching for the potential phase coexistence in experiments [28]. Apart from the presently discussed two types of transitions, the effects of an odd particle on other types of SPTs (such as the prolate to oblate transition) remain to be studied [16]. Related work is in progress.
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