Influence of the neck parameter on the fission dynamics within the two-center shell model parametrization

Li-Le Liu(刘丽乐)†  Xi-Zhen Wu(吴锡真)‡  Yong-Jing Chen(陈永静)  Cai-Wan Shen(沈彩万)§  Zhu-Xia Li(李祝霞)†  Zhi-Gang Ge(葛志刚)  Neng-Chuan Shu(舒能川)

1China Nuclear Data Center, China Institute of Atomic Energy, Beijing 102413, China  
2School of Science, Huzhou University, Huzhou 313000, China

Abstract: The influence of the neck parameter on the fission dynamics at low excitation energy is studied based on the three-dimensional Langevin approach in which the nuclear shape is described with the two-center shell model (TCSM) parametrization, and the elongation, the mass asymmetry and the fragment deformation are set to be the generalized coordinates of the Langevin equation. We first study the influence of the neck parameter on the scission configuration. We find that there is almost no obvious correlation between the neck parameter \(\epsilon\) and the mass asymmetry \(\eta\) at the scission point indicating that \(\epsilon\) has no obvious impact on the fragment mass distribution. The elongation \(Z_0/R_0\) and its correlation with the mass asymmetry \(\eta\) at the scission point are obviously influenced by the neck parameter \(\epsilon\), which has a strong effect on the total kinetic energy (TKE) distribution of fragments. The pre-neutron emission fragment mass distributions for 14 MeV \(n^+\) U and \(\ Pu\) are calculated and then based on these results the post-neutron emission fragment mass distributions are obtained by using the experimental data of prompt neutron emission. The calculated post-neutron emission fragment mass distributions can reproduce the experimental data well. The TKE distributions for 14 MeV \(n^+\) U fission are calculated for \(\epsilon=0.25,0.35,0.45\), and the results show that the TKE distribution cannot be described very well for the three cases. However, the trend of the calculated TKE distribution with \(\epsilon\) is just as that is expected from the scission configuration calculations and the results with \(\epsilon=0.35\) present a better agreement with the experiment data compared with the other two cases.

Keywords: nuclear fission, two-center shell model, fragment mass distribution, scission configuration

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I. INTRODUCTION

The phenomena and the mechanism of nuclear fission have been continuously studied for more than eighty years since its discovery, however, a comprehensive model for describing a variety of fission observables has not been presented owing to the extremely complicated fission process. Up to now many methods, such as the phenomenological approach [1-3], the scission point model [4-6] and dynamical models [7-28], have been proposed to calculate the mass yields as well as the total kinetic energy (TKE) distribution of fission fragments, and most of these methods have a high predictive power of calculating mass yields in the large region of nuclides. Nevertheless, the TKE distribution has not been calculated with comparable accuracy, as the TKE of fragments is quite sensitive to the scission configuration which is still far from being completely understood.

The dynamical process of fissioning nucleus evolving from the ground state to the scission point can be viewed as an evolution of the nuclear shape, which is usually described with the multidimensional Langevin equation [7-22] where the generalized coordinates represent the deformation parameters of nuclear shape. A few powerful shape parametrizations [29] were developed to describe the shape of fissioning nucleus, including the two-center shell model (TCSM) parametrization [30]. Based on the TCSM, the three and four dimensional Langevin approach were adopted by several groups [12,13,16,18,20,21] to study fission dynamics at low excitation energies with a fixed neck parameter in the most dynamical calculations for a certain fissioning system. As is known that the neck parameter is considerably correl-
ated with the shape of the fissioning system when the system is largely elongated and consequently influences the neck radius by which the scission point is defined within the Langevin approach. Accordingly, the neck parameter will possibly influence the fission observables such as the mass and the total kinetic energy dependence of the fragments. However, the knowledge about the influence of the neck parameter on the fission dynamics and the scission configuration as well as the fission observables is still lacking.

In our previous work, the three-dimensional Langevin approach within the TCSM parametrization was applied to study the fission dynamics at low excitation energies \cite{21,31}, in which the generalized coordinates are the elongation \( Z_0/R_0 \), the mass asymmetry \( \eta \) and the fragment deformation \( \delta \) with a fixed \( \epsilon \) such as \( \epsilon =0.35 \). Based on the model, the influence of the neck parameter \( \epsilon \) on the fission dynamics is investigated in the present work by setting the \( \epsilon \) to be different values within a reasonable range in the Langevin calculations. We first study the influence of the neck parameter on the scission configuration. We then investigate the influence of the neck parameter \( \epsilon \) on the mass distribution and the TKE distribution of fission fragments, taking the case of 14 MeV \( n+^{235}U \) fission as example. Based on the calculated pre-neutron emission fragment distributions with \( \epsilon =0.35 \), we further calculate the post-neutron fragment mass distributions of 14 MeV \( n+^{233,235,238}U \) and \( ^{238}Pu \) fission by taking account of the prompt neutron emission, and the results can reproduce the experimental data well.

This paper is organized as follows. In Sec. II, a brief introduction of the model is described. The calculated results and discussions are shown in Sec. III. Last, a summary of the present work and future prospects are presented in Sec. IV.

II. METHODS

A. The shape parametrization within the two-center shell model

In this work, the shape of nuclear surface is described with the two-center shell model (TCSM) proposed by J. Maruhn and W. Greiner \cite{30}, in which the nuclear surface is an equipotential surface retaining the same potential and enclosing the same volume as the spherical nucleus throughout nuclear fission under the assumption of volume conservation. The shape of nuclear surface could be obtained by setting the potential \( V(\rho, z) \) equal to the constant potential \( \frac{1}{2}m_0\omega_r^2R_0^2 \) (\( h\omega = 41 \text{ MeV}\cdot\text{A}^{-1} \)). In the TCSM, the central potential \( V(\rho, z) \) is expressed in cylinder coordinates as

\[
V(\rho, z) = \begin{cases}
\frac{1}{2}m_0\omega_r^2\rho^2 + \frac{1}{2}m_0\omega_z^2\rho^2, & z < z_1, \\
\frac{1}{2}m_0\omega_r^2\rho^2(1 + c_1z_1 + d_1z_1^2) + \frac{1}{2}m_0\omega_z^2\rho^2(1 + g_1z_1^2), & z_1 < z < 0, \\
\frac{1}{2}m_0\omega_r^2\rho^2(1 + c_2z_2^2 + d_2z_2^4), & 0 < z < z_2, \\
\frac{1}{2}m_0\omega_r^2\rho^2 + \frac{1}{2}m_0\omega_z^2\rho_0^2z_2, & z > z_2,
\end{cases}
\]  

with \( z_i = z - z_i \). The above potential consists of two smoothly connected oscillator potentials where the positions of the centers locate at \( z_1 \) and \( z_2 \), respectively, and a modified oscillator potential between the centers with considerable deviations caused by the introduction of a variable barrier and by the need of joining the fragments continuously. Figure 1 shows the nuclear shape within the TCSM and the corresponding potential along the symmetry axis \( z \). There are totally 5 free deformation parameters introduced in the model: the elongation parameter \( Z_0/R_0 = (z_2 - z_1)/R_0 \), where \( R_0 \) denotes the radius of the spherical compound nucleus, the fragment deformation parameter \( \delta_1 = (3\beta_1 - 3)/(1 + 2\beta_1) \) (\( \beta_i = a_i/b_i \), \( i = 1, 2 \)), the mass asymmetry \( \eta \) defined by \( \eta = (V_2 - V_1)/(V_2 + V_1) \) (\( V_1 \) and \( V_2 \) are the volumes of the left and right part separated at \( z = 0 \)), and the neck parameter \( \epsilon \) which is defined as the ratio of the actual barrier height \( E \) to the fixed barrier \( E' \) of the deformed oscillator potential located at \( z = 0 \) shown in Fig. 1. In the present work, the left and right fragment deformation are assumed to be the same, i.e., \( \delta_1 = \delta_2 = \delta \).

A series of nuclear shapes corresponding to different elongation \( Z_0/R_0 \) and fragment deformation \( \delta \) are shown
in Fig. 2, in which both the mass asymmetry and neck parameter are fixed. It can be seen that the negative value of the fragment deformation \( \delta \) corresponds to the oblate shape for the prefragment and the positive \( \delta \) corresponds to the prolate shape. With the increase of the elongation \( Z_0/R_0 \), for the large \( \delta \) cases such as \( \delta = 0.2,0.4 \), the neck radius decreases slowly and the more elongated shape is generated in the scission region. As is seen from Fig. 2, when \( \delta = 0.4 \) the system is assumed to correspond to the superlong channel, i.e., the symmetric fission channel for the major actinide nuclei. In contrary, for the cases with \( \delta = 0.0, -0.2, -0.4 \), the system corresponds to the more compact shape in the scission region and separates into two fragments with smaller elongation depending on the \( \delta \).

Figure 3 shows a series of nuclear shapes corresponding to different elongation \( Z_0/R_0 \) and neck parameter \( \epsilon \). One can see that the nuclear shape is insensitive to the neck parameter \( \epsilon \) at the smaller elongation, however, the shape of neck part changes largely with \( \epsilon \) for \( Z_0/R_0 \) larger than 2.0. Consequently, the neck radius decreases very fast and correspondingly, the system separates into fragments fast with the \( \epsilon \) increasing when the elongation \( Z_0/R_0 \) is larger than 2.5. It indicates that the nuclear shape with the smaller \( \epsilon \) will be more elongated at the scission point than that with the larger \( \epsilon \).

![Figure 2](image-url)  
*Fig. 2. (Color online) The nuclear shapes for different values of elongation \( Z_0/R_0 \) and fragment deformation \( \delta \) within the TCSM parametrization (\( \eta = 0.0, \epsilon = 0.35 \)).*

![Figure 3](image-url)  
*Fig. 3. (Color online) The nuclear shapes for different values of elongation \( Z_0/R_0 \) and neck parameter \( \epsilon \) within the TCSM parametrization (\( \eta = 0.0, \delta = 0.2 \)).*

## B. The Langevin approach

The time evolution of collective degrees of freedom of the fissioning nucleus can be viewed as that of a Brownian particle in the heat bath in the stochastic approach. In this work the multi-dimensional Langevin equation is adopted to describe the dynamics of the collective coordinates and has the following form,

\[
\frac{dq_i}{dt} = (m^{-1})_{ij} p_j, \\
\frac{dp_i}{dt} = -\frac{\partial V}{\partial q_i} + \frac{1}{2} \delta (m^{-1})_{ik} p_j p_k - \gamma_i (m^{-1})_{jk} p_j p_k + g_{ij} \Gamma_j(t),
\]  

(2)

where the collective coordinates \( q_i \) represent \( Z_0/R_0, \delta, \eta \) within the TCSM parametrization, and \( p_i \) is the generalized momentum conjugate to \( q_i \). In Eq. (2) and in the following equations the summation convention for repeated indices is taken. In the above equation, \( V \) denotes the potential energy of deformation, \( (m^{-1})_{ij} \) is the inverse of inertia tensor \( m_{ij} \), and \( \gamma_i \) is the friction tensor. For the random force term, the normalized random force \( \Gamma_j(t) \) is obtained by using a Gaussian random generator under the assumption of the white noise, and the strength \( g_{ij} \) is calculated via the fluctuation-dissipation theorem:

\[
g_{ik} g_{jk} = \gamma_{ij} T^*,
\]

(3)

where the effective temperature \( T^* \) is related to the general nuclear temperature \( T \) [32],

\[
T^* = \frac{\hbar \sigma}{2 \coth \frac{\hbar \sigma}{2T}},
\]

(4)

and we use the value 2 MeV for \( \hbar \sigma \) suggested in Ref. [16]. The temperature \( T \) is obtained from the intrinsic excitation energy \( E_{int} \), which is calculated at each step along the Langevin trajectory based on the conservation of energy as follows:

\[
E_{int}(q) = E^* - \frac{1}{2} (m^{-1})_{ij} p_i p_j - V(q, T = 0) = a T^2,
\]

(5)

where \( E^* \) denotes the excitation energy at the initial state, which is the sum of the incident neutron energy and the binding energy, and \( a \) is the level density parameter. Based on the deformation-dependent potential energy, the inertia tensor and friction tensor and the random force simulation, the above Langevin equation could be solved by the second-order Runge-Kutta numerical method. Thus, the generalized coordinates and momenta at each time \( t = n \Delta t \), i.e., the Langevin trajectory, would be calculated when an initial condition and scission condition are given. In this work, we take the initial condition at \( (Z_0/R_0 = 0.5, \delta = 0.2, \eta = 0.0) \) around the first saddle point,
and the scission point is determined by a fixed neck radius to be 0.5 fm. In the Langevin calculations, we take the neck parameters $\epsilon = 0.25$, 0.35 and 0.45, respectively, in order to study the influence of the neck parameter on the fission dynamics. The number of the Langevin trajectories reaches at least $2.5 \times 10^5$ per fissioning system in order to guarantee the enough statistics for the calculated results.

In the Langevin calculations, the potential energy, the inertia tensor and friction tensor are obtained based on the prepared meshes to save the computation time. The mesh values $(Z_0/R_0, \delta, \eta)$ are taken to be

$$Z_0/R_0 = -0.32(0.14)0.02, \quad \delta = -0.45(0.03)0.81, \quad \eta = -0.62(0.04)0.62.$$  

C. The potential energy, inertia tensor and friction tensor

The potential energy is calculated with the macroscopic-microscopic model in the present work, in which the finite range liquid drop model [33,34] is used to calculate the macroscopic energy. The microscopic energy contains the shell correction and pairing correction, which are evaluated using the Strutinsky method [35] and the BCS method [36], respectively, based on the single-particle levels obtained from the TCSM. In addition, the potential energy is dependent on the nuclear temperature as it was given in Ref. [37],

$$V(q, T) = V_{mac}(q) + V_{mic}(q; T = 0)\phi(T), \quad (6)$$

$$\phi(T) = \exp(-aT^2/E_d), \quad (7)$$

with the level density parameter $a = ACN/10$ MeV$^{-1}$. In order to describe well the ratio of the contribution of the asymmetric fission to the symmetric fission, we take the value of 60 MeV for the shell damping parameter $E_d$ in the present work.

The Werner-Wheeler method [38] is adopted to calculate the inertia tensor which is expressed in the following form:

$$m_{ij}(q) = \pi \rho_m \int_{z_{min}}^{z_{max}} \rho_z^2(z, q) \left(\frac{\partial A_i}{\partial q_j} + \frac{1}{8} \frac{\partial A_i}{\partial q_j} \partial A_j \right) dz, \quad (8)$$

$$A_i = \frac{1}{\rho_z^2(z, q)} \frac{\partial}{\partial q_j} \int_{z_{min}}^{z_{max}} \rho_z^2(z, q) dz', \quad (9)$$

where $\rho_z(z, q)$ is the transverse extension of the nucleus at the position $z$ along the symmetry axis, and $q = |q|$ represents the deformation parameter within the TCSM. $\rho_m$ denotes the mass density of the fissioning nucleus and $A'_j$ is the differentiation of $A_i$ with respect to $z$.

The wall-and-window model [39-41] is applied to obtain the friction tensor. For the compact nuclear shape without neck, the wall friction tensor is written as follows:

$$\gamma_{ij}^{wall}(q) = \frac{1}{2} \pi \rho_m \bar{v} \int_{z_{min}}^{z_{max}} dz \left( \frac{\partial \rho_z^2}{\partial q_i} + \frac{\partial \rho_z^2}{\partial z} \frac{\partial D_{ij}}{\partial q_j} \right) \left[ \rho_z^2 + \frac{1}{4} \left( \frac{\partial \rho_z^2}{\partial z} \right)^2 \right]^{-1/2}, \quad (10)$$

where the average velocity of inner nucleons $\bar{v}$ is related to the Fermi velocity by $\bar{v} = \frac{1}{2} \nu_f$. When the nucleus is highly deformed and the neck becomes obviously identified, the window dissipation needs to be taken into account. Thus, the corresponding friction tensor is

$$\gamma_{ij}^{wall2}(q) = \gamma_{ij}^{wall}(q) + \gamma_{ij}^{window}(q), \quad (11)$$

$\gamma_{ij}^{window}(q) = \frac{\rho_m \bar{v}}{2} \Delta \nu \frac{\partial R_{12}}{\partial q_i} \frac{\partial R_{12}}{\partial q_j}, \quad (13)$

where $D_{ij}$ (r=L,R for the left and right part, respectively) is the position of the mass center of the prefragment relative to the mass center of the whole system. $\Delta \nu$ is the area of the window located at the position of the smallest neck radius. $R_{12}$ denotes the distance between the centers of mass of two parts.

A smooth transition between the pure wall friction and the wall-and-window friction proposed by Nix and Sierk [42] is used for the whole fission process and expressed as

$$\gamma_{ij} = \cos \left( \frac{\pi r_N^2}{2 b^2} \right) \gamma_{ij}^{wall} + \sin \left( \frac{\pi r_N^2}{2 b^2} \right) \gamma_{ij}^{wall2}, \quad (14)$$

where $r_N$ is the neck radius and $b$ denotes the lesser of the transverse semi-axis of the two prefragments.

III. THE CALCULATED RESULTS

A. The influence of the neck parameter on the scission configuration

The scission configuration is of fundamental import-
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In addition, the correlations between the elongation $Z_0/R_0$ and the mass asymmetry $\eta$ at the scission point with three different $\epsilon$ cases are shown in the bottom panel of Fig. 4. It can be seen that the $Z_0/R_0$ decreases with the $\epsilon$ increasing, but the shapes of the correlation for the three $\epsilon$ cases are similar which results in the overall increase of the TKE for all fragment mass region with the $\epsilon$ increasing. Moreover, we find that there is a hollow near $\eta=0.10$-0.25 and the depth of the hollow increases with $\epsilon$ increasing, especially, the depth for $\epsilon=0.25$ is obviously shallower than the other two cases. Consequently, a bump around $\eta=0.10-0.25$ (corresponding to the heavy fragment mass region around $A=130$ - 147) will appear in the TKE distribution with its height and peak depending on the $\epsilon$.

The correlations between the elongation $Z_0/R_0$ and the fragment deformation $\delta$ and that between $\delta$ and $\eta$ at the scission point with the $\epsilon$ fixed at 0.25, 0.35 and 0.45, respectively are shown in the upper and middle panels of Fig. 5, respectively. It can be seen that the averaged $Z_0/R_0$ increases nearly linearly with the $\delta$ at the scission point for the fixed $\epsilon$, and the correlation between $\delta$ and $\eta$ is not linear but is similar with that of $Z_0/R_0$ and $\eta$ as there is linear relationship between $Z_0/R_0$ and $\delta$. More significantly, the figure shows that with the $\epsilon$ increasing the curve shifts downwards with the similar slopes and it means that its net effect is increasing the TKE with increasing $\epsilon$. 

![Fig. 4.](insert fig4 here) (Color online) The distribution of the elongation $Z_0/R_0$ at the scission point (top) and the correlation between the elongation and mass asymmetry at the scission point with the neck parameter $\epsilon$ fixed at 0.25, 0.35 and 0.45, respectively (bottom).

![Fig. 5.](insert fig5 here) (Color online) The correlation between the elongation and the fragment deformation (upper panel) and the correlation between the fragment deformation and the mass asymmetry (middle panel) at the scission point with the neck parameter $\epsilon$ fixed at 0.25, 0.35 and 0.45, respectively. The bottom panel shows the averaged nuclear shape of $^{236}$U for $\delta=0.0, 0.15, 0.3$ at the scission point ($\epsilon=0.35$).
Moreover, we show in the bottom panel of Fig. 5 the averaged nuclear shapes for \( \delta = 0.0, 0.15, 0.3 \) at the scission point with the \( \epsilon \) fixed at 0.35, and the corresponding shape parameters \( \{Z_0/R_0, \delta, \eta\} = [2.558, 0.0, 0.187], [2.826, 0.15, 0.171], [3.143, 0.3, 0.097] \). The scission shapes for different values of \( \delta \) are consistent with the behavior of the correlation between the \( Z_0/R_0 \) and the \( \delta \).

The neck radius is an important input for defining the scission point. In order to understand the dynamical effect of \( \epsilon \) on the evolution of neck radius, we further study the evolution of the neck radius of the fissioning nucleus from the ground state to the scission point. Figure 6 presents the contour plot of the neck radius as a function of \( Z_0/R_0 \) and \( \epsilon \) with \( \eta = 0.0 \) and \( \delta = 0.2 \). It shows that the neck radius is insensitive to the \( \epsilon \) for the \( Z_0/R_0 \) smaller than 2.0 which is consistent with that shown in Fig. 3, however, the neck radius decreases obviously with the \( \epsilon \) increasing when the elongation is larger than 2.0. And with the elongation increasing, the neck radius becomes more sensitive to the \( \epsilon \), especially around the scission line shown by the white dotted curve, where the elongation reduces with the \( \epsilon \) increasing which causes the dependence of the TKE on the \( \epsilon \) as is discussed previously. We have also made calculations of the neck radius with the other \( \eta \) and \( \delta \) and we find that the behavior of the neck radius as a function of the \( Z_0/R_0 \) and the \( \epsilon \) is similar for the \( \eta \) and \( \delta \) fixed at other values. Similar to Fig. 6 we show the results for \( \eta = 0.17 \) and \( \eta = 0.35 \) in Fig. 7, the behaviors of which are similar to the case of \( \eta = 0.0 \) shown in Fig. 6. It implies that there is no obvious impact of the \( \epsilon \) on the fragment mass distribution.

B. The influence of the neck parameter on the fission fragment distributions

In this section, we investigate the influence of the neck parameter \( \epsilon \) on the mass distribution and the total kinetic energy (TKE) distribution of fission fragments, taking the case of 14 MeV \( n + ^{235}\text{U} \) fission as example. Figure 8 shows the calculated fission fragment mass distributions using the Langevin approach with the \( \epsilon \) taken to be 0.25, 0.35 and 0.45, respectively, and there is little difference between these results, as is expected from the study given in the previous section. However, a little better fitting to the ENDF/BVIII.0 [43] with \( \epsilon = 0.35 \) case is seen in Fig. 8 compared with the other two cases. Using the 3D Langevin model with the \( \epsilon \) fixed at 0.35 as that used in the previous works, we calculate the pre-neutron fragment mass distributions of 14 MeV \( n + ^{233,235,238}\text{U} \) and \( ^{239}\text{Pu} \) fission, based on which, the post-neutron fragment mass distributions are obtained by taking account of the experimental data of prompt neutron emission \( \nu(A) \) if they are available. Here, the experimental data of \( \nu(A) \) at \( E_n = 14.5 \) MeV [44] are directly adopted for \( n + ^{238}\text{U} \) fission. For 14 MeV \( n + ^{233,235}\text{U} \) and \( ^{239}\text{Pu} \) cases, due to a lack of enough experimental data, we adopt the experimental data \( \nu_{th}(A) \) of thermal neutron induced fission [45-48] which could cover almost the whole fragment mass region, and then evaluate the \( \nu(A) \) for \( E_n = 14 \) MeV by assuming \( \nu(A) = \nu_{th}(A) \cdot \frac{E_n}{E_{th}} \) in which the averaged neut-
ron multiplicity $\nu$ ($E_n=14$ MeV) and $\nu_{th}$ are from ENDF/B-VIII.0. The calculated pre-neutron and post-neutron fragment mass distributions in 14 MeV $n^{233,235,238}\text{U}$ and $^{239}\text{Pu}$ fission are shown in Fig. 9 together with the evaluated post-neutron mass yields from ENDF/B-VIII.0. One can see that the calculated post-neutron fragment mass distributions with $\epsilon=0.35$ (red curve) are overall consistent with the evaluated data both in the peak position and peak width, indicating the power of the present model in describing the fission fragment mass distribution for the major actinides. It should be noted that the calculated mass yields are that of the first-chance fission, which is found to be similar to that of the multi-chance fission at the excitation energy of 20 MeV [49]. In the present study only the fission of the major actinides is involved and the fission of superheavy nuclei will be investigated in the future work.

In this work, the TKE of fragments consists of the precission kinetic energy defined as the collective kinetic energy of the fissioning system in fission direction at the scission point, and the Coulomb repulsion energy between two fragments which is approximately treated as that between two charged point particles located at the centers of mass of two fragments. Figure 9(a) shows the TKE distribution with $\epsilon$ fixed at 0.25, 0.35 and 0.45, respectively, compared with the experimental data. The calculated TKE with $\epsilon=0.35$ agrees with the experimental data to a certain extent, however, the calculated results are several MeV lower when the heavy fragment mass number $A_H$ is around 130, and are several MeV higher around $A_H=140$. One can see that the $\epsilon$ has a significant influence on the TKE calculation, which shows that with the increase of $\epsilon$, the TKE increases and even the peak
The position of the TKE distribution is shifted toward the right side. In order to further study the dependence of the TKE on the neck parameter \( \epsilon \), the corresponding Coulomb repulsion energy at the scission point and the prescission kinetic energy are shown in Figs. 9(b) and 9(c), respectively. It can be seen that the overall behavior of the dependence of the Coulomb repulsion energy on the \( \epsilon \) is similar to that of the prescission kinetic energy, which indicates that the influence of the \( \epsilon \) on the TKE mainly results from the Coulomb repulsion energy which is quite sensitive to the scission configuration. The behavior of the TKE distribution and its dependence on the neck parameter \( \epsilon \) can be understood very well by the results about the scission configuration given in the previous section. The slightly increase of the prescission kinetic energy with the \( \epsilon \) decreasing may be due to the increase of the elongation of the fissioning nucleus around the scission point and simultaneously the decrease of Coulomb energy leading to a larger collective kinetic energy.

**IV. SUMMARY AND DISCUSSION**

In the present paper the influence of the neck parameter on the fission dynamics at low excitation energy is studied based on the three-dimensional Langevin approach within the TCSM parametrization.

We first study the influence of the neck parameter on the scission configuration. We find that there is almost no obvious correlation between the neck parameter \( \epsilon \) and the mass asymmetry parameter \( \eta \) at the scission point, which clearly indicates that the \( \epsilon \) has no obvious impact on the fragment mass distribution. While, the elongation \( Z_0/R_0 \) and its correlation with the mass asymmetry \( \eta \) at the scission point are obviously influenced by the neck parameter \( \epsilon \), which leads to the increase of the fragment total kinetic energy (TKE) and the change of the shape of the TKE distribution with the \( \epsilon \) increasing.

We then investigate the influence of the neck parameter \( \epsilon \) on the mass distribution and the TKE distribution of fission fragments, taking the case of 14 MeV \( n^{235}_+U \) fission as example. The fragment mass distribution is found to be insensitive to the \( \epsilon \) within reasonable range as expected. Based on the calculated pre-neutron emission fragment distributions with \( \epsilon = 0.35 \), we further calculate the post-neutron fragment mass distributions of 14 MeV \( n^{233,235,238}_+U \) and \( ^{239}_{\text{Pu}} \) fission by taking account of the experimental data of prompt neutron emission from the fragments. The results obtained are overall consistent with the post-neutron mass yields from ENDF/B-VIII.0, indicating the power of the present model in describing the fission fragment mass distribution for the major actinides. The \( \epsilon \) has an important influence on the calculation results of the TKE distribution, which can be understood very well by the influence of the \( \epsilon \) on the scission configuration as it is well know that the TKE mainly contributed from the Coulomb repulsion energy strongly depends on the scission configuration. However, there are large deviations between the TKE calculated using the present model and the experimental data. It seems to us that a perfect description of TKE distribution cannot be obtained by a simple fitting of the \( \epsilon \) and more efforts are needed by increasing the dimension of the Langevin calculations and improving the shape description of the potential well for the TCSM.

**References**

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