

R^2 corrections to holographic heavy quarkonium dissociation*

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Abstract: In this study, we investigate the R^2 corrections to the dissociation of heavy quarkonium in the Gauss-Bonnet gravitational background. We analyze the impact of the Gauss-Bonnet parameter λ_{GB} on the spectral function of charmonium and bottomonium and examine the effect of λ_{GB} on the dissociation of heavy quarkonium. Our results show that λ_{GB} reduces the peak height and increases the peak width of the spectral function, suggesting that λ_{GB} enhances the dissociation of heavy quarkonium. We also discuss the variation in the dissociation of heavy quarkonium with the ratio of shear viscosity to entropy density and observe that the dissociation is easier in more perfect plasma. Additionally, we observe that the temperature decreases the peak height and widens the peak, thereby accelerating the dissociation.

Keywords: AdS/CFT correspondence, quarkonium dissociation, quark-gluon plasma

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I. INTRODUCTION

Experimental studies at the Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC) offer unique opportunities to investigate quantum chromodynamics (QCD) matter under extreme conditions [1–4]. The high energies produced in heavy ion collisions create a hot and dense environment, leading to the formation of a new state of matter called quark-gluon plasma (QGP), as expected, owing to the deconfinement phase transition. Heavy quarkonium (J/Ψ and $\Upsilon(1S)$) plays a crucial role in probing the properties of QGP and providing valuable insights into the behavior of strongly coupled plasma. The suppression of heavy quarkonia serves as evidence of their interaction with QGP and has attracted significant interest from researchers [5, 6]. The thermal dissociation of heavy quarkonium refers to the process in which a particle peak in the spectral function disappears.

The results of lattice QCD suggest that J/Ψ could exist as distinct resonances even up to temperatures of approximately $1.6T_c$ (T_c is the critical temperature) and then dissociate in the temperature range of $1.6T_c$ to $1.9T_c$

on anisotropic lattices [7]. Further research indicates that on isotropic lattices, J/Ψ gradually dissociates and disappears at approximately $3T_c$, according to lattice QCD [8]. Additionally, in a potential model, the 1S state of charmonium has been observed to dissociate at $1.2T_c$, and the excited states melt below T_c [9]. Furthermore, lattice QCD predicts that the lower bound of the $\Upsilon(1S)$ state dissociates at approximately $2.3T_c$ [10], suggesting that the ground state of bottomonium can survive in a range from approximately $0.42T_c$ to $2.09T_c$ [11]. The thermal behaviors of quarkonium states have been discussed using QCD sum rules [12, 13].

AdS/CFT correspondence [14–16] could also provide valuable insights into the dissociation of heavy quarkonium. Studies on the dissociation of scalar glueballs and scalar mesons at finite temperature from a holographic perspective have been conducted [17, 18]. Additionally, the influence of a magnetic field on the dissociation of heavy quarkonium has been explored in [19–21]. The effects of temperature and chemical potential on the melting of heavy quarkonium have been analyzed in several studies [22–26]. Furthermore, discussions on the masses and decay constants of heavy quarkonium have been

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presented in [27, 28]. Studies have also been conducted on the quasinormal modes of heavy quarkonium within the framework of holographic QCD models [29–31]. The analysis of the spectral functions of heavy quarkonium in a rotating background has been explored in [32–34]. Moreover, the effects of anisotropy on the dissociation of heavy quarkonium have been discussed in [35]. The spectra of light-flavor hadrons in the dynamical holographic QCD model have been examined in [36]. For the dilepton decays of vector mesons, the authors of [37] calculate the relationship between production rates and the spectral function of mesons within the holographic model. The spectral function of the J/Ψ meson in the soft wall model is discussed in [38]. The dissociation of scalar and vector mesons in two-flavor holographic QCD is investigated in [39]. Additionally, the analysis of the spectral functions of fermions in instantonic plasma is conducted in [40].

String theory widely recognizes that there may be higher derivative corrections owing to the complex interplay between string or quantum effects [41]. The corrections, such as the curvature squared corrections (R^2), are associated with the leading $1/N_c$ corrections when a D7-brane is present [42–44]. R^2 corrections can contribute to a more accurate description of interactions, including quantum gravitational effects or higher-dimensional operators. The R^2 term often represents higher derivative corrections or modifications to the action. The Gauss-Bonnet term naturally emerges when exploring higher-dimensional theories of gravity. In these frameworks, the action typically involves higher derivative corrections, and the Gauss-Bonnet term is one of the simplest corrections that can be made to the gravitational action. These higher derivative corrections are important for understanding various physical phenomena, including their impact on the ratio of shear viscosity to entropy density (η/s) in gravity theories with R^2 corrections [45–47]. Furthermore, higher derivative corrections have been studied in relation to the quark potential [48, 49], drag force [50], jet quenching parameter [51, 52], and energy loss of light quarks [53]. The Gauss-Bonnet term contributes to the formation of black hole solutions, exhibiting various asymptotic behaviors and thermodynamic properties. In addition to the Graviton-Dilaton-Maxwell model, research has been conducted on the QCD thermodynamics and phase diagrams through Einstein-Gauss-Bonnet gravity models in recent years [54–56]. The inclusion of the Gauss-Bonnet term offers new insights into the behavior of strongly-coupled gauge theories. Additionally, higher derivative corrections have been examined in the context of analytic structure of thermal correlators, transport coefficients [57–59], and other quantities [60–68].

The authors of Ref. [23] investigate the melting of heavy mesons in the context of AdS/QCD at finite temperature. However, the spectral functions of heavy

quarkonium within Gauss-Bonnet gravity have not yet been explored. We aim to expand on the findings of Ref. [23] by considering both finite temperature and finite λ_{GB} cases. In this study, we focus on investigating R^2 corrections to the spectral functions of heavy quarkonium in the Gauss-Bonnet gravity and examining the influence of the Gauss-Bonnet parameter λ_{GB} on the dissociation of heavy quarkonium. The results of Refs. [45–47] show that η/s decreases as λ_{GB} increases, indicating that the fluid behaves more like a perfect fluid when increasing λ_{GB} . Thus, studying the connection between η/s and the dissociation of heavy quarkonium may be an interesting research.

The remainder of this paper is organized as follows. In Sec. II, we discuss the holographic model with higher derivative correction. In Sec. III, we describe the spectral function with higher derivative correction. In Sec. IV, we present the conclusion and discussion.

II. HOLOGRAPHIC MODEL WITH HIGHER DERIVATIVE CORRECTION

When the R^2 correction is present, the Gauss-Bonnet term is generally introduced as a correction to Einstein's general relativity. R^2 corrections are associated with the leading $1/N_c$ corrections when a D7-brane is present [42–44]. The effective action to leading order can be expressed as [45, 46]

$$I = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left[R + \frac{12}{L^2} + L^2(m_1 R^2 + m_2 R_{\mu\nu} R^{\mu\nu} + m_3 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}) \right], \quad (1)$$

where R , $R_{\mu\nu}$, and $R_{\mu\nu\rho\sigma}$ denote the Ricci scalar, Ricci tensor, and Riemann tensor, respectively. G_5 represents the Newton constant. In higher-dimensional theories, adding the Gauss-Bonnet term is important for ensuring consistency and stability in the theory. This addition aids in avoiding certain types of singularities in the equations of motion and guarantees that the theory remains well-defined. L denotes the AdS radius at leading order in m_i , where $m_i \sim \frac{\alpha'}{L^2} \ll 1$. It should be mentioned that only m_3 is unambiguous, while m_1 and m_2 could be arbitrarily altered from the field redefinition [45, 46]. To avoid this problem, we can consider Gauss-Bonnet gravity, where m_i is fixed by the parameter λ_{GB} . Gauss-Bonnet gravity is a special case of the action (1), and the action with the λ_{GB} term in Gauss-Bonnet gravity can be given by

$$I = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \times \left[R + \frac{12}{L^2} + \frac{\lambda_{\text{GB}}}{2} L^2 (R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}) \right], \quad (2)$$

where λ_{GB} is valid in the $-\frac{7}{36} < \lambda_{\text{GB}} \leq \frac{9}{100}$ range. The lower bound of the range is from the requirement that the boundary energy density should be positive-definite [69].

The metric of Gauss-Bonnet gravity is [70]

$$ds^2 = \frac{L^2}{z^2} \left(-n^2 f(z) dt^2 + d\vec{x}^2 + \frac{1}{f(z)} dz^2 \right), \quad (3)$$

with

$$f(z) = \frac{1}{2\lambda_{\text{GB}}} \left[1 - \sqrt{1 - 4\lambda_{\text{GB}} \left(1 - \frac{z^4}{z_h^4} \right)} \right], \quad (4)$$

and

$$n^2 = \frac{1}{2} (1 + \sqrt{1 - 4\lambda_{\text{GB}}}), \quad (5)$$

where \vec{x} denotes the boundary coordinates x_1, x_2 , and x_3 , z is the coordinate of fifth dimension, z_h denotes the horizon. The temperature is [70]

$$T = \frac{n}{\pi z_h}. \quad (6)$$

III. SPECTRAL FUNCTION WITH HIGHER DERIVATIVE CORRECTION

In this section, we examine the spectral function of charmonium and bottomonium in Gauss-Bonnet gravity. The formulas for the spectral function can be derived by referring to the work in Ref. [20]. We can represent a heavy quarkonium using the vector field $V_m = (V_\mu, V_z)$, which is dual to the gauge theory current $J^\mu = \bar{\Psi} \gamma^\mu \Psi$. The gravitational action is defined as follows [20]:

$$I = \int d^4x dz \sqrt{-g} e^{-\phi(z)} \left[-\frac{1}{4g_5^2} F_{mn} F^{mn} \right], \quad (7)$$

where $F_{mn} = \partial_m V_n - \partial_n V_m$. In this study, for simplicity, we assume the value of g_5^2 to be 1. The dilaton field $\phi(z)$ can assume the following form [20]:

$$\phi(z) = w^2 z^2 + Mz + \tanh \left(\frac{1}{Mz} - \frac{w}{\sqrt{\Gamma}} \right), \quad (8)$$

where w and Γ denote the quark mass and string tension,

respectively. M represents the mass scale, which is related to the non-hadronic decay. These parameters can be fixed by fitting the spectrum of masses [20]. The values of these parameters for charmonium and bottomonium are

$$\begin{aligned} w_c &= 1.2 \text{ GeV}, \quad \sqrt{\Gamma_c} = 0.55 \text{ GeV}, \quad M_c = 2.2 \text{ GeV}; \\ w_b &= 2.45 \text{ GeV}, \quad \sqrt{\Gamma_b} = 1.55 \text{ GeV}, \quad M_b = 6.2 \text{ GeV}. \end{aligned} \quad (9)$$

The spectral function of heavy mesons can be obtained from the membrane paradigm [71]. We rewrite the background form (Eq. (3)) of Gauss-Bonnet gravity as

$$ds^2 = -g_{tt} dt^2 + g_{xx_1} dx_1^2 + g_{xx_2} dx_2^2 + g_{xx_3} dx_3^2 + g_{zz} dz^2. \quad (10)$$

From Eq. (7), we can obtain the equation of motion

$$\partial^m \left(\frac{\sqrt{-g}}{h(z)} F_{mm} \right) = 0, \quad (11)$$

where $h(z) = e^{\phi(z)}$.

The conjugate momentum of the gauge field for z -foliation is

$$j^\mu = -Q F^{z\mu}. \quad (12)$$

The background of Gauss-Bonnet gravity in \vec{x} -direction is isotropic. Here, we can consider that the equations of motion have longitudinal fluctuations along (t, x_3) and transverse fluctuations along (x_1, x_2) . The longitudinal components of Eq. (11) are

$$\begin{aligned} -\partial_z j^t - \frac{\sqrt{-g}}{h(z)} g^{tt} g^{xx_3} \partial_{x_3} F_{x_3 t} &= 0, \\ -\partial_z j^{x_3} + \frac{\sqrt{-g}}{h(z)} g^{tt} g^{xx_3} \partial_t F_{x_3 t} &= 0, \\ \partial_{x_3} j^{x_3} + \partial_t j^t &= 0. \end{aligned} \quad (13)$$

Applying the Bianchi identity, we obtain

$$\partial_z F_{x_3 t} - \frac{h(z)}{\sqrt{-g}} g_{zz} g_{xx_3} \partial_t j^z - \frac{h(z)}{\sqrt{-g}} g_{tt} g_{xx_3} \partial_{x_3} j^t = 0. \quad (14)$$

The conductivity of the longitudinal channel and its derivative are

$$\begin{aligned} \bar{\sigma}_L(\omega, \vec{p}, z) &= \frac{j^{x_3}(\omega, \vec{p}, z)}{F_{x_3 t}(\omega, \vec{p}, z)}, \\ \partial_z \bar{\sigma}_L &= -i\omega \sqrt{\frac{g_{zz}}{g_{tt}}} \left[\Sigma(z) - \frac{\bar{\sigma}_L^2}{\Sigma(z)} \left(1 - \frac{p_3^2}{\omega^2} \frac{g^{xx_3}}{g^{tt}} \right) \right], \end{aligned} \quad (15)$$

where the momentum $p = (\omega, 0, 0, p_3)$, and $\Sigma(z) = \frac{1}{h(z)} \sqrt{\frac{-g}{g_{zz}g_{tt}}} g^{xx_3}$.

Similarly, we can obtain the result of the transverse channel as

$$\partial_z \bar{\sigma}_T = i\omega \sqrt{\frac{g_{zz}}{g_{tt}}} \left[\frac{\bar{\sigma}_T^2}{\Sigma(z)} - \Sigma(z) \left(1 - \frac{p_3^2}{\omega^2} \frac{g^{xx_3}}{g^{tt}} \right) \right]. \quad (16)$$

For zero momentum ($p_3^2 = 0$), we can find $\bar{\sigma} = \bar{\sigma}_T = \bar{\sigma}_L$, and the form is

$$\partial_z \bar{\sigma} = i\omega \sqrt{\frac{g_{zz}}{g_{tt}}} \left[\frac{\bar{\sigma}^2}{\Sigma(z)} - \Sigma(z) \right]. \quad (17)$$

From the Kubo formula, we find that AC conductivity σ is relevant with the Retarded Green function

$$\sigma(\omega) = -\frac{G_R(\omega)}{i\omega} \equiv \bar{\sigma}(\omega, z=0). \quad (18)$$

Thus, we can obtain the spectral function

$$\rho(\omega) \equiv -\text{Im}G_R(\omega) = \omega \text{Re}\bar{\sigma}(\omega, 0). \quad (19)$$

Using the parameters of Eq. (9), we can calculate the spectral function of charmonium and bottomonium numerically. In calculations, we set AdS radius L to be 1.

In Fig. 1, we present the spectral functions for charmonium at $T = 0.15$ GeV for various values of λ_{GB} . The quasiparticle state is characterized by the bell-shaped pro-

file of the spectral function. The first and second peaks correspond to the 1S state (J/Ψ) and the 2S state, respectively. From the results shown in Fig. 1, it is evident that increasing λ_{GB} leads to a reduction of the 2S state.

In Fig. 2, we plot the spectral functions for charmonium at $T = 0.3$ GeV for different values of λ_{GB} . The peak width is inversely proportional to the decay rate, suggesting stability. We clearly observe that increasing λ_{GB} causes the spectral function peak to decrease in height and broaden in width, indicating the promotion of the dissociation effect of J/Ψ .

In Fig. 3, we present the spectral functions for bottomonium at $T = 0.2$ GeV with different values of λ_{GB} . The first and second peaks represent the 1S state ($\Upsilon(1S)$) and 2S state, respectively. The results shown in Fig. 3 indicate that the 2S state gradually diminishes as λ_{GB} increases.

In Fig. 4, we show the spectral functions for bottomonium at $T = 0.3$ GeV for various λ_{GB} values. The peak corresponds to the 1S state ($\Upsilon(1S)$). We observe that the peak height decreases as λ_{GB} increases, whereas the peak width increases with increasing λ_{GB} . This indicates that λ_{GB} enhances the dissociation effect of $\Upsilon(1S)$.

From the results observed in Figs. 1–4, we can conclude that λ_{GB} promotes the dissociation of heavy quarkonium in Gauss-Bonnet gravity. In Refs. [45–47], the authors investigate the effect of the parameter λ_{GB} on the ratio of shear viscosity to entropy density (η/s) in Gauss-Bonnet gravity, $\frac{\eta}{s} = \frac{1}{4\pi}(1 - 4\lambda_{\text{GB}})$. They find that η/s decreases as λ_{GB} increases, indicating that the fluid behaves more like a perfect fluid when increasing λ_{GB} .

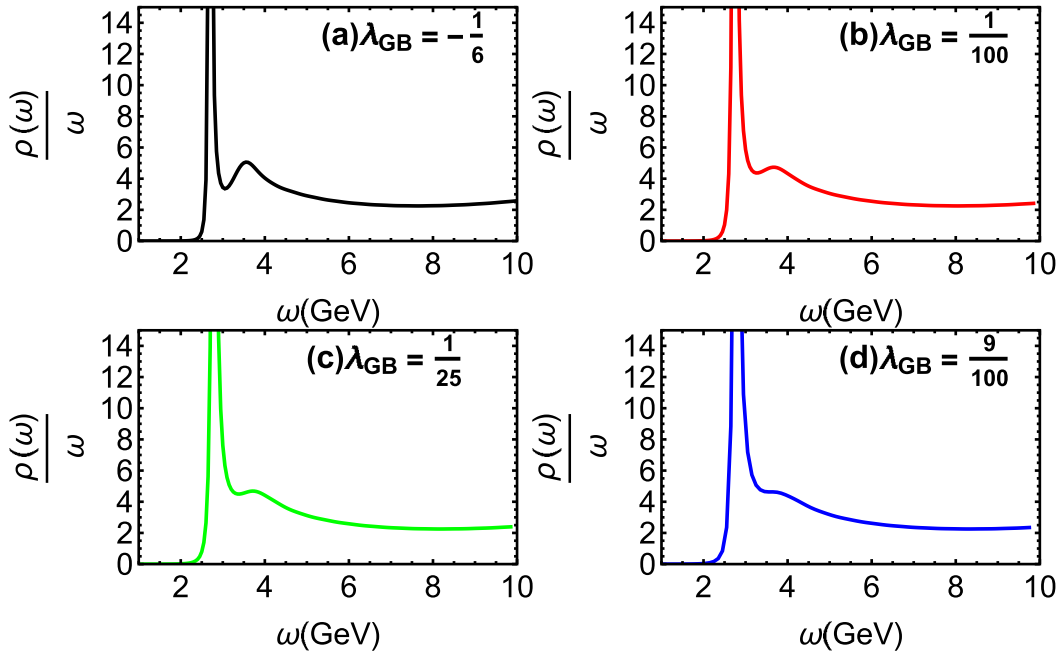


Fig. 1. (color online) Spectral functions of charmonium at $T = 0.15$ GeV for different values of λ_{GB} .

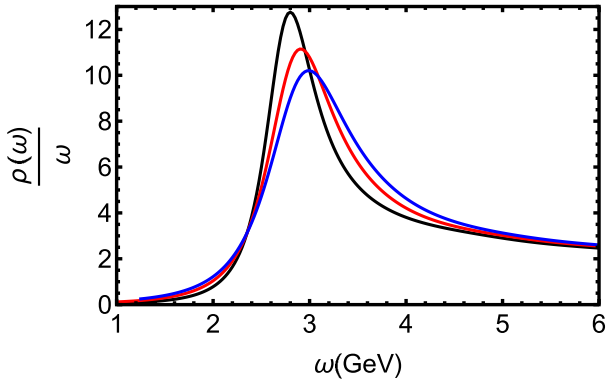


Fig. 2. (color online) Spectral functions of charmonium at $T = 0.3$ GeV for different values of λ_{GB} . The black, red, and blue lines denote $\lambda_{GB} = -\frac{1}{6}$, $\frac{1}{100}$, and $\frac{9}{100}$, respectively.

Consequently, we can conclude that the dissociation of heavy quarkonium is easier in more perfect plasma. Moreover, η/s decreases 38.56% when increasing λ_{GB} from $-1/6$ to $9/100$. The peak of J/Ψ decreases by 24% and $\Upsilon(1S)$ decreases by 64.29% when increasing λ_{GB} from $-1/6$ to $9/100$.

In Ref. [31], the authors calculate the configuration entropy of bottomonium in Gauss-Bonnet gravity and find that λ_{GB} enhances the dissociation effect. In Ref. [64], the authors examine the impact of λ_{GB} on the entropic force of the heavy quarkonium. They find that λ_{GB} increases the entropic force and enhances the dissociation of heavy quarkonium.

The qualitative results of our study are consistent with the results of Refs. [31, 64].

In [72], the authors examine the confinement/deconfinement transition within the soft wall model, incorporating Gauss-Bonnet corrections. They determine that the phase transition temperature $T_c \sim 0.19$ GeV when the chemical potential and Gauss-Bonnet term are zero. The authors also note that the effect of the Gauss-Bonnet term on the phase transition temperature is minimal. Therefore, in this study, we adopt the phase transition temperature $T_c \sim 0.19$ GeV at $\lambda_{GB} = \frac{1}{100}$ for simplicity.

In Fig. 5, we analyze the spectral functions of charmonium at $\lambda_{GB} = \frac{1}{100}$ for different temperatures. From Fig. 5 (a), we observe that the $2S$ state and J/Ψ coexist at

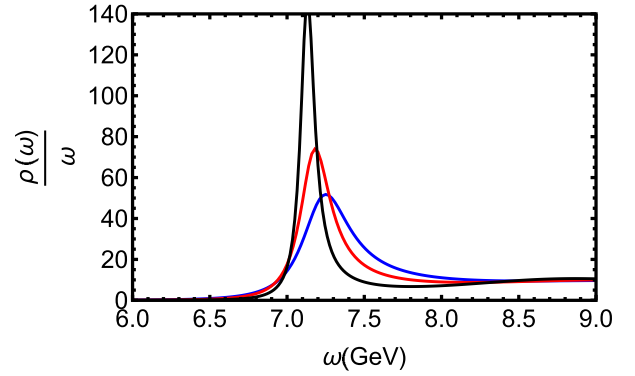


Fig. 4. (color online) Spectral functions of bottomonium at $T = 0.3$ GeV for different values of λ_{GB} . The black, red, and blue lines denote $\lambda_{GB} = -\frac{1}{6}$, $\frac{1}{100}$, and $\frac{9}{100}$, respectively.

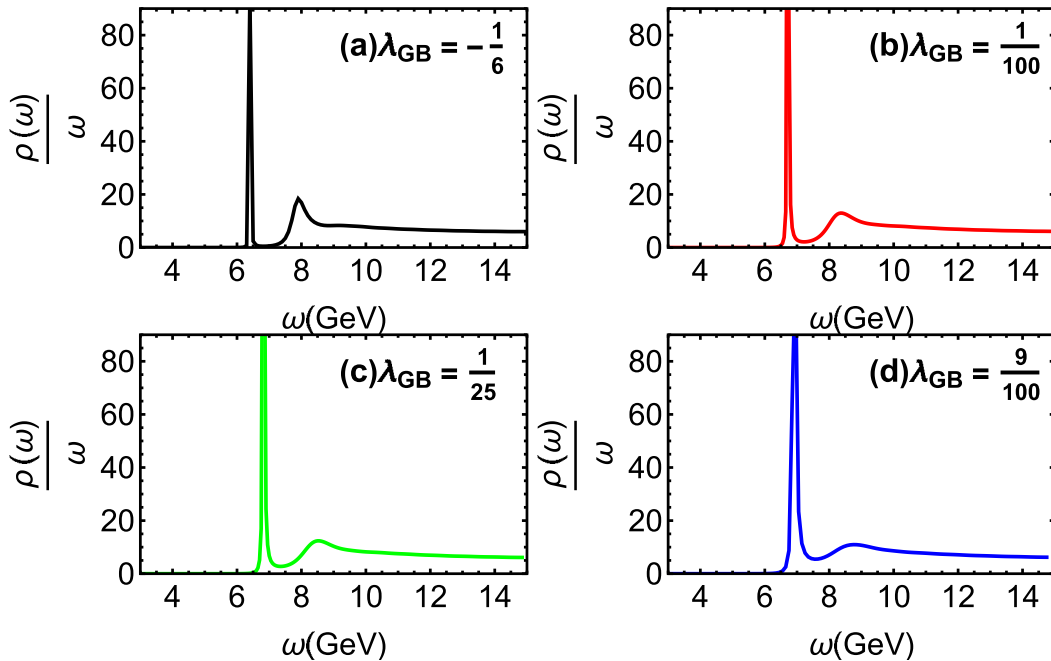


Fig. 3. (color online) Spectral functions of bottomonium at $T = 0.2$ GeV for different values of λ_{GB} .

low temperatures. As λ_{GB} increases, the $2S$ state gradually disappears, with total melting occurring at $T = 0.175 \text{ GeV}$ ($T = 0.92T_c$), as shown in Fig. 5 (b). This finding is consistent with results of the potential model [9]. Additionally, J/Ψ disappears as the temperature increases, with complete melting occurring at $T = 0.6 \text{ GeV}$ ($T = 3.16T_c$), as depicted in Figs. 5 (c) and (d). This res-

ult aligns closely with findings from lattice QCD [8].

In Fig. 6, we examine the spectral functions of bottomonium at $\lambda_{\text{GB}} = \frac{1}{100}$ for various temperatures. From Fig. 6 (a), we observe that the $2S$ state and $\Upsilon(1S)$ coexist at low temperatures. As λ_{GB} increases, the $2S$ state gradually diminishes, with total melting happening at

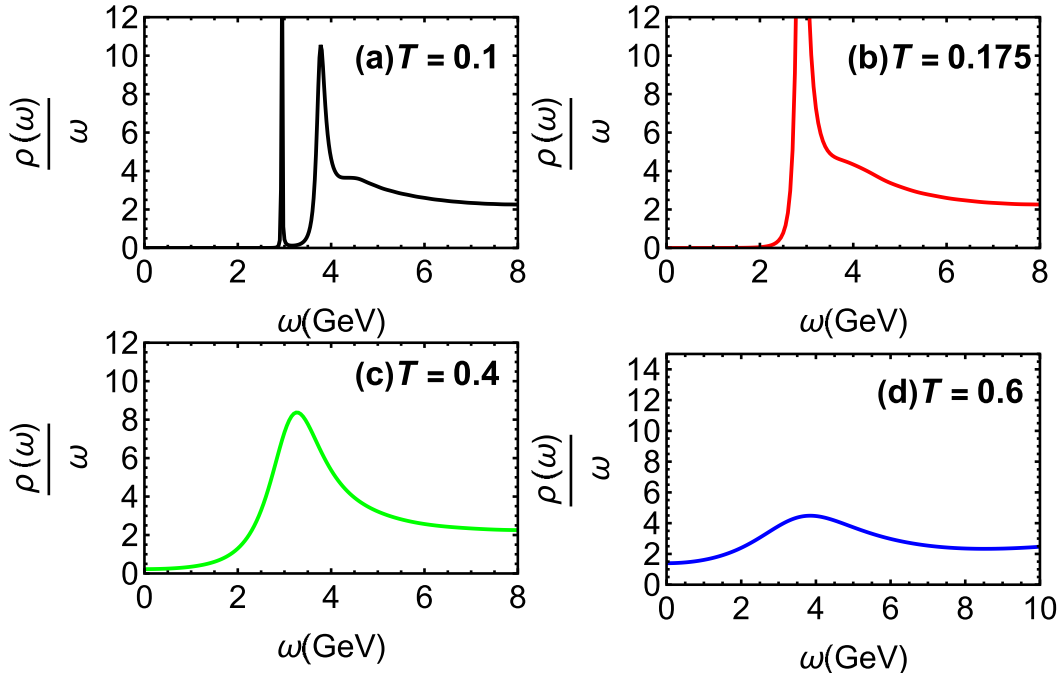


Fig. 5. (color online) Spectral functions of charmonium at $\lambda_{\text{GB}} = \frac{1}{100}$ for different values of T . T is in units GeV.

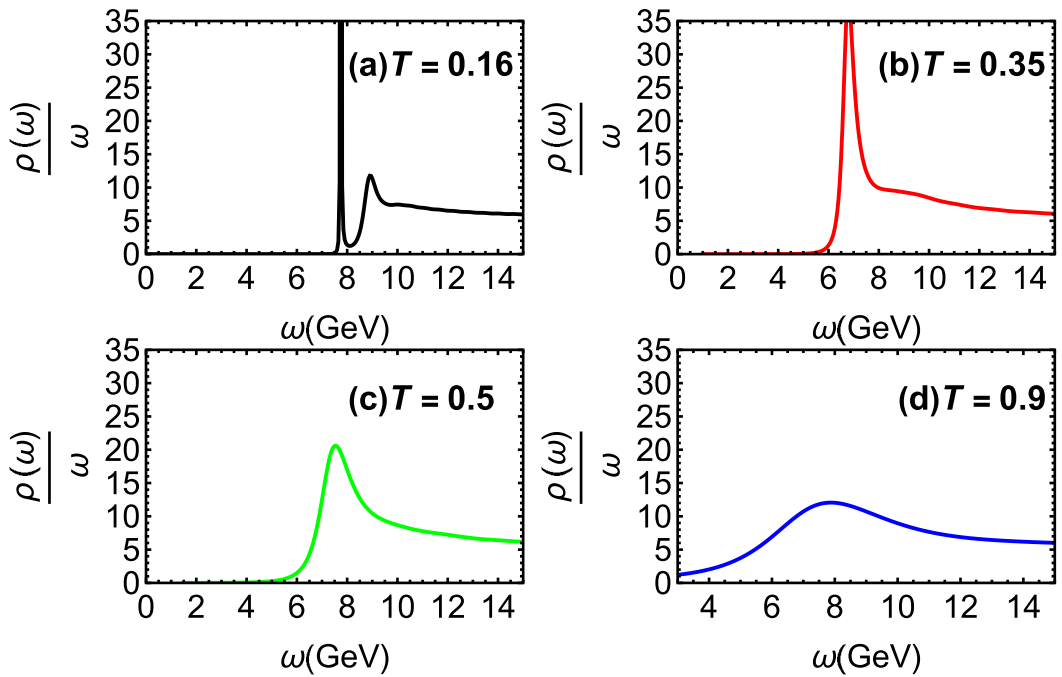


Fig. 6. (color online) Spectral functions of bottomonium at $\lambda_{\text{GB}} = \frac{1}{100}$ for different values of T . T is in units GeV.

$T = 0.35$ GeV ($T = 1.84T_c$), as shown in Fig. 6 (b). This indicates that $\Upsilon(1S)$ also begins to melt at $T = 1.84T_c$, which closely corresponds with the results from lattice QCD [10]. Furthermore, $\Upsilon(1S)$ continues to disappear as the temperature increases, with complete melting occurring at $T = 0.9$ GeV ($T = 4.74T_c$), as shown in Figs. 6 (c) and (d). From the results of Fig. 5 and Fig. 6, we can conclude that the temperature promotes the dissociation effect of heavy quarkonium.

IV. CONCLUSION AND DISCUSSION

In this study, we investigate the R^2 corrections to the spectral functions in Gauss-Bonnet gravity, specifically focusing on the effect of the Gauss-Bonnet parameter λ_{GB} on the dissociation of heavy quarkonium.

Evidently, increasing λ_{GB} causes the peak of the spec-

tral functions to decrease in height and broaden in width. These findings indicate that the dissociation effect of heavy quarkonium is promoted. Thus, we can conclude that λ_{GB} promotes the dissociation of heavy quarkonium in Gauss-Bonnet gravity. Moreover, as the temperature increases, the peak height decreases, and the peak width increases. This suggests that the temperature promotes the dissociation effect of heavy quarkonium. We also discuss the variation in the dissociation of heavy quarkonium with the ratio of shear viscosity to entropy density and observe that the dissociation is easier in more perfect plasma.

We expect this study to provide valuable insights into the behavior of strongly coupled plasma. Another interesting aspect to study is the effect of R^4 corrections on the spectral functions. We hope to report such a study in the future.

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