

Weak magnetic effect in quark-gluon plasma and local spin polarization*

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Abstract: We propose the weak magnetic effect, which emerges as the dissipative correction to the quark phase space distribution function in quark-gluon plasma close to local thermal equilibrium, as a novel contribution to the observed Lambda hyperon local spin polarization. A finite field strength that is consistent with previous estimates of the magnetic field in heavy-ion collisions can be used to explain the experimentally observed Lambda local spin polarization through all centrality classes. Moreover, the weak magnetic effect plays an unambiguous role in the ordering between the second-order and third-order modulations of experimental Lambda local spin polarization.

Keywords: heavy-ion collisions, magnetic field, Lambda local spin polarization

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I. INTRODUCTION

Thermodynamic characteristics can be effectively inferred through experimental measurements once a system reaches local thermal equilibrium. As indicated by the equilibrium density operator, $\lg \hat{\rho} \sim \alpha \hat{N} - \beta_\mu \hat{p}^\mu + \omega_{\mu\nu} \hat{M}^{\mu\nu}/2$ (cf. Ref. [1]), the constants of motion ($\hat{N}, \hat{p}^\mu, \hat{M}^{\mu\nu}$), which usually correspond to gauge symmetry group and the Poincaré group generators, are conjugate to the thermodynamic variables: chemical potential over temperature ($\alpha = \mu/T$), fluid four-velocity over temperature ($\beta_\mu = u_\mu/T$), and the thermal vorticity tensor ($\omega_{\mu\nu} = \omega_{\mu\nu}^{\text{th}} \equiv -\frac{1}{2}(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu)$). Since these constants of motion are experimental observables, the chemical potential, temperature, and other physical quantities can be obtained by measuring charge, momentum, and angular momentum of particles in a thermal system.

Such measurements have been made extensively, particularly in high-energy heavy-ion experiments. For instance, the observed hadron spectra – momentum distribution – reveals the temperature, baryon chemical potential, and flow velocity of a quark-gluon plasma (QGP) [2, 3]. Other differential measurements, including the hadron collective flow and flow correlations, indicate that the space-time distribution of temperature in the QGP fireball originates hydrodynamically from an almond-shape geometry [4–6].

Recently, the study has been advanced to the analysis of hadron spin polarization to elucidate the rotational nature of QGP [7–12]. Indeed, assuming that QGP

achieves local thermal equilibrium, spin-1/2 particles are expected to be polarized owing to thermal vorticity [13–17]. Following the Cooper-Frye formula [13, 18],

$$P^\mu(\mathbf{p}) = -\frac{1}{8m} \epsilon^{\mu\alpha\beta\sigma} p_\sigma \frac{\int d\Sigma \cdot p n_F (1 - n_F) \omega_{\alpha\beta}}{\int d\Sigma \cdot p n_F}, \quad (1)$$

with n_F the Fermi-Dirac distribution, $\epsilon^{\mu\alpha\beta\sigma}$ the totally antisymmetric Levi-Civita symbol, and $\omega_{\mu\nu} = \omega_{\mu\nu}^{\text{th}}$, the global spin polarization of Lambda hyperons emitted from a hyper-surface Σ can be successfully characterized [9–11, 19–21].

However, thermal vorticity fails to describe the differential spectrum of the Lambda spin polarization, which is often referred to as the local spin polarization (see Ref. [15] for a recent review). In particular, when the thermal vorticity is projected along the beam axis (z -axis), it yields an azimuthal distribution of the Lambda spin polarization with a sign that is opposite to experimental observations [22–27]. This discrepancy has triggered extensive research into spin generation through QGP beyond thermal vorticity and the local thermal equilibrium condition [28–37].

In this Letter, with respect to the conversion of quark spin from QGP fluid, we introduce the dissipative effect induced by a weak external magnetic field. It has been noticed that for a field strength $|eB| \ll m_\pi^2$, where m_π is the pion mass, and when the bulk evolution of QGP is barely

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affected, quarks can be driven slightly out of local thermal equilibrium [38, 39]. The out-of-equilibrium effect leads to a correction in the quark distribution function, which, for quark species with electric charge Q , is [39]

$$\delta f_{\text{EM}}(x, \mathbf{p}) = n_F(1 - n_F) \frac{\sigma_{\text{el}}}{T\chi_{\text{ch}}} Q p_\mu F^{\mu\nu} \beta_\nu, \quad (2)$$

where σ_{el} is the electrical conductivity, χ_{ch} is an effective charge susceptibility, and $F^{\mu\nu}$ is the electromagnetic field strength tensor. Equation (2) is a scalar function solution where the quark spin is averaged. Correspondingly, when the weak magnetic field is considered, the conversion of quark spin is modified, and the Cooper-Frye formula in Eq. (1) becomes

$$P^\mu(\mathbf{p}) = -\frac{1}{8m} \epsilon^{\mu\alpha\beta\sigma} p_\sigma \frac{\int d\Sigma \cdot p [n_F(1 - n_F) + (1 - 2n_F)\delta f_{\text{EM}}] \omega_{\alpha\beta}}{\int d\Sigma \cdot p (n_F + \delta f_{\text{EM}})}. \quad (3)$$

In realistic heavy-ion collisions, since the magnetic field is created and orientated out of the reaction plane, the $p_\mu F^{\mu\nu} u_\nu$ factor in Eq. (2) results in a dipole structure in the azimuthal angle distribution. Therefore, the extra δf_{EM} does not contribute to the global spin polarization in terms of local spin polarization. However, it is potentially significant. Eq. (3) is the major result of this Letter.

We use the natural units throughout the letter. We use the $g^{\mu\nu} = (+, -, -, -)$ matrix convention and take $\epsilon^{0123} = 1$ for the Levi-Civita symbol.

II. DERIVATION OF EQ. (3)

In the presence of an electromagnetic field, the evolution of quarks with spin degrees of freedom in QGP medium satisfies the Boltzmann-Vlasov equation [40–42]:

$$p^\mu \partial_\mu \mathcal{F} + Q F^{\mu\nu} p_\mu \frac{\partial \mathcal{F}}{\partial p^\nu} = -\mathcal{C}[\mathcal{F}] = -(p \cdot u) \frac{\mathcal{F} - \mathcal{F}_{\text{eq}}}{\tau_R}. \quad (4)$$

Note that the quark distribution function is a Hermitian matrix in the spin subspace, $\mathcal{F} = \mathcal{F}^\dagger$. For QGP close to local thermal equilibrium, the collision kernel can be linearized via the relaxation time approximation. In principle, relaxation time τ_R is a function of $p \cdot u$, depending on microscopic dynamics. In this Letter, we take the so-called quadratic ansatz reported in Ref. [43]: $\tau_R = \bar{\tau} p \cdot u / T$, where $\bar{\tau}$ is a free parameter determined by transport coefficients¹⁾. Because quarks in QGP are dominated by strong and electromagnetic forces, the collision kernel, as well as the parameter $\bar{\tau}$, are expected even with

respect to the charge conjugation symmetry. Accordingly, subject to a charge conjugation transformation, Eq. (4) applies to anti-quarks. In the following, we focus on the derivation of quarks, while results associated with anti-quarks can be inferred upon charge conjugate symmetry.

Using the Chapman-Enskog method, with the knowledge of the equilibrium quark distribution function [13],

$$\mathcal{F}_{\text{eq}} = \frac{1}{2m} \bar{U}(p) X(x, p) U(p), \quad (5)$$

Eq. (4) can be solved in steps [39, 40]. In Eq. (5), $U(p)$ is the amalgamated spinor solution for free spin up and spin down fermions: $U(p) = (u_+(p), u_-(p))$ and $\bar{U}(p) = U^\dagger \gamma^0$. The matrix $X(x, p)$ is identified as $X(x, p) \equiv \left(e^{\beta \cdot p} \exp \left[-\frac{1}{2} \omega_{\mu\nu} \Sigma^{\mu\nu} \right] + 1 \right)^{-1}$, where $\Sigma^{\mu\nu} = \frac{i}{4} [\gamma^\mu, \gamma^\nu]$ is the relativistic rotation generator realized through the gamma matrix. At the leading order of $|eB|/T^2$, $\mathcal{F} = \mathcal{F}_{\text{eq}} + \delta \mathcal{F}_{\text{EM}}$, where the dissipative correction is

$$\delta \mathcal{F}_{\text{EM}} = -\frac{\bar{\tau}}{T} Q F^{\mu\nu} p_\mu \frac{\partial}{\partial p^\nu} \mathcal{F}_{\text{eq}} = \frac{1}{2m} \bar{U}(p) Y(x, p) U(x, p), \quad (6)$$

with

$$\begin{aligned} Y(x, p) &\equiv -\frac{\bar{\tau}}{T} Q F^{\mu\nu} p_\mu \frac{\partial X}{\partial p^\nu} \\ &= \frac{\bar{\tau}}{T} Q F^{\mu\nu} p_\mu \beta_\nu e^{\beta \cdot p} X^2 \exp \left(-\frac{1}{2} \omega_{\alpha\beta} \Sigma^{\alpha\beta} \right). \end{aligned} \quad (7)$$

To derive Eq. (6), we used the Dirac equation $(\not{p} - m)U(p) = 0$, which implies that $F^{\mu\nu} p_\mu \frac{\partial}{\partial p^\nu} U(p) = 0$ and $F^{\mu\nu} p_\mu \frac{\partial}{\partial p^\nu} \bar{U}(p) = 0$. The hermiticity of $\delta \mathcal{F}_{\text{eq}}$ can be verified with respect to the condition $Y^\dagger = \gamma^0 Y \gamma^0$.

In accordance with the dissipative correction in the conserved current of hydrodynamics, the dissipative correction in the phase-space distribution function satisfies the Landau's matching condition. In particular, for a QGP medium with local charge neutrality, the leading order dissipative correction to the current is driven by the external electromagnetic field:

$$\Delta j^\mu = \sigma_{\text{el}} E^\mu = \sum Q_f \int \frac{d^3 \mathbf{p}}{p^0} p^\mu \text{tr}_2 [\delta \mathcal{F}_{\text{EM}} - \delta \bar{\mathcal{F}}_{\text{EM}}], \quad (8)$$

where the trace tr_2 is taken in the spin subspace and the summation takes considers flavors and colors. To evaluate the trace, using trace cyclicity, the condition that $U(p)\bar{U}(p) = \sum_{\text{spin}} u(p)\bar{u}(p) = \not{p} + m$, and the vanishing trace

¹⁾ We have numerically tested the effect of linear ansatz, for which the relaxation time is independent of momentum, $\tau_R = \bar{\tau}$. With respect to local spin polarization of the lambda hyperon, the results are not changed qualitatively.

of three gamma matrices, for quarks with electric charge Q ,

$$\begin{aligned} \text{tr}_2 \delta \mathcal{F}_{\text{EM}} &= \frac{1}{2} \text{tr}_2 Y = \frac{\bar{\tau}}{2T} Q F^{\mu\nu} p_\mu \beta_\nu e^{\beta \cdot p} \text{tr}_2 \\ &\times \left[X^2 \exp \left(-\frac{1}{2} \omega_{\alpha\beta} \Sigma^{\alpha\beta} \right) \right] \\ &= \frac{\bar{\tau}}{2T} Q F^{\mu\nu} p_\mu \beta_\nu e^{\beta \cdot p} \sum_{m,n=1} (-1)^{m+n+2} e^{-(m+n)\beta \cdot p} \text{tr}_2 \\ &\times \left[\exp \left(\frac{m+n-1}{2} \omega_{\alpha\beta} \Sigma^{\alpha\beta} \right) \right] \\ &= -\frac{\bar{\tau}}{2T} Q F^{\mu\nu} p_\mu \beta_\nu (4n'_F + \frac{\omega_{\alpha\beta} \omega^{\alpha\beta}}{2} n''_F). \end{aligned} \quad (9)$$

In the above expression, n'_F prime indicates a derivative with respect to $p \cdot u/T$. For instance, $n'_F = -n_F(1-n_F)$. A couple of comments are in order. First, we have used the approximated identity in the derivation [13]: $\text{tr}_2 \left[\exp \left(\frac{n}{2} \omega_{\alpha\beta} \Sigma^{\alpha\beta} \right) \right] \approx 4 + \frac{n^2}{2} \omega^{\alpha\beta} \omega_{\alpha\beta}$. Secondly, substituting the trace back to the matching condition, Eq. (8) yields an expression that relates $\bar{\tau}$ to the electrical conductivity σ_{el} . In the limit $\omega \rightarrow 0$, one arrives at the standard relation $\sigma_{\text{el}} = \bar{\tau} \chi_{\text{el}}$ [39]. Accordingly, up to a factor of two, which amounts to a summation over spin, the first term in Eq. (9) can be identified as the scalar function δf_{EM} . In sum, as expected, in a non-rotating system, the spin averaged quark distribution function reduces to $\frac{1}{2} \text{tr}_2 \mathcal{F} = n_F + \delta f_{\text{EM}}$.

The Cooper-Frye formula in Eq. (1) is a variant of the the Pauli-Lubanski pseudo-vector,

$$\Pi_\mu^{\text{PL}} = -\frac{1}{2m} \epsilon_{\mu\rho\sigma\tau} p^\tau S^{\rho\sigma}, \quad (10)$$

where $S^{\rho\sigma}$ corresponds to the angular momentum operator. Spin polarization of particles emitted from QGP, in particular, can be described when Π_μ^{PL} is evaluated on a hypersurface that meets the freeze out condition in heavy-ion collisions and as the averaged expectation of particles in phase space. As has been shown previously [13], because $S^{\mu\nu}$ has contributions only from spin, it can be determined from the spin density tensor in relation to the quark distribution function in a QGP that is out of local thermal equilibrium [40]:

$$\begin{aligned} s^{\lambda\rho\sigma}(x) &= \frac{1}{2} \int \frac{d^3\mathbf{p}}{2p^0} \text{tr}_2 (\mathcal{F} \bar{U}(p) \{\gamma^\lambda, \Sigma^{\rho\sigma}\} U(p)) \\ &= \int \frac{d^3\mathbf{p}}{2p^0} [p^\lambda \Theta^{\rho\sigma} + p^\rho \Theta^{\sigma\lambda} + p^\sigma \Theta^{\lambda\rho}], \end{aligned} \quad (11)$$

where the rank-2 tensor function is

$$\Theta^{\mu\nu}(x) \equiv \text{tr}_2 [(X+Y)\Sigma^{\mu\nu}] = [n_F(1-n_F) + (1-2n_F)\delta f_{\text{EM}}] \omega^{\mu\nu}. \quad (12)$$

Accordingly, the average with respect to the quark spectrum is

$$P_\mu = \langle \Pi_\mu^{\text{PL}} \rangle = -\frac{1}{2m} \epsilon_{\mu\rho\sigma\tau} p^\tau \frac{1}{\text{tr}_2 \mathcal{F}} \frac{ds^{0,\rho\sigma}}{d^3\mathbf{p}} = -\frac{1}{4m} \epsilon_{\mu\rho\sigma\tau} p^\tau \frac{\Theta^{\rho\sigma}}{\text{tr}_2 \mathcal{F}}. \quad (13)$$

Following the Cooper-Frye prescription [44] and integrating over a hypersurface, Eqs. (12) and (13) lead to Eq. (3). Note that the derivation does not rely on the explicit form of $\omega^{\mu\nu}$. Hence, Eq. (3) applies when other contributions beyond the thermal vorticity tensor, including the shear-induced polarization, are considered.

III. LOCAL SPIN POLARIZATION IN HEAVY-ION COLLISIONS

In realistic heavy-ion collisions, the collision region must be an electromagnetic field pointing out of the reaction plane, namely, $\mathbf{B} = B_y \hat{y}$. Although the field strength is expected to be initially strong, it drops drastically [45–52]. The magnetic field likely becomes weak when the QGP medium is formed and begins to expand hydrodynamically [51, 52]. Although the magnetic effect in QGP is too weak to influence the bulk medium evolution, it has been shown to be significant for some signatures, including thermal photon spectrum anisotropy [38, 39] and thermal di-lepton polarization [53].

In a weak magnetic field, as indicated by Eq. (3), although quark polarization by QGP rotation remains unchanged, the quark spin configuration in phase space can be dramatically different. To demonstrate this effect, we apply Eq. (3) to the hydrodynamical modeling of QGP evolution and calculate the spin polarization of s -quarks emitted from the chemical freeze-out hypersurface. Because the spin of a Lambda hyperon is dominantly carried by the s -quark, as u -, d - and s -quark coalesce [54], the calculation is expected to provide a good approximation to the polarization of the Lambda hyperon. Of course, as a consequence of approximate chiral symmetry breaking during the coalescence, we leave the s -quark mass m_s as a free parameter.

With respect to the isobar collision systems with $\sqrt{s_{NN}} = 200$ GeV, we conduct hydrodynamical simulations using the 3+1D MUSIC program [55]. Initial conditions are chosen and calibrated according to the centrality class determined by charged particle multiplicity and a longitudinal profile that reproduces rapidity dependent direct flow [56]. To capture the thermal vorticity associated with the elliptic and triangular initial geometry, we deform the optical Glauber model distribution with a fi-

nite ellipticity and triangularity following the prescription introduced in Ref. [57]. In addition to the standard set of parameters that have been used in hydrodynamical modeling, such as the shear and the bulk viscosities, we used the mean value of electrical conductivity $\sigma_{\text{el}}/T = 1$, with respect to the p QCD expectation $\sigma_{\text{el}}/T \in [0.2, 2]$ [58, 59]¹⁾. To avoid complexity from unknown space-time evolution, for the magnetic field, we take $\mathbf{B} = B\Gamma(\eta_s)\hat{y}$, with B representing a constant parameter and the space-time rapidity dependence $\Gamma(\eta_s)$ determined from the retarded potential solution [50]. In this way, $|eB|$ is the time-average field strength in the center of the QGP.

Lambda local spin polarization along the beam axis can be quantified when it is decomposed into sine Fourier modulations. Under the local thermal equilibrium condition, the local thermal vorticities in QGP are expected from the medium response to the elliptic and triangular geometries. As shown in Fig. 1, we solve the Lambda local spin polarization for RuRu collisions with impact parameter $b = 7$ fm and $m_s = 0.8$ GeV. Indeed, in Fig. 1 (a), where $w_{\mu\nu}$ is identified as $w_{\mu\nu}^{\text{th}}$, without a magnetic field, the negative second-order and third-order modulations of the local spin polarization are obtained. It is interesting to notice that the magnitude for the second order is larger, which is consistent with the hierarchy between elliptic and triangular flow.

To resolve the sign problem of the Lambda local spin polarization, a number of contributions have been proposed beyond thermal vorticity [31–35]. Of particular interest, the shear tensor $\xi_{\mu\nu} = \frac{1}{2}(\partial_\mu\beta_\nu + \partial_\nu\beta_\mu)$, which appears in systems in local thermal equilibrium, gives rise to the so-called shear-induced polarization $\omega_{\mu\nu}^{\text{SIP}}$. Corresponding to the two types of formulation, $\omega_{\mu\nu}^{\text{SIP}}(\text{LY})$ by Liu and Yin [32] and $\omega_{\mu\nu}^{\text{SIP}}(\text{BBP})$ by Becattini, Bucciattini, and Palermo [33], with the replacement $\omega_{\mu\nu} \rightarrow \omega_{\mu\nu}^{\text{th}} + \omega_{\mu\nu}^{\text{SIP}}$ in Eq. (3), we re-calculate the Lambda local spin polarization. Results are shown in Fig. 1 (b) and (c).

In summary, Fig. 1 shows the following. First, in all cases, with the magnetic field introduced, both modulations in the local spin polarization increase monotonically. Eventually, with a weak magnetic field, they become positive regardless of the contribution from shear-induced polarization. We emphasize that the sign change of the local polarization reflects the more significant influence of δf_{EM} in the weighted integral with respect to local vorticity at the freeze-out hypersurface. That is, in Eq. (2), δf_{EM} can overwhelm n_F in terms of the conversion of local spin. However, it does not imply $|\delta f_{\text{EM}}| > |n_F|$. Secondly, the second-order modulation appears more sensitive to the magnetic field than third-order modulation, while sensitivity is reduced by shear-in-

duced polarization. Lastly, and importantly, although the shear-induced polarization can make the local Lambda spin polarization positive [60], the second-order modulation being greater than third-order modulation is an unambiguous signature associated with the magnetic field. All of these qualitative features are not affected by the s -quark mass although for smaller m_s , spin conversion from the QGP is easier, which effectively reduces the required field strength.

Experimental observables, especially centrality dependent Lambda local spin polarization, can be well captured in the presence of a weak magnetic field. Although our current calculation is approximate, we achieve excellent characterizations with $m_s = 0.8$ GeV and $w_{\mu\nu} = \omega_{\mu\nu}^{\text{th}} + \omega_{\mu\nu}^{\text{SIP}}(\text{BBP})$. As shown in Fig. 2 (a), the STAR measured $n = 2$ modulation of the Lambda local spin polarization in the isobar systems can be reproduced [61], with the centrality dependent $|eB|$ extracted and given in Fig. 2 (b) (blue symbols). The error bars in Fig. 2 (b) are from experimental uncertainties and $\sigma_{\text{el}}/T \in [0.2, 2]$. For comparison, in Fig. 2 (b), the estimated field strength from the direct photon v_2 in AuAu collisions are shown as well [39]. It is interesting to notice that the extracted field strength grows towards peripheral collisions. The third-order modulation of the Lambda local spin polarization can be described in the same field strength, and the results are shown as the green shaded band in Fig. 2 (a).

In the centrality class 20% – 60%, the STAR collaboration also measured the transverse momentum dependent local spin polarization for the Lambda hyperon [61]. Although the ordering in Fig. 2 (a) is not obvious, as shown in Fig. 3, from the p_T dependent Lambda local spin polarization, in the low p_T region, a larger $n = 2$ modulation is observed than in $n = 3$. As we emphasized previously, this ordering indicates the presence of a weak magnetic field. Moreover, our theoretical calculation gives a consistent description of the p_T dependence. The deviations at large p_T , where hydrodynamics becomes invalid and perturbative contributions to spin local polarization can be important, should not be a surprise.

IV. SUMMARY AND DISCUSSION

For the conversion of quark spin from QGP, and consequently, the spin polarization of Lambda hyperon, we observed a novel effect originating from the weak magnetic field. Unlike coupling via a finite magnetic moment of the hyperon μ_Λ , namely, $\omega_{\mu\nu} \rightarrow \omega_{\mu\nu} + QF_{\mu\nu}\mu_\Lambda/T$ [14], this novel effect reflects a small dissipative correction in the quark phase space distribution. The weak magnetic effect can be crucial to the observed Lambda local spin

1) While from lattice calculations, one expects a smaller electrical conductivity (one order of magnitude smaller), the combined factor $\frac{\sigma_{\text{el}}}{T} \frac{eB}{m_\pi^2}$, remains small in our calculations, which does not change the formulation. On the other hand, it should be aware that the exact value of the electrical conductivity in realistic QGP systems is undetermined as it depends on strong coupling constant, QGP local thermal equilibration, etc.

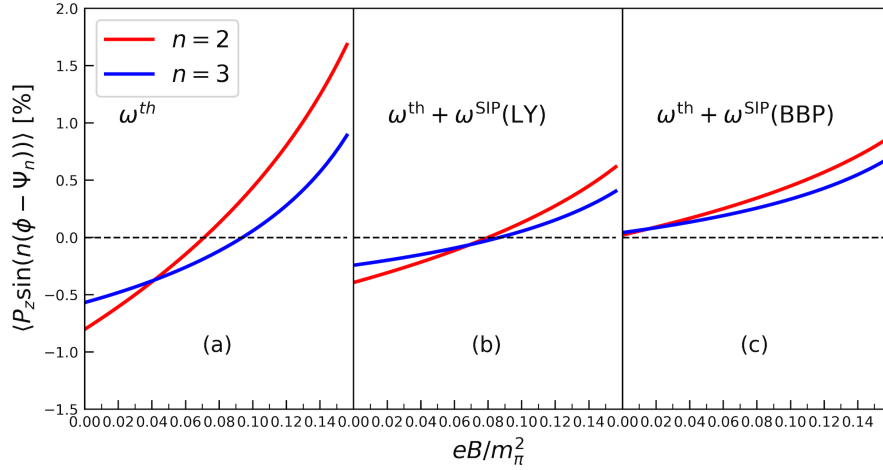


Fig. 1. (color online) Dependence of the second-order modulation (red) and the third-order modulation (blue) of Lambda local spin polarization on the magnetic field.

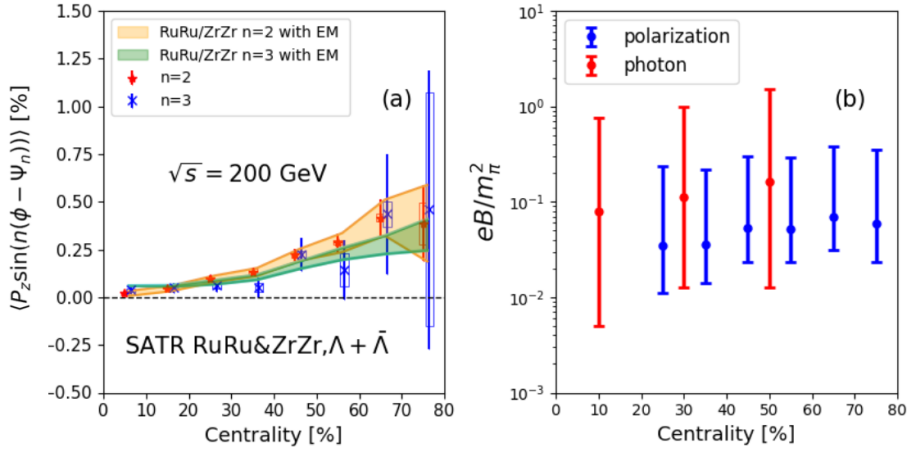


Fig. 2. (color online) (a) Centrality dependent Lambda local spin polarization from the isobar collisions: symbols are experimental results from the STAR collaboration [61] while the shaded band corresponds to theoretical results with magnetic field for $\omega = \omega^{\text{th}} + \omega^{\text{SIP}}$ (BBP). (b) Centrality dependence of the magnetic field extracted according to the Lambda local spin polarization (blue symbols) and the direct photon elliptic flow (red symbols) [39].

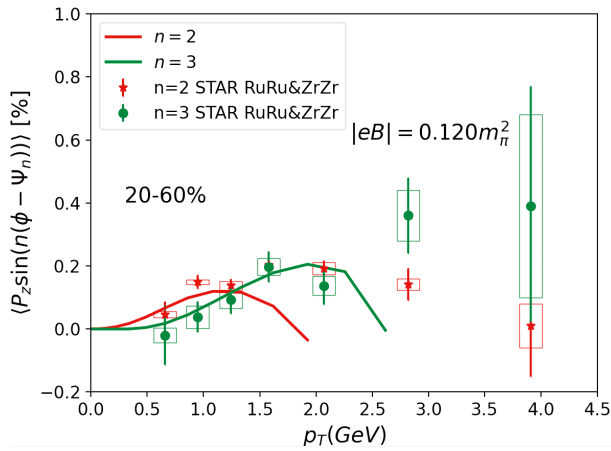


Fig. 3. (color online) Transverse momentum dependent Lambda local spin polarization for the $n=2$ (red) and $n=3$ (green) modes of the isobar collisions in the 20%–60% centrality class.

polarization, not only because it correctly captures the sign observed in experiments, but also because it determines the ordering of the second- and third-order modulations of the local spin polarization. Using hydrodynamical simulations, the experimentally measured Lambda local spin polarization can be well understood. The required fields in different centralities, as shown in Fig. 2 (b), are consistent with the estimations based on direct photon elliptic flow. These are indeed weak fields as they satisfy $|eB| \ll m_\pi^2$. In fact, using the mean value from Fig. 2 (b), the time integrated value can be obtained as $\mathcal{B} = \int dt eB \lesssim 50 \text{ MeV}$, which is too weak to induce splitting in global polarization between Λ and $\bar{\Lambda}$ [62].

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