

S, T, U Parameters in The B-LSSM*

Sheng-Kai Cui (崔生恺)^{1†} Ke-Sheng Sun (孙科盛)^{2‡} Yu-Li Yan (阎玉立)^{3§} Jin-Lei Yang (杨金磊)^{3,4,5¶}
Tai-Fu Feng (冯太傅)^{1,3,4,5,6*}

¹Department of Physics, Guangxi University, Nanning, 530004, China

²Department of Physics, Baoding University, Baoding 071000, China

³Department of Physics, Hebei University, Baoding 071002, China

⁴Hebei Key Laboratory of High-precision Computation and Application of Quantum Field Theory, Baoding, 071002, China

⁵Hebei Research Center of the Basic Discipline for Computational Physics, Baoding, 071002, China

⁶Department of Physics, Chongqing University, Chongqing, 401331, China

Abstract: Using the pinch technique, we compute the one-loop vertices of weak interactions in the B-LSSM and incorporate their pinch contributions into the gauge boson self-energies. Compared to the definitions of the S , T , and U parameters in the Standard Model based on the $SU(2)_L \otimes U(1)_Y$ group, the corresponding parameters in the local B-L gauge symmetry (B-LSSM) are modified. We provide these redefined S , T , and U parameters and demonstrate the convergence of the results. In the framework of the low-energy effective Lagrangian for weak interactions, the S , T , and U parameters can be expressed as functions of certain parameters in the B-LSSM. The updated experimental and fitting results constrain the parameter space of the B-LSSM strongly.

Keywords: Pinch Technique, Electroweak Interaction, B-LSSM

DOI: 10.1088/1674-1137/ae5590

CSTR:

I. INTRODUCTION

The measurement results for the mass of the W boson, as reported by the ATLAS experimental group in 2023 [1], are in close agreement with the theoretical predictions of the Standard Model (SM). This high-precision agreement provides a stringent benchmark for probing theories beyond the SM. In particular, models with extended gauge sectors, such as the B-LSSM, must accommodate this result, which can lead to significant constraints on their parameter space. In comparison to the SM, the inclusion of the additional $U(1)_{B-L}$ gauge group in the B-LSSM leads to the emergence of a more substantial Z' boson, which in turn modifies the original definition of the Weinberg angle. The electroweak radiative corrections of observable quantities can be obtained through the oblique parameter method, and the superfields in the B-LSSM exert a notable influence on these oblique parameters. In this paper, we derive the gauge-in-

variant gauge boson self-energies in the B-LSSM. Based on experimental results, we impose constraints on the parameters of the model.

The B-LSSM [2–18] is based on the gauge symmetry group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_{B-L}$, where B represents the baryon number and L represents the lepton number. The B-LSSM not only provides an explanation for the existence of a small mass for left-handed neutrinos but also offers a solution to the little hierarchy problem [19] in the Minimal Supersymmetric Standard Model (MSSM). The gauge invariance of $U(1)_{B-L}$ allows for the conservation of R-parity, which is typically assumed in the MSSM to prevent proton decay. When this symmetry is spontaneously broken, R-parity conservation can still be maintained [20]. This model helps to understand the origin of R-parity and the potential ways it could be broken in supersymmetric models. Additionally, compared to the MSSM, the B-LSSM provides more candidates for dark matter [21, 22].

Received 9 October 2025; Accepted 19 March 2026

* The work was supported by the Natural Science Foundation of Guangxi Autonomous Region with Grant No. 2022GXNSFDA035068. The Natural Science Foundation of Hebei Province under Grant No. A2022104001, the Foundation of Baoding University under Grant No. 2023Z01

[†] E-mail: cuisk261@163.com

[‡] E-mail: sunkesheng@126.com

[§] E-mail: yychanghe@sina.com

[¶] E-mail: jlyang@hbu.edu.cn

^{*} E-mail: fengtf@hbu.edu.cn



Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI. Article funded by SCOAP³ and published under licence by Chinese Physical Society and the Institute of High Energy Physics of the Chinese Academy of Sciences and the Institute of Modern Physics of the Chinese Academy of Sciences and IOP Publishing Ltd

The S , T , and U parameters [23–33] present an extension of the method developed by Peskin [34], building on the work of Kennedy and Lynn [35], to handle radiative corrections in electroweak interaction processes. Although the B-LSSM introduces an additional $U(1)_{B-L}$ gauge group compared to the SM, the oblique parameter framework remains a powerful tool at the electroweak scale. The new gauge sector modifies the observables directly via tree-level gauge kinetic mixing and mass matrix deformations, which can be elegantly encoded into the effective S , T , and U parameters.

When calculating physical observables or considering physical processes, their gauge invariance is typically taken into account. It becomes critical when extracting physical information from S-matrix elements, as gauge invariance cannot always be guaranteed. The oblique parameters discussed in this paper must account for gauge invariance. To address this issue, the pinch technique (PT), a well-established method based on the background field method (BFM), is employed. This technique has found widespread adoption and application. For more details, see references [36–49].

While many studies on oblique parameters rely heavily on Standard Model Effective Field Theory (SMEFT) global fits, it is important to recognize the conceptual distinction between these effective coefficients and the full theoretical calculations. To provide a more fundamental theoretical foundation, this work aims to avoid "over-fitting" the full theory directly to these effective parameters. Instead, we start from the formal theory of the B-LSSM to rigorously derive the gauge-invariant analytic expressions at the one-loop level, focusing on the physical trends and sensitivities revealed by the theoretical calculation itself.

Based on this robust framework, our numerical analysis strategy focuses on characterizing the dominant new physics sources. First, we systematically analyze how the mass mixing in the key supersymmetric sectors (namely the squark and neutralino-chargino sectors) influences the electroweak precision observables at the loop level. Second, for the additional $U(1)_{B-L}$ gauge parameters, we employ a complementary approach, utilizing the effective Lagrangian to impose direct constraints at the tree level. This dual approach lays the necessary methodological foundation and serves as a physical template for understanding how different symmetry-breaking sectors collectively impact low-energy precision measurements.

The structure of this paper is outlined as follows: In Section II, we provide a brief introduction to the B-LSSM and the construction of the Z' boson. In Section III, we present the gauge-invariant self-energies in the B-LSSM, derived using the pinch technique. In Section IV, we introduce the gauge-invariant S , T , and U parameters in the B-LSSM, and based on these results, we further constrain the gauge mixing angle s' by incorporating effect-

ive Lagrangian techniques. In Section V, we proved the divergence cancellation of two sectors. In Section VI, we analyze the experimental data's constraints on the parameter space. Finally, in Section VII, we present our conclusions.

II. THE B-L SSM

The superpotential of the B-LSSM is given by

$$W = Y_u^{ij} \hat{Q}_i \hat{H}_2 \hat{U}_j^c + \mu \hat{H}_1 \hat{H}_2 - Y_d^{ij} \hat{Q}_i \hat{H}_1 \hat{D}_j^c - Y_e^{ij} \hat{L}_i \hat{H}_1 \hat{E}_j^c + Y_{\nu,ij} \hat{L}_i \hat{H}_2 \hat{\nu}_j^c - \mu' \hat{\eta}_1 \hat{\eta}_2 + Y_{x,ij} \hat{\nu}_i^c \hat{\eta}_1 \hat{\nu}_j^c, \quad (1)$$

where i, j are generation indices, $\hat{\eta}_1 \sim (1, 1, 0, -1)$, $\hat{\eta}_2 \sim (1, 1, 0, 1)$ are two chiral singlet superfields and $\hat{\nu}$ are three generations of right-handed neutrinos. The gauge group of $U(1)_{B-L}$ spontaneously broke without the simultaneous breaking of R-parity. The soft breaking terms of the B-LSSM are given by

$$\begin{aligned} \mathcal{L}_{soft} = & \left[-\frac{1}{2}(M_1 \tilde{\lambda}_B \tilde{\lambda}_B + M_2 \tilde{\lambda}_W \tilde{\lambda}_W + M_3 \tilde{\lambda}_g \tilde{\lambda}_g + 2M_{BB'} \tilde{\lambda}_B \tilde{\lambda}_{B'} \right. \\ & + M_{B'} \tilde{\lambda}_{B'} \tilde{\lambda}_{B'}) - B_\mu H_1 H_2 - B_{\mu'} \tilde{\eta}_1 \tilde{\eta}_2 + T_{u,ij} \tilde{Q}_i \tilde{u}_j^c H_2 \\ & + T_{d,ij} \tilde{Q}_i \tilde{d}_j^c H_1 + T_{e,ij} \tilde{L}_i \tilde{e}_j^c H_1 + T_V^{ij} H_2 \tilde{\nu}_i^c \tilde{L}_j + \\ & T_x^{ij} \tilde{\eta}_1 \tilde{\nu}_i^c \tilde{\nu}_j^c + h.c. \left. \right] - m_{\tilde{\nu},ij}^2 (\tilde{\nu}_i^c)^* \tilde{\nu}_j^c - m_{\tilde{q},ij}^2 \tilde{Q}_i^* \tilde{Q}_j \\ & - m_{\tilde{u},ij}^2 (\tilde{u}_i^c)^* \tilde{u}_j^c - m_{\tilde{\eta}_1}^2 |\tilde{\eta}_1|^2 - m_{\tilde{\eta}_2}^2 |\tilde{\eta}_2|^2 - m_{\tilde{d},ij}^2 (\tilde{d}_i^c)^* \tilde{d}_j^c \\ & - m_{\tilde{L},ij}^2 \tilde{L}_i^* \tilde{L}_j - m_{\tilde{e},ij}^2 (\tilde{e}_i^c)^* \tilde{e}_j^c - m_{H_1}^2 |H_1|^2 - m_{H_2}^2 |H_2|^2. \end{aligned} \quad (2)$$

The terms with tilde denotes the supersymmetric partner of the corresponding chiral superfield. To obtain the masses of the physical neutral Higgs bosons, The Higgs fields are usually redefined as:

$$\begin{aligned} H_1^1 &= \frac{1}{\sqrt{2}}(v_1 + \text{Re}H_1^1 + i\text{Im}H_1^1), \\ H_2^2 &= \frac{1}{\sqrt{2}}(v_2 + \text{Re}H_2^2 + i\text{Im}H_2^2), \\ \tilde{\eta}_1 &= \frac{1}{\sqrt{2}}(u_1 + \text{Re}\tilde{\eta}_1 + i\text{Im}\tilde{\eta}_1), \\ \tilde{\eta}_2 &= \frac{1}{\sqrt{2}}(u_2 + i\text{Re}\tilde{\eta}_2 + i\text{Im}\tilde{\eta}_2). \end{aligned} \quad (3)$$

In accordance with this definition, the majority of the symmetries pertaining to the $SU(2)_L \otimes U(1)_Y \otimes U(1)_{B-L}$ groups have been disrupted, with only the residual electromagnetic symmetry observed in the electromagnetic symmetry groups $U(1)_{B-L}$ remaining intact. The group $U(1)_{B-L}$ introduces new gauge boson Z' and the corresponding gauge coupling constant g_B . In addition, two

Abelian groups give rise to a new effect absent in the MSSM or other SUSY models with just one Abelian gauge group: the gauge kinetic mixing. Here, we write the covariant derivative:

$$D_\mu = \partial_\mu - i \begin{pmatrix} Y & Y_B \end{pmatrix} \begin{pmatrix} \tilde{g} & \tilde{g}_{YB} \\ \tilde{g}_{BY} & \tilde{g}_B \end{pmatrix} \begin{pmatrix} \tilde{A}_\mu^Y \\ \tilde{A}_\mu^B \end{pmatrix}, \quad (4)$$

where \tilde{A}_μ^Y and \tilde{A}_μ^B denote the gauge fields associated with the two $U(1)$ gauge groups. Y and Y_B denote the hypercharge and $B-L$ charge respectively. It is easier to work with non-canonical covariant derivatives instead of off-diagonal field-strength tensors:

$$D_\mu = \partial_\mu - i \begin{pmatrix} Y & Y_B \end{pmatrix} \begin{pmatrix} \tilde{g} & \tilde{g}_{YB} \\ \tilde{g}_{BY} & \tilde{g}_B \end{pmatrix} R^T R \begin{pmatrix} \tilde{A}_\mu^Y \\ \tilde{A}_\mu^B \end{pmatrix}. \quad (5)$$

We insert the unitary matrix into the definition equation, which redefines both the coupling coefficient and gauge fields:

$$\begin{pmatrix} \tilde{g} & \tilde{g}_{YB} \\ \tilde{g}_{BY} & \tilde{g}_B \end{pmatrix} R^T = \begin{pmatrix} g_1 & g_Y \\ 0 & g_B \end{pmatrix}, \\ R \begin{pmatrix} \tilde{A}_\mu^Y \\ \tilde{A}_\mu^B \end{pmatrix} = \begin{pmatrix} A_\mu^Y \\ A_\mu^B \end{pmatrix}. \quad (6)$$

Now we can give the mass matrix in the basis (A^Y, B^3, A^B) :

$$M_N^2 = \frac{1}{8} \begin{pmatrix} g_1^2 v^2 & g_1 g_2 v^2 & g_1 g_Y v^2 \\ g_1 g_2 v^2 & g_2^2 v^2 & g_2 g_Y v^2 \\ g_1 g_Y v^2 & g_2 g_Y v^2 & g_Y^2 v^2 + g_B^2 u^2 \end{pmatrix}. \quad (7)$$

The mass matrix defined in Eq. (7) can be diagonalized by unitary matrix R_N :

$$R_N = \begin{pmatrix} c_W & -s_W & 0 \\ s_W & c_W & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c' & -s' \\ 0 & s' & c' \end{pmatrix} \\ = \begin{pmatrix} c_W & -s_W c' & s_W s' \\ s_W & c_W c' & -c_W s' \\ 0 & s' & c' \end{pmatrix}, \quad (8)$$

with

$$\begin{pmatrix} \gamma \\ Z \\ Z' \end{pmatrix} = R_N^T \begin{pmatrix} A^Y \\ B^3 \\ A^B \end{pmatrix}. \quad (9)$$

If we do some inverse operations on the mass matrix:

$$\frac{1}{4} \begin{pmatrix} 0 & 0 & 0 \\ 0 & G^2 v^2 & -G g_{YB} v^2 \\ 0 & -G g_{YB} v^2 & g_{YB}^2 v^2 + 4g_B^2 u^2 \end{pmatrix} \\ = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c' & -s' \\ 0 & s' & c' \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & m_Z^2 & 0 \\ 0 & 0 & m_{Z'}^2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c' & s' \\ 0 & -s' & c' \end{pmatrix} \\ = \begin{pmatrix} 0 & 0 & 0 \\ 0 & c'^2 m_Z^2 + s'^2 m_{Z'}^2 & c' s' (m_Z^2 - m_{Z'}^2) \\ 0 & c' s' (m_Z^2 - m_{Z'}^2) & s'^2 m_Z^2 + c'^2 m_{Z'}^2 \end{pmatrix}, \quad (10)$$

where G is electroweak coupling coefficient $G = \frac{e}{s_W c_W}$, $v^2 = v_1^2 + v_2^2$ and $u^2 = u_1^2 + u_2^2$. Then, g_B and g_{YB} can be represented by some observable measurements:

$$g_{YB} = G \frac{c' s' (m_Z^2 - m_{Z'}^2)}{c'^2 m_Z^2 + s'^2 m_{Z'}^2}, \quad g_B = \frac{G}{2} \frac{x m_Z m_{Z'}}{c'^2 m_Z^2 + s'^2 m_{Z'}^2}. \quad (11)$$

Where $x = \frac{v}{u}$.

III. PINCH TECHNIQUE

The gauge-invariant self-energies of the Z and Z' bosons can be extracted from any neutral-current process. Following the approach of Papavasiliou [49], we consider the scattering process $e^+ e^- \rightarrow \mu^+ \mu^-$. The S -matrix element of this process should remain independent of the gauge-fixing parameters ξ_i at any order of perturbative calculation. The propagators for the gauge bosons γ , Z and Z' which include the gauge-fixing parameters, are given by:

$$\Delta_{\mu\nu}\gamma(q) = \frac{-i}{q^2} \left[g_{\mu\nu} - (1 - \xi_\gamma) \frac{q_\mu q_\nu}{q^2} \right], \quad (12)$$

$$\Delta_{\mu\nu}Z(q) = \frac{-i}{q^2} \left[g_{\mu\nu} - (1 - \xi_Z) \frac{q_\mu q_\nu}{q^2 - \xi_Z m_Z^2} \right], \quad (13)$$

$$\Delta_{\mu\nu}Z'(q) = \frac{-i}{q^2} \left[g_{\mu\nu} - (1 - \xi_{Z'}) \frac{q_\mu q_\nu}{q^2 - \xi_{Z'} m_{Z'}^2} \right]. \quad (14)$$

Traditionally, the S -matrix element T can be decom-

posed into three parts based on the Mandelstam variables s , t and the external masses m_e , m_μ . In terms of self-energy, vertex, and box contributions, T_1 , T_2 , and T_3 correspondingly contribute to the total expression:

$$T(s, t, m_e, m_\mu) = T_1(t) + T_2(t, m_e, m_\mu) + T_3(s, t, m_e, m_\mu) \quad (15)$$

Although the total T is gauge-independent, the individual components T_1 , T_2 and T_3 are gauge dependent. By employing the pinch technique, we recast these into gauge-invariant quantities \hat{T}_1 , \hat{T}_2 , and \hat{T}_3 :

$$T(s, t, m_e, m_\mu) = \hat{T}_1(t) + \hat{T}_2(t, m_e, m_\mu) + \hat{T}_3(s, t, m_e, m_\mu). \quad (16)$$

From this, we derive the gauge-invariant self-energy \hat{T}_1 .

The PT is a systematic approach. The form of the Ward identity changes depending on the specific physical process. For instance, when a gluon mediates strong interactions, the Ward identity takes the form:

$$\begin{aligned} k^\mu \gamma_\mu &\equiv \not{k} = (\not{p} + \not{k} - m_i) - (\not{p} - m_i) \\ &= S_i^{-1}(p+k) - S_i^{-1}(p). \end{aligned} \quad (17)$$

For charged W^\pm or neutral Z bosons coupled to fermions, the Ward identity becomes:

$$\begin{aligned} k^\mu \gamma_\mu P_L &\equiv \not{k} P_L = (\not{p} + \not{k} - m_i) P_L - P_R (\not{p} - m_j) + m_i P_L - m_j P_R \\ &= S_i^{-1}(p+k) P_L - S_j^{-1}(p) P_R + m_i P_L - m_j P_R. \end{aligned} \quad (18)$$

Here, $P_{R,L} = \frac{1 \pm \gamma_5}{2}$. The terms $S_i^{-1}(p+k)$ and $S_j^{-1}(p)$ vanish on shell, leaving behind $m_i P_L - m_j P_R$. When charged W^\pm couples to fermions, the masses m_i and m_j differ. For the Z boson coupling to a fermion-antifermion pair of equal mass, $m_i = m_j$.

Additionally, the vertex in Fig. 1(b) is decomposed into two components:

$$\Gamma_{\mu\nu\alpha} = \Gamma_{\mu\nu\alpha}^{P\xi} + \Gamma_{\mu\nu\alpha}^\xi, \quad (19)$$

where

$$\begin{aligned} \Gamma_{\mu\nu\alpha}^{P\xi} &= \frac{1}{\xi} [(q+k)_\nu g_{\mu\alpha} + k_\mu g_{\nu\alpha}], \\ \Gamma_{\mu\nu\alpha}^\xi &= 2q_\mu g_{\nu\alpha} - 2q_\nu g_{\mu\alpha} - (2k+q)_\alpha g_{\mu\nu} \\ &\quad + (1 - \frac{1}{\xi}) [k_\nu g_{\mu\alpha} + (k+q)_\mu g_{\nu\alpha}]. \end{aligned} \quad (20)$$

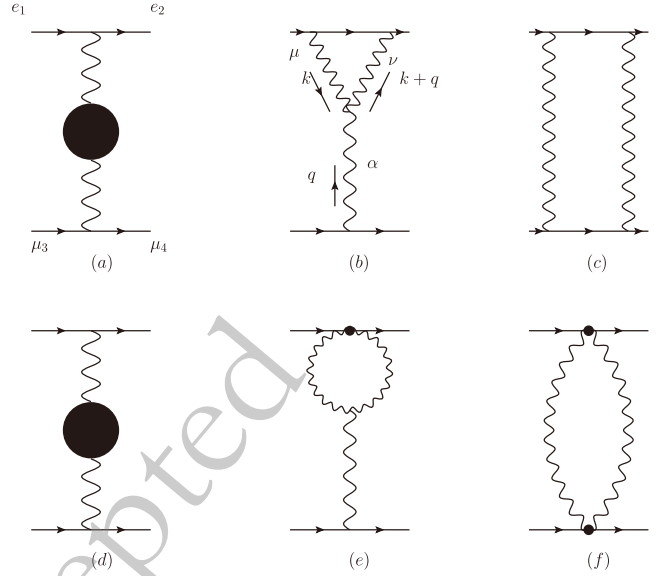


Fig. 1. Diagram (a)-(c) show representative one-loop contributions to the S-matrix of the four-fermion process. Diagram (a) is the conventional gauge-dependent self-energy. The pinch parts extracted from diagrams (b) and (c)—shown separately in (e) and (f)—are systematically absorbed into (a), thereby converting it into the gauge-invariant effective self-energy displayed in (d).

The term $\Gamma_{\mu\nu\alpha}^\xi$ satisfies the Ward identity

$$q^\alpha \Gamma_{\mu\nu\alpha}^\xi = \Delta_{\mu\nu}^{-1}(k, \xi) - \Delta_{\mu\nu}^{-1}(k+q, \xi). \quad (21)$$

Gauge boson propagators depend on the gauge-fixing parameters:

$$\Delta_{\mu\nu}^i(q) = \mathcal{D}_{\mu\nu}^i(q) - \frac{q_\mu q_\nu}{M_i^2} D^i(q), \quad (22)$$

where

$$\begin{aligned} D^i(q, \xi) &= \frac{i}{q^2 - \xi_i M_i^2}, \\ \mathcal{D}_{\mu\nu}^i(q) &= [g_{\mu\nu} - \frac{q_\mu q_\nu}{M_i^2}] \frac{-i}{q^2 - M_i^2}. \end{aligned} \quad (23)$$

$\mathcal{D}_{\mu\nu}^i(q)$ are the W^\pm , Z and Z' propagators ($\xi \rightarrow \infty$). Selecting the Feynman gauge ($\xi = 1$) can greatly simplify calculations[49]. Then, the pinch part of Fig. 1(c) vanished. Furthermore, when extracting the Pinch contribution of self energy, the following formula will be used:

$$\begin{aligned} g_\nu^\alpha &= \Delta_{\nu\mu}^i(q, \xi_i) \Delta_i^{-1}(q, \xi_i)^{\mu\alpha} \\ &= \Delta_{\nu\mu}^i(q, \xi_i) \mathcal{D}_i^{-1}(q)^{\mu\alpha} - i q_\nu q^\alpha D_i(q, \xi), \\ q_\mu &= -i [q^2 D_i(q, \xi_i) q_\mu + M_i^2 q^\nu \Delta_{\nu\mu}^i(q, \xi_i)], \end{aligned} \quad (24)$$

where

$$\mathcal{D}_i^{-1}(q)^{\mu\alpha} = i\{g^{\mu\alpha}(q^2 - M_i^2) - q^\mu q^\alpha\}. \quad (25)$$

If $i = \gamma$,

$$\Delta_{\mu\nu}^{\gamma^{-1}}(q) = iq^2[g^{\mu\nu} + (1 - \frac{1}{\xi_\gamma})\frac{q^\mu q^\nu}{q^2}]. \quad (26)$$

In the B-LSSM, the pinch part of the self-energy of W^\pm is similar to the result in the SM, here are the results:

$$\begin{aligned} \Pi_{WW}^P(q^2) = & -4g_2^2(q^2 - M_W^2)\{s_W^2 I_{W\gamma}(q^2) \\ & + c_W^2(c'^2 I_{WZ}(q^2) + s'^2 I_{WZ'}(q^2))\}. \end{aligned} \quad (27)$$

Where,

$$I_{ij}(q^2) = i\mu^{4-n} \int \frac{d^n k}{(2\pi)^n} \frac{1}{(k^2 - m_i^2)(k+q)^2 - m_j^2}. \quad (28)$$

The calculation of gauge-independent self-energies of γ, Z and Z' needs to utilize the following allocation:

$$\frac{g_2}{2}\gamma^\mu P_L = (-i)s_W(e\gamma e)^\mu - ic_W c'(eZe)^\mu + ic_W s'(eZ'e)^\mu, \quad (29)$$

Where (eZe) and $(eZ'e)$ denote the vertex of the corresponding particles. By combining Eq. (24) and Eq. (29), the following results can be obtained:

$$\begin{aligned} \Pi_{\gamma\gamma}^P(q^2) &= -4g_2^2 s_W^2 q^2 I_{WW}(q^2), \\ \Pi_{\gamma Z}^P(q^2) &= -2g_2^2 s_W c_W c'(2q^2 - M_Z^2) I_{WW}(q^2), \\ \Pi_{\gamma Z'}^P(q^2) &= -2g_2^2 s_W c_W s'(2q^2 - M_{Z'}^2) I_{WW}(q^2), \\ \Pi_{ZZ}^P(q^2) &= -4g_2^2 c_W^2 c'^2 (q^2 - M_Z^2) I_{WW}(q^2), \\ \Pi_{Z Z'}^P(q^2) &= -4g_2^2 c_W^2 s'^2 (q^2 - M_{Z'}^2) I_{WW}(q^2), \\ \Pi_{Z' Z'}^P(q^2) &= 2g_2^2 c_W^2 s' c'(2q^2 - M_Z^2 - M_{Z'}^2) I_{WW}(q^2). \end{aligned} \quad (30)$$

By utilizing PT, we successfully derive gauge-invariant self-energies for the W^\pm, Z, Z' and γ bosons, which are essential for subsequent calculations.

IV. FORMALISM OF OBLIQUE CORRECTIONS

A. Formalism of S T U parameters in the B-LSSM

The implementation of renormalization schemes that utilize the number of observables equivalent to the number of free parameters may prove challenging in practice. This is due to the necessity of solving a more extensive set of equations than those encountered in the SM, which may result in the generation of highly intricate and unwieldy analytical formulae [54].

The oblique parameters are the products of renormalization under the SM, and thus warrant examination to ascertain whether such a quantity remains well-defined in the B-LSSM. A significant indicator is whether the divergence of oblique parameters is cancelled. From a technical standpoint, it appears that the requisite divergent elimination is guaranteed by the symmetry of the group, since $Tr[T^3 Y] = 0$. As can be observed in the Table 1, the self-energy functions, namely, the unrenormalized vacuum polarisation functions with coupling constants factored out, namely, the (Π 's), will cancel out any divergence after traversing all the superfields. It is therefore reasonable to continue using the original definition of Peskin and Takeuchi[34]:

$$\begin{aligned} \alpha S &\equiv 4e^2[\Pi'_{33}(0) - \Pi'_{3Q}(0)], \\ \alpha T &\equiv \frac{e^2}{s_W^2 c_W^2 \widetilde{m}_Z^2} [\Pi_{11}(0) - \Pi_{33}(0)], \\ \alpha U &\equiv 4e^2[\Pi'_{11}(0) - \Pi'_{31}(0)], \end{aligned} \quad (31)$$

where $\widetilde{m}_Z^2 = c'^2 m_Z^2 + s'^2 m_{Z'}^2$.

In accordance with Peskin's approach, the one-particle-irreducible (1PI) self-energies of the photon, Z and Z' in the B-LSSM are expressed as a linear combina-

Table 1. The charges and the mixed products of each superfield in the B-LSSM

	Q	L	H_d	H_u	d	u	e	ν	η	$\bar{\eta}$
Y	$\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	-1	0	0	0
T^3	$\frac{1}{2} \quad 0$ $0 \quad -\frac{1}{2}$	$\frac{1}{2} \quad 0$ $0 \quad -\frac{1}{2}$	$\frac{1}{2} \quad 0$ $0 \quad -\frac{1}{2}$	$\frac{1}{2} \quad 0$ $0 \quad -\frac{1}{2}$	0	0	0	0	0	0
Y_B	$\frac{1}{6}$	$-\frac{1}{2}$	0	0	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{2}$	-1	1
$T^3 \times Y$	$\frac{1}{12} \quad 0$ $0 \quad -\frac{1}{12}$	$-\frac{1}{2} \quad 0$ $0 \quad \frac{1}{2}$	$-\frac{1}{2} \quad 0$ $0 \quad \frac{1}{2}$	$\frac{1}{2} \quad 0$ $0 \quad -\frac{1}{2}$	0	0	0	0	0	0
$T^3 \times Y_B$	$\frac{1}{12} \quad 0$ $0 \quad -\frac{1}{12}$	$-\frac{1}{2} \quad 0$ $0 \quad \frac{1}{2}$	0	0	0	0	0	0	0	0
$Y \times Y_B$	$\frac{1}{36}$	$\frac{1}{4}$	0	0	$\frac{1}{18}$	$-\frac{1}{9}$	$-\frac{1}{2}$	0	0	0

tion of the $T_3 - Q - B$ -mixed eigenstate self-energy.

$$\Pi_{AA} = e^2 \Pi_{QQ}. \quad (32a)$$

$$\Pi_{ZA} = \frac{e^2 c'}{s_W c_W} (\Pi_{3Q} - s_W^2 \Pi_{QQ}) + e s' g_E \Pi_{QB}. \quad (32b)$$

$$\Pi_{Z'A} = -\frac{e^2 s'}{s_W c_W} (\Pi_{3Q} - s_W^2 \Pi_{QQ}) + e c' g_E \Pi_{QB}. \quad (32c)$$

$$\begin{aligned} \Pi_{ZZ} = & \frac{e^2 c'^2}{s_W^2 c_W^2} (\Pi_{33} - 2s_W^2 \Pi_{3Q} + s_W^4 \Pi_{QQ}) \\ & + 2 \frac{e s' c'}{s_W c_W} g_E (\Pi_{3B} - s_W^2 \Pi_{QB}) + s'^2 g_E^2 \Pi_{BB}. \end{aligned} \quad (32d)$$

$$\begin{aligned} \Pi_{Z'Z'} = & \frac{e^2 s'^2}{s_W^2 c_W^2} (\Pi_{33} - 2s_W^2 \Pi_{3Q} + s_W^4 \Pi_{QQ}) \\ & - 2 \frac{e s' c'}{s_W c_W} g_E (\Pi_{3B} - s_W^2 \Pi_{QB}) + c'^2 g_E^2 \Pi_{BB}. \end{aligned} \quad (32e)$$

$$\begin{aligned} \Pi_{ZZ'} = & -\frac{e^2 c' s'}{s_W^2 c_W^2} (\Pi_{33} - 2s_W^2 \Pi_{3Q} + s_W^4 \Pi_{QQ}) \\ & + 2 \frac{e(c'^2 - s'^2)}{s_W c_W} g_E (\Pi_{3B} - s_W^2 \Pi_{QB}) + c' s' g_E^2 \Pi_{BB}. \end{aligned} \quad (32f)$$

Where $g_E = g_B + \frac{Y}{Y_B} g_{YB}$, Q indicates the electric charge, T_3 indicates the charge of W_3 and B indicates the charge of $U(1)_{B-L}$ group. The 1PI self energy of W^\pm is given as:

$$\Pi_{WW} = \frac{e^2}{s_W^2} \Pi_{11}. \quad (33)$$

We can linearly combine the above equations and define effective self-energy $\tilde{\Pi}_{ZA}$ and $\tilde{\Pi}_{ZZ}$ as follows to eliminate the self-energies "containing" U_1 charge B : $\tilde{\Pi}_{ZA} = c'(32b) - s'(32c)$, $\tilde{\Pi}_{ZZ} = c'^2(32e) - 2c' s'(32i) + s'^2(32g)$. Combining with (32a), one can obtain:

$$\Pi_{AA} = e^2 \Pi_{QQ}. \quad (34a)$$

$$\tilde{\Pi}_{ZA} = \frac{e^2}{s_W c_W} (\Pi_{3Q} - s_W^2 \Pi_{QQ}). \quad (34b)$$

$$\tilde{\Pi}_{ZZ} = \frac{e^2}{s_W^2 c_W^2} (\Pi_{33} - 2s_W^2 \Pi_{3Q} + s_W^4 \Pi_{QQ}). \quad (34c)$$

Combining the definition of oblique parameters Eq.

(31), Eq. (33) and Eq. (34), one can obtain the expression of oblique parameters under the B-LSSM:

$$\begin{aligned} \alpha S = & 4s_W c_W [\tilde{\Pi}'_{ZZ}(0) - \frac{c_W^2 - s_W^2}{s_W c_W} \tilde{\Pi}'_{ZY}(0) - \Pi'_{\gamma\gamma}(0)], \\ \alpha T = & \frac{\Pi_{WW}(0)}{m_W^2} - \frac{\tilde{\Pi}_{ZZ}(0) + \frac{2s_W}{c_W} \tilde{\Pi}_{ZY}(0) + \frac{s_W^2}{c_W^2} \Pi'_{\gamma\gamma}(0)}{m_Z^2}, \\ \alpha U = & 4s_W^2 [\Pi'_{WW}(0) - c_W^2 \tilde{\Pi}'_{ZZ}(0) - 2s_W c_W \tilde{\Pi}'_{ZY}(0) - s_W^2 \Pi'_{\gamma\gamma}(0)]. \end{aligned} \quad (35)$$

B. The S T U parameters in the effective-Lagrangian

For the case of Z couples with two fermions, neutral-current of Z particle can be expressed as follows:

$$\begin{aligned} J_Z^\mu = & \left[\frac{e}{s_W c_W} (T^3 - s^2 Q) c' + g_E Y_B s' \right] \bar{\psi} \gamma^\mu P_L \psi \\ & + \left[\frac{e}{s_W c_W} (-s^2 Q) c' + g_E Y_B s' \right] \bar{\psi} \gamma^\mu P_R \psi. \end{aligned} \quad (36)$$

Using Eq. (11) and $g_E = g_B + \frac{Y}{Y_B} g_{YB}$, the neutral-current coupling between the Z boson and SM leptons in the B-LSSM can be written as follows:

$$\begin{aligned} \mathcal{L}_{N,Zff} = & G(c' - s' \frac{c' s' (m_Z^2 - m_{Z'}^2)}{c'^2 m_Z^2 + s'^2 m_{Z'}^2}) \\ & \times Z^\mu \bar{f} \gamma_\mu [T^3 P_L - (s_W^2 - \frac{s' (c_W^2 c' s' (m_Z^2 - m_{Z'}^2) + \frac{x m_Z m_{Z'}}{2})}{c'^2 m_Z^2 + s'^2 m_{Z'}^2})] f. \end{aligned} \quad (37)$$

Current experimental constraints on the Z' boson impose a lower bound on the ratio $M_{Z'}/g_B > 6$ TeV [50, 51]. Theoretical analyses indicate that direct contributions from Z' -mediated neutral currents to electroweak-scale observables are suppressed to a precision level below $\mathcal{O}(10^{-3})$, as determined by the interplay of the Z' -boson mass scale and its coupling strength. Consequently, these neutral current effects can be safely neglected in calculations of electroweak precision parameters within the current experimental resolution. This suppression arises dominantly from the decoupling behavior of the heavy Z' , whose contributions to low-energy processes scale as $\sim (M_Z^2/M_{Z'}^2) \times g_B^2$. The stringent experimental bound on $M_{Z'}/g_B$ ensures that such terms remain subleading compared to SM electroweak radiative corrections. Using the effective-Lagrangian techniques given by [52]

$$\begin{aligned} \mathcal{L}_{N,Zee} = & \frac{e}{s_W c_W} (1 + \frac{\alpha T}{2}) Z^\mu \bar{f} \gamma_\mu [T^3 P_L \\ & - (\hat{s}_W^2 + \frac{\alpha S}{4(\hat{c}_W^2 - \hat{s}_W^2)} - \frac{\hat{c}_W^2 \hat{s}_W^2 \alpha T}{\hat{c}_W^2 - \hat{s}_W^2})] f, \end{aligned}$$

$$\mathcal{L}_{C,Wev} = -\frac{e}{\sqrt{2}s_W}(1 - \frac{\alpha S}{4(\hat{c}_W^2 - \hat{s}_W^2)} + \frac{\hat{c}_W^2 \alpha T}{2(\hat{c}_W^2 - \hat{s}_W^2)} + \frac{\alpha U}{8\hat{s}^2})W^{\pm\mu}\bar{f}\gamma_\mu P_L f. \quad (38)$$

Where \hat{s}_W and \hat{c}_W are defined by

$$\hat{s}_W \hat{c}_W m_Z = \frac{1}{2} ev = s_W c_W m_Z^{SM}. \quad (39)$$

Comparing Eq. (37) and Eq. (38), one can obtain:

$$\begin{aligned} \alpha T &= 2\left[\frac{\hat{s}_W \hat{c}_W}{s_W c_W}(c' - s' \frac{c' s'(m_Z^2 - m_Z'^2)}{c'^2 m_Z^2 + s'^2 m_Z'^2}) - 1\right], \\ \alpha S &= 4\hat{c}_W^2 \hat{s}_W^2 \alpha T + 4(\hat{c}_W^2 - \hat{s}_W^2)(s_W^2 - \hat{s}_W^2) \\ &\quad - \frac{s' c_W^2 c' s'(m_Z^2 - m_Z'^2) + \frac{x m_Z m_Z'}{2}}{c' (c'^2 m_Z^2 + s'^2 m_Z'^2)}, \\ \alpha U &= 8\hat{s}_W^2 \left(\frac{\hat{s}_W}{s_W} - 1 + \frac{\alpha S}{4(\hat{c}_W^2 - \hat{s}_W^2)} - \frac{\hat{c}_W^2 \alpha T}{2(\hat{c}_W^2 - \hat{s}_W^2)}\right). \quad (40) \end{aligned}$$

It can be demonstrated that the definition of \hat{c}_W is indeed equivalent to the the Sirlin definition [53], based on the values of m_W and m_Z . In comparison to the intrinsic definition of c_W at the tree level, these can be connected by means of the equation:

$$\begin{aligned} \Delta s &= \hat{s}_W - s_W, \\ \hat{c}_W^2 &= \frac{m_W^2}{m_Z^2}, \\ \hat{c}_W^2 &= \frac{m_W^2}{c'^2 m_Z + s'^2 m_Z'} = 1 - s_W^2 \\ &= 1 - \hat{s}_W^2 + 2\hat{s}_W \Delta s = \hat{c}_W^2 + 2\hat{s}_W \Delta s. \quad (41) \end{aligned}$$

By comparing the three equations in Eq. (41), one can obtain:

$$s'^2 \left(\frac{m_Z^2}{m_Z'^2} - 1\right) = -2 \frac{\hat{s}_W}{\hat{c}_W^2} \Delta s. \quad (42)$$

Combining Eq. (40) with Eq. (42), the following results can be derived through a perturbative analysis:

$$\alpha T = 2 \frac{\Delta s}{\hat{s}_W} - s'^2. \quad (43)$$

$$\alpha S + \alpha U = 8\hat{s}_W \Delta s - 4s' \frac{\hat{c}_W \hat{s}_W}{e} g_B. \quad (44)$$

$$\begin{aligned} &2\hat{s}_W^2 \alpha S + (\hat{s}_W^2 - \hat{c}_W^2) \alpha U \\ &= 4\hat{c}_W^2 \hat{s}_W^2 \alpha T + 8\hat{s}_W (\hat{s}_W^2 - \hat{c}_W^2) \Delta s. \quad (45) \end{aligned}$$

The aforementioned formulas are conducive to the analysis of relationship between s' , Δs and g_B .

C. Gauge-invariant electroweak interaction parameters

We have already provided the pinch contributions of all self energy-graphs of gauge particles in Eq. (27) and Eq. (30), and also provided specific expressions for the S T U parameters in (34) and Eq. (35). Combining the above four equations, we can obtain pinch parts of the S T U parameters. The gauge-invariant S T U parameters can be represented as the sum of the pinch parts and the conventional parts:

$$\alpha(S)_{GI} = \alpha(S)_C + 8g_2^2 s_W c_W I'_{WW}(0)(c'^2 m_Z^2 + s'^2 m_Z'^2). \quad (46)$$

$$\begin{aligned} \alpha(T)_{GI} &= \alpha(T)_C + 4g_2^2 [s_W^2 I_{W\gamma}(0) + c_W^2 (c'^2 I_{WZ}(0) \\ &\quad + s'^2 I_{WZ'}(0)) - I_{WW}(0)]. \quad (47) \end{aligned}$$

$$\begin{aligned} \alpha(U)_{GI} &= \alpha(U)_C + 16s_W^2 g_2^2 \{m_Z^2 [s_W^2 I'_{W\gamma}(0) \\ &\quad + c_W^2 (c'^2 I'_{WZ}(0) + s'^2 I'_{WZ'}(0)) \\ &\quad - I'_{WW}(0)] - [s_W^2 I_{W\gamma}(0) + c_W^2 (c'^2 I_{WZ}(0) \\ &\quad + s'^2 I_{WZ'}(0)) - I_{WW}(0)]. \quad (48) \end{aligned}$$

The subscript C here follows the definition in the literature [55], the S , T and U parameters exclude the pinch terms that ensure gauge independence. Formally speaking, our results are extensions of Degraasi's results [55] based on the SM. The divergences in both expressions cancel out. This confirms the assertion made in the first subsection of this section.

V. THE DIVERGENCE CANCELLATION OF S T U PARAMETERS IN THE NEUTRALINO-CHARGINO SECTOR AND SQUARK SECTOR

In the context of one-loop diagram calculations, particularly in the context of renormalization, the elimination of divergences serves as a crucial indicator for the verification of calculation results. This section presents the divergence cancellation of the S, T, and U parameters in the neutralino-chargino and squark sectors, with the objective of further validating the theoretical framework.

A. Neutralino-Chargino sector

The calculation of the vacuum polarization diagrams of fermions as loop particles is presented in Appendix B, along with a processing of all the coefficients involved.

As a consequence of the non-divergence of the derivative of the B_0 function, the divergence of the S and U parameters is directly proportional to C_1 . By combining equa-

tions Eq. (34) and Appendix B, the divergences of the S T and U parameters (div_S , div_T and div_U) can be expressed as:

$$\begin{aligned}
div_S &\sim -\frac{G^2}{4} \left[-\sum_{i,j}^2 (U_{j2}^* U_{i2} U_{i2}^* U_{j2} + V_{j2}^* V_{i2} V_{i2}^* V_{j2}) + \sum_{i,j}^7 (N_{j3}^* N_{i3} - N_{j4}^* N_{i4})^2 \right] = 0. \\
div_U &\sim -\frac{c_W^2 G^2}{4} \left\{ \sum_{i,j}^{7,2} [4U_{j1}^* N_{i2} N_{i2}^* U_{j1} + 2U_{j2}^* N_{i3} N_{i3}^* U_{j2} + 2\sqrt{2}(U_{j1}^* N_{i2} N_{i3}^* U_{j1} + h.c.) + 4V_{j1}^* N_{i2} N_{i2}^* V_{j1} + 2V_{j2}^* N_{i4} N_{i4}^* V_{j2} \right. \\
&\quad \left. - 2\sqrt{2}(V_{j1}^* N_{i2} N_{i4}^* V_{j2} + h.c.)] - \sum_{i,j}^2 [4U_{j1}^* U_{i1} U_{i1}^* U_{j1} + 4V_{j1}^* V_{i1} V_{i1}^* V_{j1} + U_{j2}^* U_{i2} U_{i2}^* U_{j2} + V_{j1}^* V_{i1} V_{i2} V_{j2}^* \right. \\
&\quad \left. + 2c_W^2 (U_{j1}^* U_{i1} U_{i2}^* U_{j2} + V_{j1}^* V_{i1} V_{i2}^* V_{j2}) + 2(1 + 2s_W^2)(U_{j2}^* U_{i2} U_{i1}^* U_{j1} + V_{j2}^* V_{i2} V_{i1}^* V_{j1}) \right] - \sum_{i,j}^7 (N_{j3}^* N_{i3} - N_{j4}^* N_{i4})^2 \} = 0. \\
div_T &\sim -\frac{G^2}{4m_Z^2} \{\mathcal{A}_1 - \mathcal{A}_2\}.
\end{aligned}$$

Where,

$$\begin{aligned}
\mathcal{A}_1 &= \sum_{i,j}^{2,7} (m_{\chi_i^\pm}^2 + m_{\chi_j^0}^2) [4U_{j1}^* N_{i2} N_{i2}^* U_{j1} + 2U_{j2}^* N_{i3} N_{i3}^* U_{j2} + 2\sqrt{2}(U_{j1}^* N_{i2} N_{i3}^* U_{j1} + h.c.) \\
&\quad + 4V_{j1}^* N_{i2} N_{i2}^* V_{j1} + 2V_{j2}^* N_{i4} N_{i4}^* V_{j2} - 2\sqrt{2}(V_{j1}^* N_{i2} N_{i4}^* V_{j2} + h.c.)] - \sum_{i,j}^2 (m_{\chi_i^\pm}^2 + m_{\chi_j^\mp}^2) [4c_W^4 U_{j1}^* U_{i1} U_{i1}^* U_{j1} \\
&\quad + c_{2W}^2 U_{j2}^* U_{i2} U_{i2}^* U_{j2} + 2c_W^2 c_{2W} (U_{j1}^* U_{i1} U_{i2}^* U_{j2} + h.c.) + 4c_W^4 V_{j1}^* V_{i1} V_{i1}^* V_{j1} + c_{2W}^2 2V_{j2}^* V_{i2} V_{i2}^* V_{j2} \\
&\quad + 2c_W^2 c_{2W} (V_{j1}^* V_{i1} V_{i2}^* V_{j2} + h.c.)] - \sum_{i,j}^7 (m_{\chi_i^0}^2 + m_{\chi_j^0}^2) (N_{j3}^* N_{i3} - N_{j4}^* N_{i4})^2. \\
\mathcal{A}_2 &= \sum_{i,j}^{2,7} 2m_{\chi_i^\pm} m_{\chi_j^0} [4U_{j1}^* N_{i2} V_{j1}^* N_{i2} - 2U_{j2}^* N_{i3} V_{j2}^* N_{i4} - 2\sqrt{2}U_{j1}^* N_{i2} V_{j2}^* N_{i4} + 2\sqrt{2}U_{j2}^* N_{i3} V_{i1} N_{i2}^* \\
&\quad + 4N_{j2}^* U_{i1} N_{j2}^* V_{i1} + 2N_{j3}^* U_{i2} N_{j4}^* V_{i2} - 2\sqrt{2}N_{j2}^* U_{i1} N_{j4}^* V_{i2} + 2\sqrt{2}N_{j3}^* U_{i2} N_{j2}^* V_{i1}] \\
&\quad - \sum_{i,j}^2 2m_{\chi_i^\pm} m_{\chi_j^\mp} [4c_W^4 U_{j1}^* U_{i1} V_{j1}^* V_{i1} + c_{2W}^2 U_{j2}^* U_{i2} V_{j2}^* V_{i2} + 2c_W^2 c_{2W} (U_{j1}^* U_{i1} V_{j2}^* V_{i2} + h.c.) \\
&\quad + 4c_W^4 V_{j1}^* V_{i1} U_{j1}^* U_{i2} + c_{2W}^2 V_{j2}^* V_{i2} U_{j2}^* U_{i2} + 2c_W^2 c_{2W} (V_{j1}^* V_{i1} U_{j2}^* U_{i2} + h.c.)] - \sum_{i,j}^7 2m_{\chi_i^0} m_{\chi_j^0} (N_{j3}^* N_{i3} - N_{j4}^* N_{i4})^2.
\end{aligned}$$

Using the corresponding mass matrix diagonalization formula:

$$\sum_i U_{ik}^* m_{\chi_i^\pm} V_{ij} V_{ij}^* m_{\chi_j^\mp} U_{ik} = \sum_j (M_{\chi^\pm})_{kj}^2, \quad \sum_{ij} U_{ik}^* U_{jk} V_{jl}^* V_{il} m_{\chi_i^\pm} m_{\chi_j^\mp} = (M_{\chi^\pm})_{kl}^2. \quad (49)$$

one can obtain:

$$\begin{aligned}
\mathcal{A}_1 &= 16M_2^2 + g_2^2 v^2 + 2\mu^2 + 2g_2^2 v^2 + \frac{1}{2}g_{YB}v^2 - 4c_W^4(4M_2^2 + g_2^2 v^2) + c_{2W}^2(4\mu^2 + g_2^2 v^2) - (2\mu^2 + 2g_2^2 v^2 + \frac{1}{2}g_{YB}v^2) \\
&= (16c_W^4 - 16)M_2^2 + (4c_{2W}^2 - 4)\mu^2 + (4c_W^4 + c_{2W}^2 - 5)g_2^2 v^2, \\
\mathcal{A}_2 &= 2 \times [8M_2^2 + 4\mu^2 + g_2^2 v^2 - (8c_W^4 M_2^2 + 2c_{2W}^2 \mu^2 + 2c_W^2 c_{2W} g_2^2 v^2) + 2\mu^2] = (16c_W^4 - 16)M_2^2 + (4c_{2W}^2 - 4)\mu^2 + (4c_W^4 + c_{2W}^2 - 5)g_2^2 v^2.
\end{aligned}$$

It can be seen from this that their divergence cancels out each other!

B. Squark sector

When squarks are loop particles in the one-loop self-energy diagrams, the cancellation of divergences in the S , T , and U parameters can be demonstrated as follows.

$$\begin{aligned}
div_S &\sim -\frac{4}{3\alpha} s_W c_W [C_a^{\gamma\gamma} - \frac{c_W^2 - s_W^2}{s_W c_W} \tilde{C}_a^{Z\gamma} - \tilde{C}_a^{ZZ}] = \frac{4}{\alpha} s_W c_W [(\sum_{k=1}^3 Z_{ik}^{U*} Z_{ik}^U)^2 + 4(\sum_{k=4}^6 Z_{ik}^{U*} Z_{ik}^U)^2 - (\sum_{k=1}^3 Z_{ik}^{D*} Z_{ik}^D)^2 + 2(\sum_{k=4}^6 Z_{ik}^{D*} Z_{ik}^D)^2] = 0. \\
div_U &\sim \frac{4}{\alpha} s_W^2 [C_a^{WW} - s_W^2 C_a^{\gamma\gamma} - 2s_W c_W \tilde{C}_a^{Z\gamma} - c_W^2 \tilde{C}_a^{ZZ}] = \frac{4}{\alpha} s_W^2 [-\frac{1}{2}g_2^2 \sum_{k=1}^3 (Z_{jk}^{D*} Z_{jk}^U) \sum_{l=1}^3 (Z_{il}^{U*} Z_{il}^D) + \frac{1}{4}g_2^2 (\sum_{k=1}^3 Z_{ik}^{U*} Z_{ik}^U)^2 \\
&\quad + \frac{1}{4}g_2^2 (\sum_{k=1}^3 Z_{ik}^{D*} Z_{ik}^D)^2] = 0. \\
div_T &\sim \frac{1}{m_W^2} [\sum_{i,j}^6 C_a^{WW} (m_{U_i}^2 + m_{D_j}^2) + \sum_i^6 C_b^{WU} m_{U_i}^2 + \sum_j^6 C_b^{WD} m_{D_j}^2] - \frac{1}{c'^2 m_Z^2 + s'^2 m_{Z'}^2} [\sum_i^6 \tilde{C}_a^{ZZU} m_{U_i}^2 + \sum_j^6 \tilde{C}_a^{ZZD} m_{D_j}^2 \\
&\quad + \frac{1}{2} \sum_i^6 \tilde{C}_b^{ZZU} m_{U_i}^2 + \frac{1}{2} \sum_j^6 \tilde{C}_b^{ZZD} m_{D_j}^2 + \frac{s_W}{2c_W} (\sum_i^6 \tilde{C}_a^{Z\gamma U} m_{U_i}^2 + \sum_j^6 \tilde{C}_a^{Z\gamma D} m_{D_j}^2) + \frac{1}{2} \sum_i^6 \tilde{C}_b^{Z\gamma U} m_{U_i}^2 + \frac{1}{2} \sum_j^6 \tilde{C}_b^{Z\gamma D} m_{D_j}^2] \\
&= \frac{0}{m_W^2} - \frac{0}{c'^2 m_Z^2 + s'^2 m_{Z'}^2} = 0.
\end{aligned}$$

The specific coefficients for this process have been placed in Appendix A. In the demonstration above, we utilized the unitarity relation of the transposed matrix:

$$\sum_{k=1}^6 (Z_{ik}^U Z_{jk}^{U*}) = \sum_{k=1}^6 (Z_{ik}^D Z_{jk}^{D*}) = \delta_{ij}.$$

This work focuses on the neutralino-chargino and squark sectors of the B-LSSM, which are expected to provide the dominant new-physics contributions to electroweak precision observables, and within this framework we have constructed and verified the gauge-invariant formalism. Although the Higgs sector involves non-Abelian symmetry breaking and its mixing with Goldstone bosons, which introduces specific complexities in maintaining gauge invariance, its loop contributions to the S , T , and U parameters have been shown to be extremely small already in the Standard Model. In the B-LSSM, the Higgs fields associated with the additional $U(1)$ breaking typically acquire masses well above the electroweak scale, further suppressing their contributions. Moreover, for the Higgs sector to yield a significant effect, it would generally require mixing between different

generations of scalar fields, and such mixing is usually suppressed by the large mass scales. Therefore, at this stage of systematically studying the main sources of new-physics contributions, we have not included the Higgs sector in the detailed calculations.

VI. NUMERICAL ANALYSIS

A global fit of the S , T , and U parameters incorporating recent top quark mass measurements from CMS has been provided in Ref.[56]:

$$\begin{aligned}
S &= -0.04 \pm 0.10, & T &= 0.01 \pm 0.12, & U &= -0.01 \pm 0.09, \\
\rho_{ST} &= 0.93, & \rho_{SU} &= -0.87, & \rho_{TU} &= -0.70.
\end{aligned}$$

In order to study how the parameters affect the values of the S , T and U parameters, we use the Mathematica to calculate the oblique parameters of the B-LSSM. Some parameters we assumed are: $A_t = 3$ TeV, $A_b = 3$ TeV, $g_{YB} = -0.3$, $g_B = 0.5$, $\tan\beta = 20$, $\tan\beta' = 1.5$, $m_{Z'} = 4500$ GeV, $\mu = 200$ GeV, $\mu_B = 800$ GeV, $m_B = 600$ GeV, $m_{BL} = 800$ GeV, $m_1 = 200$ GeV and $m_2 = 300$ GeV. We use this set of data as a benchmark

and run some of the variables respectively. Through a parameter scan, we can identify which model parameters are most sensitive to the S , T , and U observables.

When we run m_1 , m_2 and μ in the (200 GeV \sim 2000 GeV), (200 GeV \sim 800 GeV) and (200 GeV \sim 800 GeV) intervals respectively, we find that they are sensitive to the S and T parameters. The Fig. 3 shows that the S parameter gradually decreases with the increase of m_1 , m_2 , and μ , while the T parameter shows an upward trend with the increase of m_1 and m_2 . m_1 and m_2 are the diagonal elements in the mass matrices of neutralinos and charginos, and their increase weakens the mixing of the matrices, which is the source of the contributions of oblique parameters. m_2 is the diagonal element with the highest value in the mass matrices of neutralinos and charginos, which explains its sensitivity to oblique parameters relative to m_1 .

Fig. 4 illustrates the changes in the T and U paramet-

ers as the A_t parameter varies from 1000 GeV to 7000 GeV. The gray area indicates one standard deviation of the experimental observations. As shown, the T parameter increases with the rise in A_t , while the U parameter exhibits the opposite trend. The observed changes in the T and U parameters in Fig. 4 are primarily due to the contributions from the Feynman diagrams involving squark fields as loop particles. The A_t parameter, originating from the off-diagonal elements of the squark mass matrix, influences the mixing between the left-handed and right-handed \tilde{u} squarks (\tilde{u}_L and \tilde{u}_R), which in turn drives the squark sector's contributions to the oblique parameters.

To genuinely highlight the distinctive features of the B-LSSM as opposed to the MSSM, it is essential to illustrate how the extended gauge sector parameters influence the electroweak precision observables. In Fig. 5, we present the dependence of the S , T , and U parameters on

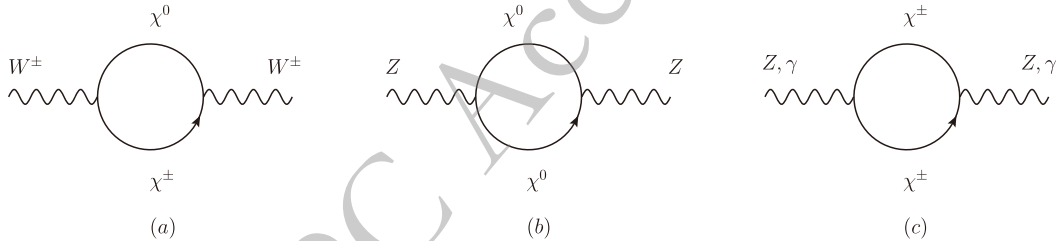


Fig. 2. Vacuum polarization diagram with Neutralino-Chargino sector

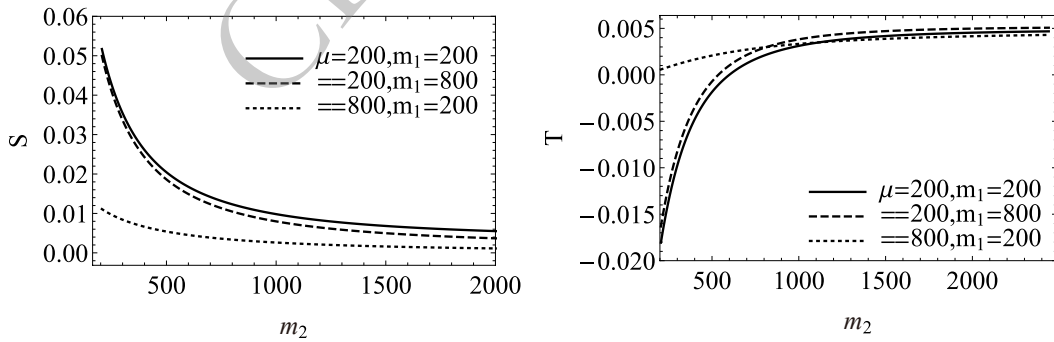


Fig. 3. S and T parameters that are more sensitive to m_1 , m_2 , and μ .

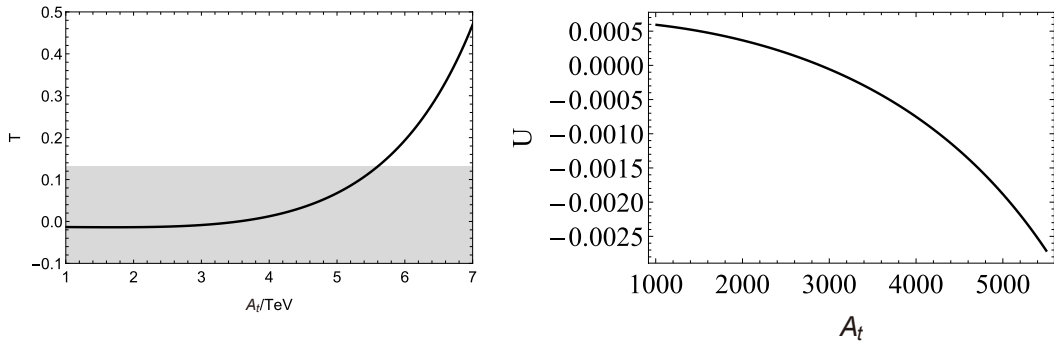


Fig. 4. T and U parameters that are more sensitive to A_t .

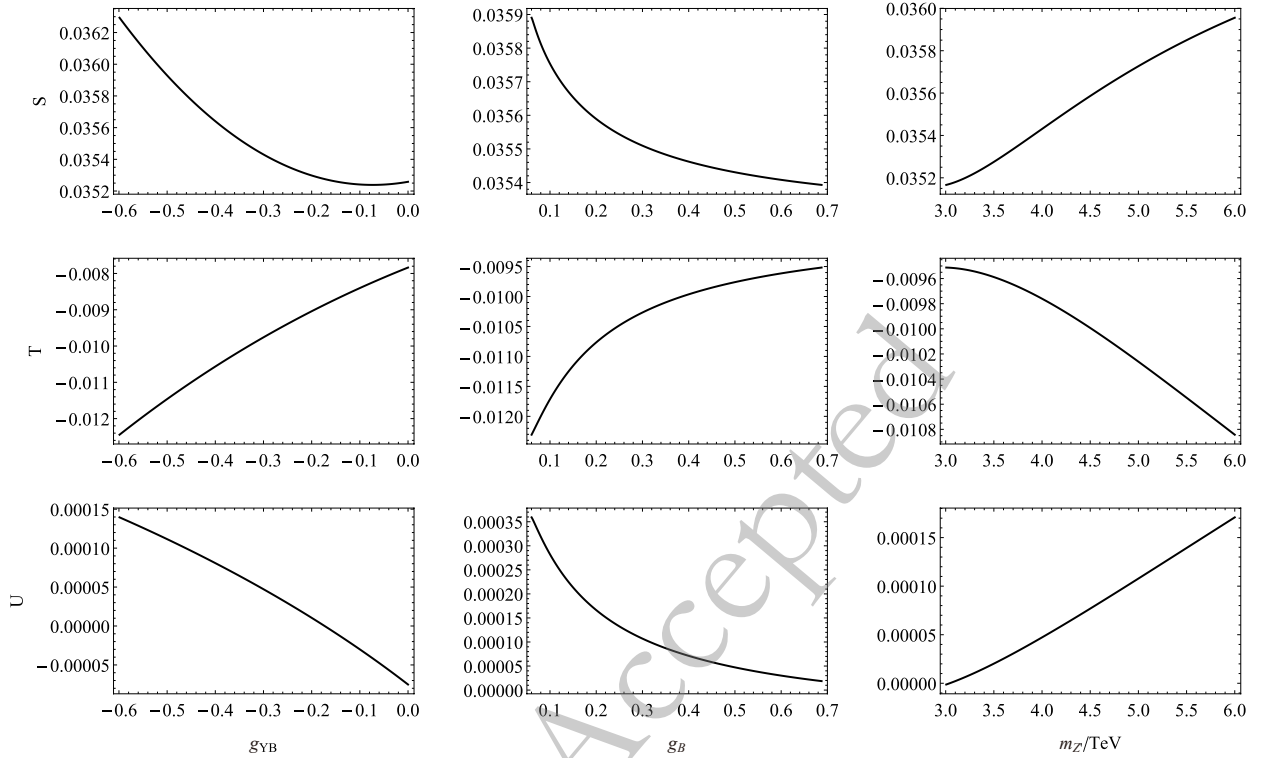


Fig. 5. The dependence of the oblique parameters S, T , and U on the B-LSSM distinctive parameters g_{YB} , g_B , and $m_{Z'}$. The SUSY-breaking parameters are fixed at the benchmark point.

the unique B-LSSM parameters: the gauge kinetic mixing coupling g_{YB} , the $U(1)_{B-L}$ gauge coupling g_B , and the extra gauge boson mass $m_{Z'}$, while keeping other supersymmetric parameters fixed at the aforementioned benchmark point.

As shown in the panels of Fig. 5, varying $m_{Z'}$ from 3000 GeV to 6000 GeV leads to a smooth, monotonic modulation of the oblique parameters. Because the Z' boson is heavy, its decoupling behavior ensures that the absolute variations remain strictly perturbative. Furthermore, the figures demonstrate the pronounced effects of the extra gauge couplings. While g_B dictates the interaction strength of the $U(1)_{B-L}$ sector, the parameter g_{YB} explicitly governs the gauge kinetic mixing between the $U(1)_Y$ and $U(1)_{B-L}$ groups. This kinetic mixing effect directly modifies the $Z-Z'$ mass matrix, systematically shifting the T parameter closer to zero while slightly reducing the S parameter. These results explicitly demonstrate that while the loop-level sparticle mass patterns set the baseline, the unique $U(1)_{B-L}$ extensions provide a finite and distinctive tree-level modulation.

We performed a random three-dimensional sampling of the S , T , and U parameters in Eq. (43-45) based on the probability distribution results obtained from the experimental fits. Each randomly selected point corresponds to a unique set of values for Δs , s'^2 , and g_B . We then conducted statistical analysis on the resulting Δs , s'^2 , and g_B values to generate probability histograms. As shown in

Fig. 6, the distribution of Δs does not exceed one standard deviation of the current experimental measurements, making it challenging to extract further useful information from the measurement of the Weinberg angle. Additionally, the probability distribution of g_B exhibits a sharp peak around $g_B = 0.08$, with a rapid decay for $g_B > 0.05$. Analysis reveals that the probability for $0.05 < g_B < 0.2$ exceeds ninety percent when compared to the entire range.

VII. CONCLUSIONS

Using the pinch technique, we derive the gauge-invariant self-energy of gauge bosons in the B-LSSM and obtain gauge-invariant oblique parameters. This approach ensures gauge symmetry when extracting information from S-matrix elements.

In models featuring an additional $U(1)$ symmetry compare to the Standard Model, the definitions of oblique parameters remain applicable. From the perspective of divergence cancellation, these definitions are comprehensive and can be extended to the models, which include various types of group expansion, thereby simplifying the complex definitions and proofs associated with the S , T , and U parameters.

In this work, the contributions to the S , T , and U parameters from two key sectors of the B-LSSM are calculated and the impacts of model parameters in the B-

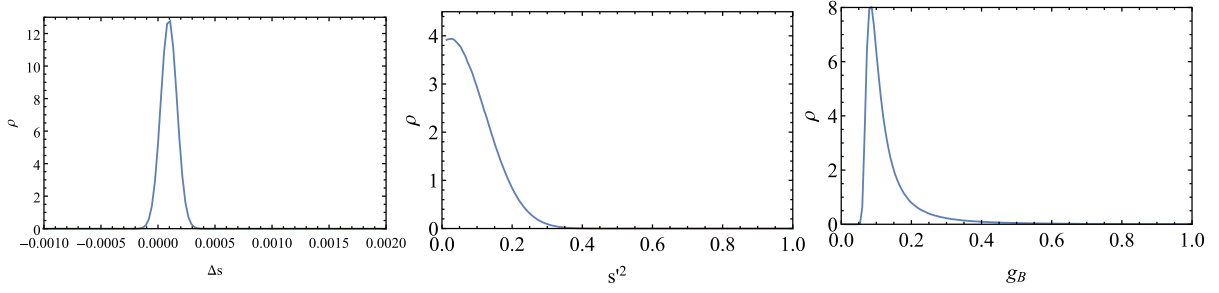


Fig. 6. By using the Monte Carlo sampling method and generating statistical histograms, the probability distributions of the three parameters can be obtained.

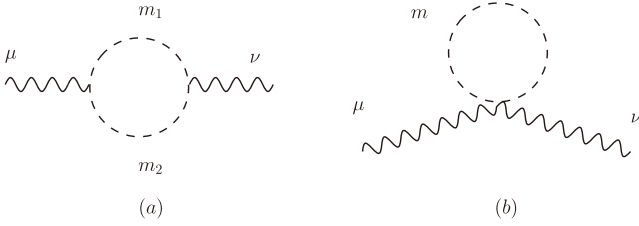


Fig. 7. Feynman diagrams with scalar loop-particle

LSSM are analyzed. The parameter space of the B-LSSM we selected aligns well with the global fitting of the S , T , and U parameters reported in the experiments.

By comparing the effective Lagrangian with the model Lagrangian and incorporating constraints from experimental fits, the coupling constant of the additional $U(1)$ group can be related to the S , T , and U parameters. This analysis allows us to derive effective constraints on these coefficients.

Finally, comparing the loop-level supersymmetric contributions with the tree-level extra gauge effects reveals a distinct physical hierarchy within the B-LSSM. Under current experimental precision, the globally fitted electroweak observables impose significantly more stringent constraints on the additional $U(1)_{B-L}$ gauge parameters (e.g., g_B and $m_{Z'}$) than on the detailed supersymmetric mass spectrum. The latter, entering primarily via loop corrections, is naturally suppressed by the decoupling of heavy sparticles. In contrast, the extra gauge sector directly modulates the observables through tree-level kinetic mixing and mass matrix deformations. Consequently, contemporary precision measurements act as a powerful and direct probe of the extended gauge sector, validating the necessity of investigating the distinctive gauge extensions beyond the minimal supersymmetric framework.

APPENDIX A: INTEGRAL FORMULAS FOR SOME FEYNMAN DIAGRAMS

We provide the integral formulas for the vacuum polarization diagrams, expressing them uniformly in terms of the standard Passarino-Veltman A and B functions. We define A functions:

$$\begin{aligned} i\pi^2 A_0(m) &= \mu^{4-n} \int d^n q \frac{1}{q^2 + m^2 - i\epsilon}, \\ i\pi^2 A_\mu(m) &= \mu^{4-n} \int d^n q \frac{q_\mu}{q^2 + m^2 - i\epsilon} = 0, \\ i\pi^2 A_{\mu\nu}(m) &= \mu^{4-n} \int d^n q \frac{q_\mu q_\nu}{q^2 + m^2 - i\epsilon} = i\pi^2 A_2(m) g_{\mu\nu}, \end{aligned} \quad (A1)$$

and B functions:

$$\begin{aligned} i\pi^2 B_0(p^2; m_1, m_2) &= \mu^{4-n} \int d^n q \frac{1}{d_0 d_1}, \\ i\pi^2 B_\mu(p^2; m_1, m_2) &= \mu^{4-n} \int d^n q \frac{q_\mu}{d_0 d_1} = i\pi^2 B_1(p^2; m_1, m_2) p_\mu, \\ i\pi^2 B_{\mu\nu}(p^2; m_1, m_2) &= \mu^{4-n} \int d^n q \frac{q_\mu q_\nu}{d_0 d_1} = i\pi^2 [B_{21}(p^2; m_1, m_2) p_\mu p_\nu \\ &\quad + B_{22}(p^2; m_1, m_2) g_{\mu\nu}]. \end{aligned} \quad (A2)$$

Where $d_0 = q^2 + m_1^2 - i\epsilon$ and $d_1 = (q+p)^2 + m_2^2 - i\epsilon$. We can list the divergence coefficients of these functions (The coefficients of $\frac{2}{\epsilon} - \gamma_E - \ln\pi$):

$$\begin{aligned} A_0(m) &\sim -m^2, \quad A_2(m) \sim \frac{m^4}{4}, \quad B_0(p^2; m_1, m_2) \sim 1, \\ B_1(p^2; m_1, m_2) &\sim -\frac{1}{2}, \quad B_{21}(p^2; m_1, m_2) \sim \frac{1}{3}, \\ B_{22}(p^2; m_1, m_2) &\sim -\frac{m_1^2 + m_2^2}{4} - \frac{p^2}{12}. \end{aligned}$$

Using the above formulas, the amplitudes of Fig. (7) can be represented as

$$\begin{aligned} \mathcal{M}_a &= -C_a \{ 4B_{22}(p^2; m_1, m_2) g_{\mu\nu} \\ &\quad + [B_0 + 4B_1 + 4B_{21}](p^2; m_1, m_2) p_\mu p_\nu \}, \\ \mathcal{M}_b &= C_b A_0(m) g_{\mu\nu}. \end{aligned} \quad (A3)$$

C_a and C_b are the coefficients from the vertices. We expand and sum the coefficients of the involved graphs.

$$\begin{aligned}
C_a^{WW} &= -\frac{1}{2}g_2^2 \sum_{k=1}^3 (Z_{ik}^{D*} Z_{jk}^U) \sum_{l=1}^3 (Z_{jl}^{U*} Z_{il}^D), & C_b^{WWD} &= \frac{1}{2}g_2^2 \sum_{k=1}^3 (Z_{ik}^{D*} Z_{ik}^D) & C_b^{WU} &= \frac{1}{2}g_2^2 \sum_{k=1}^3 (Z_{jk}^{U*} Z_{jk}^U), \\
C_a^{\gamma\gamma U} &= -\frac{4}{9}e^2 \left(\sum_{k=1}^3 Z_{ik}^{U*} Z_{jk}^U + \sum_{k=4}^6 Z_{ik}^{U*} Z_{jk}^U \right)^2, & C_a^{\gamma\gamma D} &= -\frac{1}{9}e^2 \left(\sum_{k=1}^3 Z_{ik}^{D*} Z_{jk}^D + \sum_{k=4}^6 Z_{ik}^{D*} Z_{jk}^D \right)^2, \\
C_b^{\gamma\gamma U} &= \frac{8}{9}e^2 \left(\sum_{k=1}^3 Z_{ik}^{U*} Z_{ik}^U + \sum_{k=4}^6 Z_{ik}^{U*} Z_{ik}^U \right), & C_b^{\gamma\gamma D} &= \frac{1}{18}e^2 \left(\sum_{k=1}^3 Z_{jk}^{D*} Z_{jk}^D + \sum_{k=4}^6 Z_{jk}^{D*} Z_{jk}^D \right), \\
\tilde{C}_a^{ZZU} &= -\frac{1}{36} [G(1+2c_{2W}) \sum_{k=1}^3 Z_{ik}^{U*} Z_{jk}^U - 2G(1-c_{2W}) \sum_{k=4}^6 Z_{ik}^{U*} Z_{jk}^U]^2, \\
\tilde{C}_a^{ZZD} &= -\frac{1}{36} [G(-2-c_{2W}) \sum_{k=1}^3 Z_{ik}^{D*} Z_{jk}^D + G(1-c_{2W}) \sum_{k=4}^6 Z_{ik}^{D*} Z_{jk}^D]^2, \\
\tilde{C}_b^{ZZU} &= \frac{1}{18} \{ [G(1+2c_{2W}) \sum_{k=1}^3 Z_{ik}^{U*} Z_{ik}^U]^2 + [2G(1-c_{2W}) \sum_{k=4}^6 Z_{ik}^{U*} Z_{ik}^U]^2 \}, \\
\tilde{C}_b^{ZZD} &= \frac{1}{18} \{ [G(-2-c_{2W}) \sum_{k=1}^3 Z_{jk}^{D*} Z_{jk}^D]^2 + [G(1-c_{2W}) \sum_{k=4}^6 Z_{jk}^{D*} Z_{jk}^D]^2 \}, \\
\tilde{C}_a^{Z\gamma U} &= -\frac{e}{9} \left(\sum_{k=1}^3 Z_{ik}^{U*} Z_{jk}^U + \sum_{k=4}^6 Z_{ik}^{U*} Z_{jk}^U \right) [G(1+2c_{2W}) \sum_{k=1}^3 Z_{ik}^{U*} Z_{jk}^U - 2G(1-c_{2W}) \sum_{k=4}^6 Z_{ik}^{U*} Z_{jk}^U]^2, \\
\tilde{C}_a^{Z\gamma D} &= -\frac{e}{18} \left(\sum_{k=1}^3 Z_{ik}^{D*} Z_{jk}^D + \sum_{k=4}^6 Z_{ik}^{D*} Z_{jk}^D \right) [G(-2-c_{2W}) \sum_{k=1}^3 Z_{ik}^{D*} Z_{jk}^D + G(1-c_{2W}) \sum_{k=4}^6 Z_{ik}^{D*} Z_{jk}^D]^2, \\
\tilde{C}_b^{Z\gamma U} &= \frac{2}{9} e [G(1+2c_{2W}) \left(\sum_{k=1}^3 Z_{ik}^{U*} Z_{ik}^U \right)^2 - 2G(1-c_{2W}) \left(\sum_{k=4}^6 Z_{ik}^{U*} Z_{ik}^U \right)^2], \\
\tilde{C}_b^{Z\gamma D} &= \frac{1}{9} e [G(-2-c_{2W}) \left(\sum_{k=1}^3 Z_{jk}^{D*} Z_{jk}^D \right)^2 + G(1-c_{2W}) \left(\sum_{k=4}^6 Z_{jk}^{D*} Z_{jk}^D \right)^2].
\end{aligned}$$

Where, we mark $c_{2W} = c_W^2 - s_W^2 = \cos 2\theta_W$ and

$$\tilde{C}^{ZZ} = c'^2 C^{ZZ} - 2c' s' C^{ZZ'} + s'^2 C^{Z'Z'}, \quad \tilde{C}^{Z\gamma} = c' C^{Z\gamma} - s' C^{Z'\gamma}, \quad (\text{A4})$$

APPENDIX B: SELF-ENERGIES OF CHARGINOS AND NEUTRALINOS

For the convenience of batch processing, we divid the calculation results of graphs into two parts \mathcal{M}_1 and \mathcal{M}_2 :

$$\begin{aligned}
\mathcal{M} &= -\int \frac{d^D k}{(2\pi)^D} \text{tr} \left[A^\mu \frac{i}{k-m_1} B^\nu \frac{i}{(k+\not{q})-m_2} \right] (A_L B_L + A_R B_R) \int \frac{d^D k}{(2\pi)^D} \frac{1}{d_0 d_1} [2k^\mu q^\mu + 2k^\nu q^\nu + 4k^\mu k^\nu - 2(k^2 + k \cdot q) g^{\mu\nu}] \\
&\quad + (A_L B_R + A_R B_L) \int \frac{d^D k}{(2\pi)^D} \frac{2m_1 m_2}{d_0 d_1} g^{\mu\nu}.
\end{aligned}$$

Where, $A^\mu = A_L P_L \gamma^\mu + A_R P_R \gamma^\mu$. We define

$$\begin{aligned}
\mathcal{M}_1 &= C_1 \int \frac{d^D k}{(2\pi)^D} \frac{1}{d_0 d_1} [2k^\mu q^\mu + 2k^\nu q^\nu + 4k^\mu k^\nu - 2(k^2 + k \cdot q) g^{\mu\nu}] \\
&= i \frac{1 - \ln(2\pi)\epsilon}{16\pi^2} C_1 \{ [4B_{22}(q^2) - 2DB_{22}(q^2) - 2B_{21}(q^2)q^2 - 2B_1(q^2)q^2] g^{\mu\nu} + 4[B_1(q^2) + B_{21}(q^2)] q^\mu q^\nu \},
\end{aligned}$$

$$\begin{aligned} \mathcal{M}_2 &= C_1 \int \frac{d^D k}{(2\pi)^D} \frac{2m_1 m_2}{d_0 d_1} g^{\mu\nu} = i \frac{1 - \ln(2\pi)\epsilon}{16\pi^2} C_2 2m_1 m_2 B_0(q^2) g^{\mu\nu}, \\ C_1 &= (A_L B_L + A_R B_R), \quad C_2 = (A_L B_R + A_R B_L). \end{aligned} \quad (\text{B1})$$

We can analyze their divergences:

$$\mathcal{M}_1(q^2 = 0) \sim \frac{i}{16\pi} C_1 (m_1^2 + m_2^2) g^{\mu\nu}, \quad \mathcal{M}_2(q^2 = 0) \sim 2 \frac{i}{16\pi} C_2 m_1 m_2 g^{\mu\nu}, \quad \frac{d\mathcal{M}(q^2 = 0)}{dq^2} \sim \frac{i}{16\pi} \frac{2}{3} C_1 g^{\mu\nu}. \quad (\text{B2})$$

We expand and sum the coefficients of the involved graphs.

$$\begin{aligned} C_1^{WW} &= -\frac{c_W^2 G^2}{4} [4U_{j1}^* N_{i2} N_{i2}^* U_{j1} + 2U_{j2}^* N_{i3} N_{i3}^* U_{j2} + 2\sqrt{2}(U_{j1}^* N_{i2} N_{i3}^* U_{j1} + h.c.) \\ &\quad + 4V_{j1}^* N_{i2} N_{i2}^* V_{j1} + 2V_{j2}^* N_{i4} N_{i4}^* V_{j2} - 2\sqrt{2}(V_{j1}^* N_{i2} N_{i4}^* V_{j2} + h.c.)], \\ C_2^{WW} &= -\frac{c_W^2 G^2}{4} [4U_{j1}^* N_{i2} V_{j1}^* N_{i2} - 2U_{j2}^* N_{i3} V_{j2}^* N_{i4} - 2\sqrt{2}U_{j1}^* N_{i2} V_{j2}^* N_{i4} + 2\sqrt{2}U_{j2}^* N_{i3} V_{i1} N_{i3}^* \\ &\quad + 4N_{j2}^* U_{i1} N_{j2}^* V_{i1} + 2N_{j3}^* U_{i2} N_{j4}^* V_{i2} - 2\sqrt{2}N_{j2}^* U_{i1} N_{j4}^* V_{i2} + 2\sqrt{2}N_{j3}^* U_{i2} N_{j2}^* V_{i1}], \\ C_1^{\gamma\gamma} &= -\frac{e^2}{4} [4U_{j1}^* U_{i1} U_{i1}^* U_{j1} + 4U_{j2}^* U_{i2} U_{i2}^* U_{j2} + 4(U_{j1}^* U_{i1} U_{i2}^* U_{j2} + h.c.) \\ &\quad + 4V_{j1}^* V_{i1} V_{i1}^* V_{j1} + 4V_{j2}^* V_{i2} V_{i2}^* V_{j2} + 4(V_{j1}^* V_{i1} V_{j2}^* V_{j2} + h.c.)], \\ C_2^{\gamma\gamma} &= -\frac{e^2}{4} [4U_{j1}^* U_{i1} V_{j1} V_{i1}^* + 4U_{j2}^* U_{i2} V_{j2} V_{i2}^* + 4U_{j1}^* U_{i1} V_{j2} V_{i2}^* + 4U_{j2}^* U_{i2} V_{j1} V_{i1}^* \\ &\quad + 4V_{j1}^* V_{i1} U_{j1} U_{i1}^* + 4V_{j2}^* V_{i2} U_{j2} U_{i2}^* + 4V_{j1}^* V_{i1} U_{j2} U_{i2}^* + 4V_{j2}^* V_{i2} U_{j1} U_{i1}^*], \\ \tilde{C}_{1\chi^\pm}^{ZZ} &= -\frac{G^2}{4} [4c_W^4 U_{j1}^* U_{i1} U_{i1}^* U_{j1} + c_{2W}^2 U_{j2}^* U_{i2} U_{i2}^* U_{j2} + 2c_W^2 c_{2W} (U_{j1}^* U_{i1} U_{i2}^* U_{j2} + h.c.) \\ &\quad + 4c_W^4 V_{j1}^* V_{i1} V_{i1}^* V_{j1} + c_{2W}^2 2V_{j2}^* V_{i2} V_{i2}^* V_{j2} + 2c_W^2 c_{2W} (V_{j1}^* V_{i1} V_{j2}^* V_{j2} + h.c.)], \\ \tilde{C}_{2\chi^\pm}^{ZZ} &= -\frac{G^2}{4} [4c_W^4 U_{j1}^* U_{i1} V_{j1} V_{i1}^* + c_{2W}^2 U_{j2}^* U_{i2} V_{j2} V_{i2}^* + 2c_W^2 c_{2W} (U_{j1}^* U_{i1} V_{j2} V_{i2}^* + h.c.) \\ &\quad + 4c_W^4 V_{j1}^* V_{i1} U_{j1} U_{i1}^* + c_{2W}^2 V_{j2}^* V_{i2} U_{j2} U_{i2}^* + 2c_W^2 c_{2W} (V_{j1}^* V_{i1} U_{j2} U_{i2}^* + h.c.)], \\ \tilde{C}_{1\chi^0}^{ZZ} &= -2 \frac{G^2}{4} (N_{j3}^* N_{i3} - N_{j4}^* N_{i4})^2, \\ \tilde{C}_{2\chi^0}^{ZZ} &= 2 \frac{G^2}{4} (N_{j3}^* N_{i3} - N_{j4}^* N_{i4})^2, \\ \tilde{C}_1^{Z\gamma} &= -\frac{eG}{4} [4c_W^2 U_{j1}^* U_{i1} U_{i1}^* U_{j1} + 2c_{2W} U_{j2}^* U_{i2} U_{i2}^* U_{j2} + 2c_{2W} U_{j1}^* U_{i1} U_{i2}^* U_{j2} + 4c_W^2 U_{j2}^* U_{i2} U_{i1}^* U_{j1} + 4c_W^2 V_{j1}^* V_{i1} V_{i1}^* V_{j1} \\ &\quad + 2c_{2W} V_{j2}^* V_{i2} V_{i2}^* V_{j2} + 2c_{2W} V_{j1}^* V_{i1} V_{i2}^* V_{j2} + 4c_W^2 V_{j2}^* V_{i2} V_{i1}^* V_{j1}], \\ \tilde{C}_2^{Z\gamma} &= -\frac{eG}{4} [4c_W^2 U_{j1}^* U_{i1} V_{j1} V_{i1}^* + 2c_{2W} U_{j2}^* U_{i2} V_{j2} V_{i2}^* + 2c_{2W} U_{j1}^* U_{i1} V_{j2} V_{i2}^* + 4c_W^2 U_{j2}^* U_{i2} V_{j1} V_{i1}^* + 4c_W^2 V_{j1}^* V_{i1} U_{j1} U_{i1}^* \\ &\quad + 2c_{2W} V_{j2}^* V_{i2} U_{j2} U_{i2}^* + 2c_{2W} V_{j1}^* V_{i1} U_{j2} U_{i2}^* + 4c_W^2 V_{j2}^* V_{i2} U_{j1} U_{i1}^*]. \end{aligned}$$

References

- [1] [ATLAS], ATLAS-CONF-2023-004.
[2] M. Ambroso and B. A. Ovrut, *Int. J. Mod. Phys. A* **26**, 1569 (2011), arXiv: 1005.5392[hep-th]
[3] P. Fileviez Perez and S. Spinner, *Phys. Rev. D* **83**, 035004 (2011), arXiv: 1005.4930[hep-ph]
[4] V. Barger, P. Fileviez Perez and S. Spinner, *Phys. Rev. Lett.* **102**, 181802 (2009), arXiv: 0812.3661[hep-ph]

- [5] P. Fileviez Perez and S. Spinner, *Phys. Lett. B* **673**, 251 (2009), arXiv: 0811.3424[hep-ph]
- [6] J. L. Yang, T. F. Feng and H. B. Zhang, *J. Phys. G* **47**(5), 055004 (2020), arXiv: 2003.09781[hep-ph]
- [7] J. L. Yang, H. B. Zhang, C. X. Liu, X. X. Dong and T. F. Feng, *JHEP* **08**, 086 (2021), arXiv: 2104.03542[hep-ph]
- [8] J. L. Yang, T. F. Feng, S. M. Zhao, R. F. Zhu, X. Y. Yang and H. B. Zhang, *Eur. Phys. J. C* **78**(9), 714 (2018), arXiv: 1803.09904[hep-ph]
- [9] J. L. Yang, T. F. Feng, Y. L. Yan, W. Li, S. M. Zhao and H. B. Zhang, *Phys. Rev. D* **99**(1), 015002 (2019), arXiv: 1812.03860[hep-ph]
- [10] J. L. Yang, T. F. Feng, H. B. Zhang, G. Z. Ning and X. Y. Yang, *Eur. Phys. J. C* **78**(6), 438 (2018), arXiv: 1806.01476[hep-ph]
- [11] Z. N. Zhang, H. B. Zhang, J. L. Yang, S. M. Zhao and T. F. Feng, *Phys. Rev. D* **103**(11), 115015 (2021), arXiv: 2105.09799[hep-ph]
- [12] J. L. Yang, T. F. Feng, S. K. Cui, C. X. Liu, W. Li and H. B. Zhang, *JHEP* **04**, 013 (2020), arXiv: 1910.05868[hep-ph]
- [13] J. L. Yang, T. F. Feng and H. B. Zhang, *Eur. Phys. J. C* **80**(3), 210 (2020), arXiv: 2002.09313[hep-ph]
- [14] X. X. Dong, T. F. Feng, H. B. Zhang, S. M. Zhao and J. L. Yang, *JHEP* **12**, 052 (2021), arXiv: 2106.11084[hep-ph]
- [15] X. X. Dong, T. F. Feng, S. M. Zhao and H. B. Zhang, *Eur. Phys. J. C* **80**(12), 1206 (2020), arXiv: 2005.03351[hep-ph]
- [16] X. X. Dong, S. M. Zhao, J. P. Huo, T. T. Wang and T. F. Feng, *Phys. Rev. D* **109**(5), 055019 (2024), arXiv: 2402.19131[hep-ph]
- [17] J. L. Yang, Z. J. Yang, X. Y. Yang, H. B. Zhang and T. F. Feng, *Eur. Phys. J. C* **83**(11), 1073 (2023)
- [18] D. D. Cui, T. F. Feng, Y. L. Yan, H. B. Zhang, G. Z. Ning and J. L. Yang, *Phys. Rev. D* **102**, 075002 (2020), arXiv: 2009.09598[hep-ph]
- [19] W. Abdallah, A. Hammad, S. Khalil and S. Moretti, *Phys. Rev. D* **95**, 055019 (2017), arXiv: 1608.07500[hep-ph]
- [20] C. S. Aulakh, A. Melfo, A. Rasin and G. Senjanovic, *Phys. Lett. B* **459**, 557 (1999), arXiv: hep-ph/9902409[hep-ph]
- [21] S. Khalil and H. Okada, *Phys. Rev. D* **79**, 083510 (2009), arXiv: 0810.4573[hep-ph]
- [22] L. Delle Rose, S. Khalil, S. J. D. King, C. Marzo, S. Moretti and C. S. Un, *Phys. Rev. D* **96**(5), 055004 (2017), arXiv: 1702.01808[hep-ph]
- [23] W. Grimus, L. Lavoura, O. M. Ogreid and P. Osland, *Nucl. Phys. B* **801**, 81 (2008), arXiv: 0802.4353[hep-ph]
- [24] I. Maksymyk, C. P. Burgess and D. London, *Phys. Rev. D* **50**, 529 (1994), arXiv: hep-ph/9306267[hep-ph]
- [25] G. Cacciapaglia, C. Csaki, C. Grojean and J. Terning, *Phys. Rev. D* **71**, 035015 (2005), arXiv: hep-ph/0409126[hep-ph]
- [26] L. Lavoura and J. P. Silva, *Phys. Rev. D* **47**, 2046 (1993)
- [27] S. Haywood, P. R. Hobson, W. Hollik, Z. Kunszt, G. Azuelos, U. Baur, J. van der Bij, D. Bourilkov, O. Brein and R. Casalbuoni, *et al.* doi: 10.5170/CERN-2000-004.117[arXiv:hep-ph/0003275[hep-ph]].
- [28] G. Cacciapaglia, C. Csaki, C. Grojean and J. Terning, *Phys. Rev. D* **70**, 075014 (2004), arXiv: hep-ph/0401160[hep-ph]
- [29] C. P. Burgess, S. Godfrey, H. Konig, D. London and I. Maksymyk, *Phys. Lett. B* **326**, 276 (1994), arXiv: hep-ph/9307337[hep-ph]
- [30] P. Asadi, C. Cesarotti, K. Fraser, S. Homiller and A. Parikh, *Phys. Rev. D* **108**(5), 055026 (2023), arXiv: 2204.05283[hep-ph]
- [31] H. N. Long and T. Inami, *Phys. Rev. D* **61**, 075002 (2000), arXiv: hep-ph/9902475[hep-ph]
- [32] A. Pich, I. Rosell and J. J. Sanz-Cillero, *JHEP* **01**, 157 (2014), arXiv: 1310.3121[hep-ph]
- [33] M. Rehman, M. A. Iqbal, M. E. Gomez and O. Panella, *Phys. Rev. D* **112**(5), 055029 (2025), arXiv: 2507.18527[hep-ph]
- [34] M. E. Peskin and T. Takeuchi, *Phys. Rev. D* **46**, 381 (1992)
- [35] D. C. Kennedy and B. W. Lynn, SLAC-PUB-4608.
- [36] D. Binosi and J. Papavassiliou, *Phys. Rept.* **479**, 1 (2009), arXiv: 0909.2536[hep-ph]
- [37] S. Hashimoto, J. Kodaira, Y. Yasui and K. Sasaki, *Phys. Rev. D* **50**, 7066 (1994), arXiv: hep-ph/9406271[hep-ph]
- [38] A. Denner, G. Weiglein and S. Dittmaier, *Nucl. Phys. B* **440**, 95 (1995), arXiv: hep-ph/9410338[hep-ph]
- [39] A. Denner, G. Weiglein and S. Dittmaier, *Phys. Lett. B* **333**, 420 (1994), arXiv: hep-ph/9406204[hep-ph]
- [40] A. Denner, S. Dittmaier and G. Weiglein, *Acta Phys. Polon. B* **27**, 3645 (1996)
- [41] A. Denner, S. Dittmaier and G. Weiglein, [arXiv: hep-ph/9505271[hep-ph]].
- [42] J. Papavassiliou and K. Philippides, *Phys. Rev. D* **48**, 4255 (1993), arXiv: hep-ph/9310210[hep-ph]
- [43] G. Degrossi and A. Sirlin, *Phys. Rev. D* **46**, 3104 (1992)
- [44] D. Binosi and J. Papavassiliou, *Phys. Rev. D* **66**, 111901 (2002), arXiv: hep-ph/0208189[hep-ph]
- [45] D. Binosi and J. Papavassiliou, *J. Phys. G* **30**, 203 (2004), arXiv: hep-ph/0301096[hep-ph]
- [46] J. Papavassiliou and A. Pilaftsis, *Phys. Rev. D* **53**, 2128 (1996), arXiv: hep-ph/9507246[hep-ph]
- [47] D. Binosi and J. Papavassiliou, *Phys. Rev. D* **66**, 025024 (2002), arXiv: hep-ph/0204128[hep-ph]
- [48] J. Papavassiliou and A. Pilaftsis, *Phys. Rev. D* **54**, 5315 (1996), arXiv: hep-ph/9605385[hep-ph]
- [49] J. Papavassiliou, *Phys. Rev. D* **50**, 5958 (1994), arXiv: hep-ph/9406258[hep-ph]
- [50] G. Cacciapaglia, C. Csaki, G. Marandella and A. Strumia, *Phys. Rev. D* **74**, 033011 (2006), arXiv: hep-ph/0604111[hep-ph]
- [51] M. Carena, A. Daleo, B. A. Dobrescu and T. M. P. Tait, *Phys. Rev. D* **70**, 093009 (2004), arXiv: hep-ph/0408098[hep-ph]
- [52] C. P. Burgess, S. Godfrey, H. Konig, D. London and I. Maksymyk, *Phys. Rev. D* **49**, 6115 (1994), arXiv: hep-ph/9312291[hep-ph]
- [53] A. Sirlin, *Phys. Rev. D* **22**, 971 (1980)
- [54] P. H. Chankowski, S. Pokorski and J. Wagner, *Eur. Phys. J. C* **47**, 187 (2006), arXiv: hep-ph/0601097[hep-ph]
- [55] G. Degrossi, B. A. Kniehl and A. Sirlin, *Phys. Rev. D* **48**, R3963 (1993)
- [56] S. Navas *et al.*[Particle Data Group], *Phys. Rev. D* **110**(3), 030001 (2024)