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Multinucleon Effect in the Pion Single Charge Exchange Reaction*

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Under the frame of DWBA, using multinucleon effects in transition matrix elements and optical potential, we calculate the $^{13}C(\pi^+,\pi^0)\,^{13}N(IAS)$ reaction and study the angular distribution at 165 MeV, the forward angle excitation function and the integrated cross section in this reaction. A formula for the mass dependence of the forward differential cross sections is derived.

1. INTRODUCTION

There are three charge states for the pion meson. It can induce the single charge exchange (SCX) reaction on nucleus. Many theoretical and experimental investigations concerning this reaction have been conducted in recent years[1].

However, to what extent do we understand the mechanism of this reaction? Many contradictions still exist in terms of the theory and experiments. Therefore, it is a challenge to further understand these contradictions. Any simple theoretical expression that can compare with the experiment will be helpful.

Due to the development of the technique for detecting the neutral pion meson, high precise measurements can be made for the (π^+ , π^0) reactions[2]. By using the π^0 - spectrometer, a systematic study on a number of nuclei have been conducted at LAMPF for the resonance pion energies. It is found that the forward angle differential cross sections of the reaction to the isobaric analog state can be parametrized as follows[3]

$$\frac{d\sigma^{SCX}}{d\Omega}(\theta = 0^\circ) = g(E)(N - z)A^{-\alpha(E)}, \tag{1}$$

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where α changes from 1.4 to 1.1 for pion energies between 100 MeV to 300 MeV.

Based on a semiclassical theory and the first order optical potential, and by applying isospin invariance for strong interaction, M. B. Johnson et al.[4] found that the forward cross section for this reaction is proportional to $A^{-4/3}$ in the resonance region. This A dependence is qualitatively verified by experiments.

Many scientists have calculated the isobaric analog state (IAS) transition by the DWIA formulism. However, some deviations always occur in between calculations and the experiments. Scientists have been looking for physical origins. It is found that calculating DWIA for SCX reaction is very sensitive to the neutron distribution in nuclear surface region; N. Auerbach et al. studied the influence of nuclear structure on the single charge exchange reaction; Gibbs and Kaufman investigated the medium correction in the SCX reaction; L. C. Liu calculated the SCX and elastic scatterings with a coupled channel method. All these investigations have improved in different ways the theoretical results for the SCX reaction.

However, we know that the higher order processes contribute to the pion-nucleus optical potential. Beginning with a static field theory, the higher order contributions can be calculated. Johnson and Siciliano found that the most important correction to the second order optical potential comes from the Pauli exchange term. Chiang and Johnson studied even higher order of Pauli corrections and obtained an integral equation by summing up a group of irreducible Pauli exchange diagrams. They discussed the effect of multinucleon processes on the pion-nucleus optical potential.

In this paper we examine the mechanism for SCX to the IAS from a different angle. We know that the SCX reaction can be a one-step process. But the multinucleon processes may also contribute to the reaction via nuclear core excitations. In the framework of the DWBA and by adopting the optical potential and transition matrix elements of Ref.[10], in which the higher order terms are included, we calculate the differential cross sections for the single charge exchange reactions. Using the DWIA, an analytic expression for the mass dependence of the SCX reactions is derived. By comparing our calculation with the experimental data we discuss whether the DWIA mechanism is applicable to the SCX reactions.

In Section 2 we formulate our theory for the SCX reaction. The mass dependence of the forward cross sections is derived in Section 3. We present our numerical results and discussions in Section 4.

2. REACTION THEORY

2.1 DWBA Formula

We adopt the DWBA method to describe the single charge exchange reaction induced by a pion in the intermediate energy region. In configuration space the amplitude for a transition from the nuclear ground state to the isobaric analog state can be written as

$$\langle \mathrm{IAS}|\hat{T}(\boldsymbol{K}',\boldsymbol{K})|0\rangle = \int d^3r\,\phi_{\pi^0}^{(-)*}(\boldsymbol{K}',\boldsymbol{r})\langle \mathrm{IAS}|\hat{Q}|0\rangle\phi_{\pi}^{(+)}(\boldsymbol{K},\boldsymbol{r}), \qquad (2)$$

where $\psi_{\pi^+}^{(+)}$ and $\psi_{\pi^0}^{(-)}$ are the distorted waves for an incoming pion with momentum K and an outgoing pion with momentum K', respectively. $\hat{Q}$ is an effective transition operator. In principle $\psi_{\pi^+}^{(+)}$ and $\psi_{\pi^0}^{(-)}$ can be obtained by solving the Klein-Gordon equation. It is shown[11] that the eikonal approximation is good for the distorted wave of pion in the intermediate energy region. In this paper we use the eikonal distorted waves for the pions,

$$\phi^{(+)}(K, r) = e^{iK\cdot r + i\chi(r)}. \tag{3}$$

Here X(r) refers to the phase shift function. It is obtained by the integration of the optical potential along the classical trajectory,

$$\chi(r) = \frac{1}{2K}\int_{-\infty}^{r} dz U_0(r). \tag{4}$$

Here we take U_0 as the isoscalar part of the pion-nucleus optical potential. If a local form of the optical potential is used, U_0 can be written as follows,

$$\begin{aligned} U_0(r) &= K^2\left[1 + \frac{\nabla^2}{2K^2}\right]\xi_0(r), \\ \xi_0(r) &= \xi_0^{(1)}(r) + \xi_0^{(2)}(r) + \cdots. \end{aligned} \tag{5}$$

$\xi_0^{(i)}$ corresponds to a contribution from the ith order term. We use the results of Ref.[10] for $\xi_0^{(i)}$. The transition matrix element is taken as

$$\langle \mathrm{IAS}|\hat{Q}|0\rangle = \frac{i}{\sqrt{2T}}\cdot\frac{K^2}{4\pi}\left[1 + \frac{\nabla^2}{2K^2}\right]\xi_1(r). \tag{6}$$

Here T refers to the nuclear isospin and $1/\sqrt{2T}$ comes from the isospin coupling of the wave functions. ξ_1 (r) can be expanded according to the nuclear density. Using the eikonal distortions and neglecting $q_{||}$, we find that the $\Delta L = 0$ transition,

$$\langle \mathrm{IAS}|\hat{T}|0\rangle = \frac{iK^2}{2\sqrt{2T}}\int_0^\infty b db J_0(qb)\int_{-\infty}^{\infty}\left[1 + \frac{\nabla^2}{2K^2}\right]\xi_1(r)dz \times e^{i\chi_0(b)}, \tag{7}$$

where

$$\chi(b) = \frac{1}{2K}\int_{-\infty}^{\infty} dz U_0(b, z), \tag{8}$$

Here b is the impact parameter and $J_0(qb)$ is the zero order Bessel function.

2.2 First Order Approximation–DWIA

If the spin dependent term is neglected, in the first order approximation one may write $\xi_i = \xi_i^{(1)}$ and

$$\begin{aligned} \xi_0^{(1)} &= \lambda_0^{(1)}\rho(r) \\ \xi_1^{(1)} &= \lambda_1^{(1)}\Delta\rho(r) \end{aligned} \tag{9}$$

Here $\lambda_0^{(1)}$ and $\lambda_1^{(1)}$ are the isoscalar and isovector components of the pion-nucleon scatter-

ing amplitude, respectively. They can be constructed from the pion-nucleon phase shifts. ρ (r) is the nuclear density function normalized to A. $\Delta\rho$ (r) is the valence neutron density distribution function normalized to the number of valence neutrons. If the ∇^2 term is ignored, the reaction amplitude can be written as

$$F^{IAS} \equiv \langle \mathrm{IAS}|\hat{T}|0\rangle = \frac{iK^2\lambda_1}{2\sqrt{2T}}\int_0^\infty bdbJ_0(qb)T_{tr}(b)e^{i\lambda_0 KT(b)/2}, \tag{10}$$

Here T(b) and T_{tr}(b) are the nuclear thickness and transition thickness functions, respectively,

$$\begin{aligned} T(b) &= \int_{-\infty}^{\infty} dz\rho(b,z), \\ T_{tr}(b) &= \int_{-\infty}^{\infty} dz\Delta\rho(b,z). \end{aligned} \tag{10.1}$$

Eq.(10) is the result of the DWIA

2.3 Multinucleon Effect

We introduce the multinucleon correction to the DWIA both in the optical potential for the distortions and in the effective transition matrix element. According to the discussion in Ref.[10], we sum up all the irreducible Pauli exchange diagrams and take the isoscalar part of the Pauli optical potential as the correction to the distortion potential. The correction to the transition matrix element is from the isovector part of the Pauli potential. The multinucleon corrections can be parametrized as

$$\begin{aligned} \Delta\xi_0 &= \lambda_0^{(2)}\rho^2(r)/\rho(0), \\ \Delta\xi_1 &= \lambda_1^{(2)}\Delta\rho(r)\rho(r)/\rho(0). \end{aligned} \tag{11}$$

$\lambda_0^{(2)}$ and $\lambda_1^{(2)}$ are energy dependent and are taken from the results of Ref.[10]. In the calculation of $\lambda^{(2)}$, the spin-dependent part is neglected and only the P-wave part is included in the pion-nucleon scattering amplitude. Furthermore, the local density approximation for the nuclear matrix element is adopted. It is shown [12] that the local density approximation is a good one in the (3,3) resonance region.

3.THE A DEPENDENCE OF THE FORWARD CROSS SECTION

In order to study how good the DWIA mechanism is for dealing with the SCX reaction at the resonance energies, we further investigate the A dependence of the forward SCX differential cross sections. By neglecting the mass difference between the positive pion and the neutral pion, from Eq.[10] the forward amplitude is

$$F^{IAS}(0^0) = \frac{iK^2\lambda_1}{2\sqrt{2T}}\int_0^\infty bdbT_{tr}(b)e^{i\lambda_0 KT(b)/2}, \tag{12}$$

According to the discussion of Johnson and Bethe[13] the elastic scatterings of pions mainly occur in the outer surface of nuclei in the (3,3) resonance energy region, and the nucleus can be approximated by a black disk with certain size. In Eq.(12), the integrand

is peaked at some impact parameter b_1 which is near the nuclear surface. b_1 depends on the size of nucleus and slightly on the incident energy as well. The nuclear thickness function can also be parametrized as the following form,

$$T(b) = T(b_1)e^{(b_1-b)/a}. \tag{13.1}$$

where

$$a(b_1) = T(b)/T'(b)|_{b=b_1} \tag{13.2}$$

We further assume $\Delta\rho$ (r) = (N–z) ρ (r)/A, where N and z are the number of neutrons and protons in the nucleus, respectively. Then we can approximate Eq. (12) as

$$F^{IAS}(0^0) \approx \frac{iK^2}{2\sqrt{2T}} \cdot \frac{N-z}{A} b_1 \int_0^\infty dbT(b_1)e^{(b_1-b)/a} \times e^{i\lambda_0 KT(b_1)e^{(b_1-b)/a}}/2 \tag{14}$$

The above integration can be easily performed. Taking into account the fact that 2T = N–z then we find

$$F^{IAS}(0^\circ) = \sqrt{N-z} \cdot K\frac{\lambda_1}{\lambda_0} \cdot \frac{ab_1}{A} \times [e^{i\lambda_0 KT(b_1)e^{b_1/a}} - 1] \tag{15}$$

It is well known that $\lambda_0^{(1)}$ has a very large imaginary part in the (3,3) resonance energy region. Therefore we have

$$\frac{d\sigma}{d\Omega}^{IAS}(0^0) \approx (N-z)K^2a^2\left|\frac{\lambda_1 b_1}{A\lambda_0}\right|^2. \tag{16}$$

Assuming $b_1 = r_0(A,E)A^{1/3}$, then we find

$$\frac{d\sigma}{d\Omega}^{IAS}(0^0) = (N-z)g(E,A)A^{-4/3}, \tag{17}$$

here

$$g(E,A) = K^2a^2r_0^2(E,A)\left|\frac{\lambda_1}{\lambda_0}\right|^2. \tag{18}$$

At 165 MeV incident pion energy the P-wave dominates in the pion-nucleon amplitude. If the values a = 0.7 fm and r_0 = 1.3 fm are used, we find

$$g = 32.4 \text{ mb}.$$

This result qualitatively agrees with the experiments. Since r_0(E,A) depends slightly on the incident energy and A, it results in some slight deviations for the A dependences of the forward SCX reaction cross sections from $A^{-4/3}$ at different energies. This finding is in agreement with the data.

4. NUMERICAL CALCULATION AND DISCUSSION

Using Eq.(7), we calculate the differential cross section of the SCX reaction on ^{13}C to the ground state of ^{13}N for 165 MeVπ^+. The energy dependence of the forward differential cross sections and the integrated cross sections are calculated. The choice of the ^{13}C nucleus is due to the fact that the nuclear structure of ^{13}C is relatively simple, so that the information for the reaction dynamics can be extracted. The nuclear density of ^{13}C is taken from the electron scattering experiment. The $1P_{1/2}$ pure shell model configuration for the valence neutron in ^{13}C is assumed. The harmonic oscillator parameter is determined by the r. m. s. radius of ^{13}C. The ground state of ^{13}N is considered to be the isobaric analog state of ^{13}C. The parameters in the optical potential for the distortions of pion waves and in the transition matrix elements are taken from the theoretical results described in Section 2. It is known that due to the propagation of Δ in nucleus a complex energy shift is needed in the P_{33} partial wave for the evaluation of the pion-nucleus optical potential. From the empirical results we take $E_s^- = (35.0, 0.3i)$ MeV. The $\lambda_i^{(2)}$ for 165 MeV incident positive pion are $\lambda_0^{(2)} = (0.75, 3.65i)$ fm^3 and $\lambda_1^{(2)} = (7.71, 15.1i)$ fm^3, respectively. In Fig.1 we show our calculation of the differential cross section. The solid line presents the result including the higher order corrections and the dashed line is the result of the DWIA. We find that the higher order corrections increase the differential cross section in our calcula-

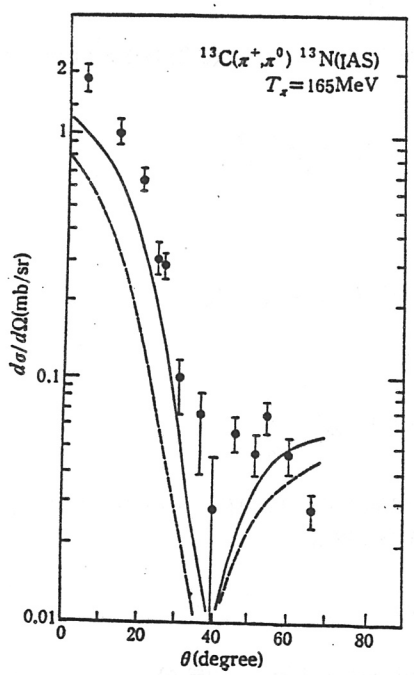

Fig. 1 Differential cross section for the $^{13}C(\pi^+, \pi^0)\ ^{13}N(IAS)$ SCX reaction at T_π = 165 MeV.

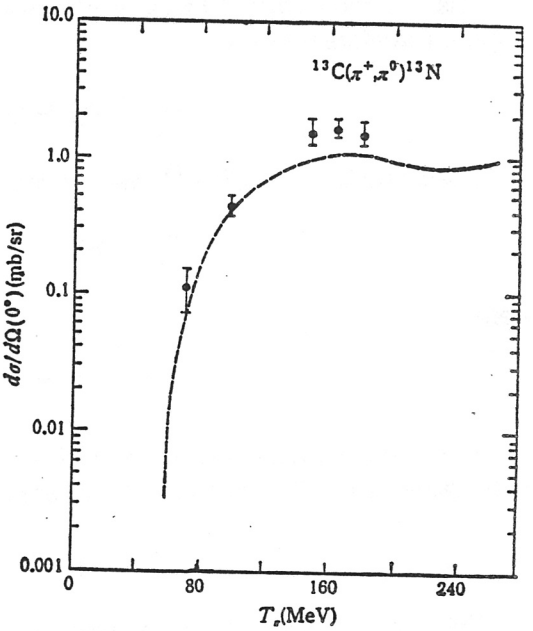

Fig. 2 Energy dependence of the forward differential cross sections for the $^{13}C(\pi^+, \pi^0)\ ^{13}N_{g.s.}$ reaction.

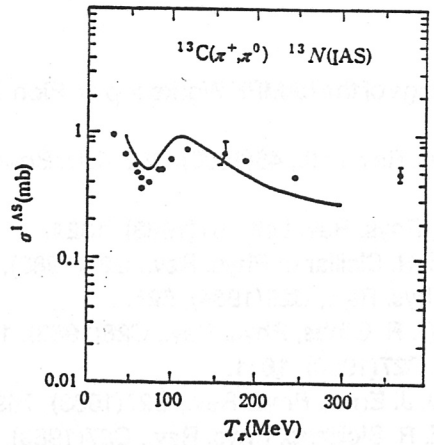

Fig. 3 Energy dependence of the integrated cross sections for the $^{13}C(\pi^+, \pi^0)\,^{13}N_{g.s.}$ reaction.

tion for the incoming pion energy 165 MeV. The shape of the angular distribution does not change much. In another word the core excitation enhances the cross section for the isobaric analog state transition.

By using the $\lambda^{(2)}$ and $\lambda^{(2)}$ of Ref.[10] we also calculated the cross sections of the IAS transitions at different energies. The excitation function for ^{13}C is shown in Fig.2. Recently the energy dependence of the integrated cross sections for the $^{13}C(\pi^+, \pi^0)\,^{13}N$(IAS) reaction was measured. Our calculation is given together with the data in Fig.3. It shows that by using the DWBA a better result is obtained in comparison with the data.

To sum up, our investigation shows that the DWBA mechanism can better describe the SCX reaction. The DWIA alone can give the A dependence of the forward cross sections. The multinucleon effect only provides some correction to the absolute value of the cross sections. In our calculation both the s- and p-wave parts in the π -N amplitude are included in the first order optical potential. In the calculation of higher order corrections only the p-wave π -N amplitude is used. This approximation is not good for the calculation of the SCX reaction around 50 MeV, where the isovector parts of the s- and p-wave π- N amplitudes almost cancel each other. Therefore, the multinucleon effect is overestimated at 50 MeV in our calculation. However, the most important contribution to the SCX reaction is from the first order optical potential. We expect that our result is not qualitatively affected by this approximation. In our calculation the spin-dependent part in the π - N amplitude is neglected. It shows that the spin dependent term is small.

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