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The Mass Gap in 2 + 1 Dimensional SU(2) Lattice Gauge Theory*

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A glueball mass in 2 + 1 dimensional SU(2) lattice gauge theory is calculated by applying variation in the Hamiltonian with exact ground state. In the range $0 \le 1/g^2 \le 7$, a good scaling behaviour $am = 2.28g^2$ is obtained, which is in agreement with weak-coupling expansion result.

Using lattice gauge theory we can predict glueball mass. We calculated the glueball mass in 2 + 1 dimensional U(1) LGT, and explained that the use of the Hamiltonian with exact ground state is effective in studying glueball masses [1,2]. This paper extends the method to SU(2) theory. 2 + 1 dimensional SU(2) gauge theory is super-renormalizable which has been discussed somewhere [4-9], and the weak coupling expansion shows that the scaling behaviour is $ma \propto g^2$ (where $g^2 = e^2 a$, e is the charge).

The lattice Hamiltonian with exact ground state is:

$$H = \frac{g^2}{2a}\sum_l E_l^2 - \frac{1}{ag^2}\sum_p \mathrm{Tr}(U_p + U_p^\dagger) - \frac{g^2\theta^2}{2a}\sum_{l,\pi,\pi'} \mathrm{Tr}(U_l^\dagger \Lambda^a U_\pi^\dagger - U_\pi \Lambda^a U_l)\mathrm{Tr}(U_l^\dagger \Lambda^a U_{\pi'}^\dagger - U_{\pi'} \Lambda^a U_l) \tag{1}$$

where $\theta = 1/(2g^4 C_N)$, C_N is the Casimir invariant of the SU(N) gauge group in the fundamental representation. Λ^a is the representation matrix of the generator T^a of SU(N) gauge group. For SU(2) gauge group, $\theta = 2/(3g^4)$, $\Lambda^a = \tau^a/2$, τ^a are the Pauli matrices, U_π, $U_{\pi'}$ are the product of three link variables other than U_l on a plaquette U_p.

The exact ground state of the Hamiltonian in eq.(1) is:

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$$|\Psi_0\rangle = e^R|0\rangle \tag{2}$$

where $R = 2/(3g^4)\sum_p \mathrm{Tr}(U_p + U_p) = 4/(3g^4) \quad \mathrm{Tr}U_p$, and $|0>$ is the state defined by $E_l|0> = 0$.

The Hamiltonian in eq.(1) can be rewritten as:

$$H = \frac{g^2}{2a} e^{-R} E_l^a e^{2R} E_l^a e^{-R} \tag{3}$$

We use the variational method to obtain the spectrum of the excited state in eq.(3), The variational state is chosen as:

$$|\Psi\rangle = \sum_{n=1}^{N} C_n |\Psi_n\rangle \tag{4}$$

where $|\psi_n>$ are a set of states which are orthogonal to $|\psi_0>$ and possess the same quantum numbers, C_n are the variational parameters, the excited energy E is determined by the following minimum condition:

$$\delta E = \delta(\langle\Psi|H|\Psi\rangle/\langle\Psi|\Psi\rangle) = 0 \tag{5}$$

The lowest excited state of SU(2) gauge theory is a $J^{pc} = 0^{++}$ state. For this excited state, we choose $|\psi_n>$ as $n \times n$ square Wilson loop, that is:

$$|\Psi_n\rangle = (\hat{\Phi}_n - v_n)|\Psi_0\rangle \tag{6}$$

$$\hat{\Phi}_n = \frac{1}{\sqrt{V}} \sum_x \mathrm{Tr}U_{np}(x) \tag{7}$$

where V is the total number of lattice site, $U_{np}(\vec{x})$ is the $n \times n$ square Wilson loop with a corner located at $\vec{x}$. To make $|\psi_n>$ orthogonal to $|\psi_0>$, v_n should be chosen as:

$$v_n = \langle\hat{\Phi}_n\rangle \tag{8}$$

where $<\ldots> \equiv <\psi_0|\ldots|\psi_0>/<\psi_0|\psi_0>$. By variation with respect to C_n in eq.(5), we can obtain the eigenvalue equation:

$$\det\|W_{mn} - \lambda D_{mn}\| = 0 \tag{9}$$

where

$$W_{mn} = -\left\langle \sum_{l,a,x} [E_l^a, \hat{\Phi}_m(0)][E_l^a, \hat{\Phi}_n(x)] \right\rangle_0 \tag{10}$$

$$D_{mn} = \left\langle \sum_x \Phi_m(0)\Phi_n(x) \right\rangle_0 - \langle\Phi_m(0)\rangle_0 \left\langle \sum_x \Phi_n(x) \right\rangle_0 \tag{11}$$

$$\lambda = 2am\beta = 2am/g^2 \tag{12}$$

In 2 + 1 dimensional LGT, when we change the link integration variables $[dU_l]$ into plaquette integration variables [dUp], the Jacobian is 1 [10], the integral measure [dUp] and ground state $|\psi_0>$ are invariant in local gauge transformation. From this we can show that the expectation of any $n \times m$ Wilson loop in ground state can be rewritten as expectation of the trace of the product of $n \times m$ plaquettes inside the loop. Therefore, the calculation of matrices W_{mn} and D_{mn} can be reduced to one-link SU(2) group integration. The ground state expectation B_n of the square of Wilson loop including n plaquettes is needed in the calculation:

$$B_n = \langle \mathrm{Tr}(U_{1p}U_{2p}\cdots U_{np})\mathrm{Tr}(U_{1p}U_{2p}\cdots U_{np})\rangle_0 \tag{13}$$

By using the SU(2) one-link integration formula:[11]

$$Z = \int_{SU(2)} dU e^{\mathrm{Tr}(UJ^+ + JU^+)} = \sum_{k=0}^{\infty} (\mathrm{Tr}JJ^+ + \det J + \det J^+)^k / [k!(k+1)!]$$

We obtain the recurrence formula:

$$B_n = (1 - 2y_2/x)B_{n-1} + 2y_2/x \tag{14}$$

where $x = 8/(3g^4)$, $y_2 = I_2(2x)/I_1(2x)$, $I_i(2x)$ is the i-th modified Bessel function, and

$$B_1 = 4(1 - 3y_2(2x)/2x)$$

When $n \geq m$, the symmetric matrix elements W_{mn}, D_{mn} are:

$$W_{mn} = 4m(n - m - 1)y_2^{n^2-m^2}(1 - B_{m^2}/4) + 8\sum_{k=1}^{m} k y_2^{n^2+m^2-2km}(1 - B_{km}/4) \tag{15}$$

$$D_{mn} = 4\sum_{k,i=1}^{m}(y_2^{n^2+m^2-2ki}B_{ki} - 4y_2^{n^2+m^2}) + (n - m - 1)^2(y_2^{n^2-m^2}B_{m^2} - 4y_2^{n^2+m^2})$$
$$+ 4(n - m - 1)\sum_{k=1}^{m}(y_2^{n^2+m^2-2km}B_{km} - 4y_2^{n^2+m^2}) \tag{16}$$

We solve the eigenvalues of eq.(9) for N = 1, 2, 3, 4, 5, 8, 15 and 30, respectively, and choose the minimum value of m to give the resulting curves β am $\sim \beta$ as shown in Fig.1. The scaling behaviour am $= 2.28g^2$ is observed in the range $1/g^2 > 1$. It is in agreement with weak coupling expansion in a considerable large range ($1 \leq 1/g^2 \leq 7$). The latest Monte Carlo result [9] is am $= (2.15 \pm 0.2)g^2$ in the range $4.5 \leq 4/g^2 \leq 5.5$. It is worthwhile to mention here that in our calculation other variational states do not alter the eigenvalue β am significantly. Similar to the U(1) case [2], $n \times n$ Wilson loop (that is the variational states we choose) is most important to decrease the eigenvalue. On the other hand, owing to the scaling behaviour of 2 + 1 dimensional SU(2) theory being $a \propto g^2$, it cannot prove that the last term ΔH in eq.(1) vanishes (but it can be shown to be finite) as $a \to 0$. Therefore the continuum limit of eq.(1) in 2 + 1 dimension is not the same as that of Wilson action, but the above results show that they possess the same scaling behaviour. In addition, we have also obtained another Hamiltonian which possesses exact ground state and the

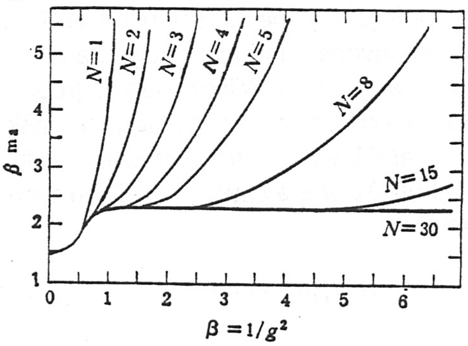

FIG. 1 $\beta=1/g^2$

same continuum limit as that of Wilson action in both 2 + 1 and 3 + 1 dimensions. The same scaling behaviour with the one in this paper can also be obtained. We will report these results later.

REFERENCES

[1] Guo S. H., Liu J. M. and Chen Q. Z., Chin. Phys. Lett., 2(1985), 409.
[2] Guo S. H., Zheng W. H. and Liu J. M., Chin. Phys. Lett., 3(1986), 445.
[3] V. F. Muller and W. Ruhl, Nucl. Phys., B230(1984), 49.
[4] H. Arosie, M. Kato and T. Fujiwara, Progr. of Theor. Phys., 70(1983), 229.
[5] E. d'Hoker, Nucl. Phys., B180(1981), 341.
[6] A. Patkos, Phys. Lett., B110(1982), 391.
[7] A. Irback and C. Peterson, Phys. Lett., B174(1986), 99.
[8] C. J. Hamer and A. C. IvingG, Z. Phys., C27(1985), 307.
[9] K. Farakos, G. Koutsoumbas and S. Sarantakos, Phys. Lett., B189(1987), 173.
[10] G. G. Batrouni, Nucl. Phys., B208(1982), 12.
[11] K. E. Erikson et al, J. Math. Phys., 22(1981), 2276.