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Nuclear Pairing Phase Transition*

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The intrinsic structure of the eigenfunctions of Cranking Shell Model Hamiltonian and the pairing phase transition at high rotational frequency are investigated with the particle-number-conserving approach. Pair-transfer matrix elements and the K-structure of the wave functions display similar behaviour under the Coriolis interaction, hence, both of them may be used to indicate the pairing phase transition. The blocking effects on pairing parameter, $\Delta = G\sqrt{<S+S>}$, are much more important than the Coriolis anti-pairing effects.

1. INTRODUCTION

Recently, it was found experimentally that in a series of Hf-isotopes the yrast bands and the negative parity bands are a macroscopic rotor for $\hbar\omega > 0.40$ MeV, namely, their moments of inertia are almost constant and close to the rigid-body value[1]. Similar behaviour was observed[2] in the high spin states of ^{80}Zr. These facts seem to imply that the nuclear pairing correlation has effectively disappeared at high spin states which brought to us the view-point that the pairing collapse would happen in high spin states (Mottelson and Valatin[3]). But as was pointed out by P. Ring et al.[4], the moment of inertia deduced from the energy spectra is of limited value to indicate a phase transition from superfluid state to normal state, and the constant moments of inertia are by no means indications of a pairing collapse. The energy gap in the quasi-particle spectrum is expected at low spins due to the pairing correlation, but a gapless superconductivity will be found at high spin states. Therefore, it is hard to judge the superconductivity from the energy spectra. Indeed, people found a pairing collapse in all self-consistent solutions of the cranked HFB equation [5]. For rare earth nuclei, the collapse would happen at $I \sim 20-$

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30 $\hbar$ for neutron and at I~ 40−50 $\hbar$ for proton. However, the results obtained without particle-number projection are not reliable[4]. In fact,calculation results with particle-number projection before variation[6] show that the gap parameter decreases very slowly with increasing I and still remains larger than 300 keV even at I~ 80 $\hbar$, the expected maximal angular momenta accommodated by rotating nuclei[7].

As for the definition of gap parameter, there have been several discussions available[8]. Usually, the gap parameter Δ is used to characterize the pairing correlation in the BCS or HFB method[9].

$$\Delta = G\langle S^+\rangle = G\sum_{\nu} U_\nu V_\nu, \tag{1}$$

where G is the average pairing strength and $S^+ = \sum_{\nu} S_\nu^+ = \sum_{\nu} a_\nu^+ a_{\bar{\nu}}^+$ is the creation operator of a pair of nucleons. Δ is also referred as pairing deformation. Obviously, definition (1) is meaningless in the particle-number-conserving approach because $<S^+> = 0$ for a state with a fixed particle number. Instead, we adopt the following definition[4,9] in the present paper.

$$\tilde{\Delta} = G\sqrt{\langle S^+S\rangle}, \tag{2}$$

This is a measurement of pairing energy, $\tilde{\Delta} = \sqrt{G|\langle H_p\rangle|}$ or $\langle H_p\rangle = -\tilde{\Delta}^2/G$. It can be proved that Δ is in coincidence with Δ in the BCS or HFB framework.

Of course, the most direct evidence for a pairing phase transition should be given by the pair transfer matrix element[4]. But unfortunately, it seems to be very difficult to measure this matrix element at high spin states [10]. Therefore, we have to rely on theoretical investigations at the moment.

Usually, the particle-number non-conservation is considered as a main defect of the BCS or HFB approximation. Many techniques of particle-number projection have been developed. The simplest one is to extract the components with the desired particle-number from the BCS wave functions (PBCS). Another recipe is to carry out the particle-number projection before variation (FBCS). The relevant references can be found in Ref.[9]. However, it needs to point out that, rather than the particle-number non-conservation, the most serious defects of the BCS or HFB approximation for treating the nuclear system are as follows:

(i) It is hard to take into account the blocking effect[11,12]. Simply due to this effect, the gap parameter Δ is sensitively configuration-dependent[13,14], however it is generally considered as a constant in the BCS treatment. The fact is that, apart from the Coriolis anti-pairing (CAP) effect, the blocking effect is another kind of anti-pairing effects[15], which is of special importance for the low-lying excited spectra.

(ii) There exist excessive spurious states[12]. The number of the spurious states is often larger than that of true states by a factor 2 or more[12]. Therefore, the conclusions deduced from the quasi-particle spectra have to be reexamined carefully.

(iii) The usability of the usual BCS or HFB treatment is intimately related to the assumption that the residual interaction between quasi-particles is negligible. However, the

analyses of experimental data indicate that this residual interaction is not very weak[15], it is very difficult (in fact it is impossible) to be dealt with in the BCS or HFB method.

The particle-number conserving (PNC) approach for treating the nuclear pairing correlation is extended to solve the eigen-problem of the cranked shell model (CSM) Hamiltonian in this paper. A brief report was given in Ref. [16]. Because the number of the valence nucleons involved in low-lying excited states is not too large ($\sim$ 10) and the average pairing strength G is not too strong ($G \lesssim d/2$, d is the average spacing between the single particle levels in the vicinity of the Fermi surface), it is not difficult to find out the accurate PNC wave functions. Once the wave functions are obtained, the various nuclear properties, such as the pairing parameter $\tilde{\Delta}$ which is of interest in this paper, and the pair transfer matrix elements, etc., can be deduced immediately. The variation of all these properties are the reflection of the variation of the intrinsic structure of the wave function. We are interested in the analysis on the K-structure of low-lying excited bands in this paper. As will be seen later, similar to the pair transfer matrix element, the K-structure (i.e., the component of the fully-paired $K^{\pi} = 0^{+}$ configurations) is an important quantity which can be used to characterize the pairing phase transition.

2. THEORY

As usual, the cranked shell model Hamiltonian for a deformed nucleus is expressed as

$$H_{\mathrm{CSM}} = H_{\mathrm{intr}} - \omega J_x = H_{sp} + H_p - \omega J_x. \tag{3}$$

where $-\omega J_x$ is the Coriolis interaction, H_{sp} is the single particle Hamiltonian (deformed shell model), and H_P the pairing interaction. Due to the Coriolis interaction, the component of the nuclear angular momentum onto the symmetry (z-axis) is no longer a constant of motion. However, H_P (and hence, H_{CSM}) is usually assumed to be invariant under $R_x(\pi)$, a rotation of 180° around x-axis, therefore,it is convenient to choose the single particle state as an eigenstate of $R_x(\pi)$. The eigenvalue of $R_x(\pi)$, signature, $r = \pm i$ for a single nucleon and $r = \pm 1$ for a system consisting of even nucleons. It can be seen that the form of H_{CSM} remains unchanged if a proper phase convention is adopted [14]. Particularly, the pairing interaction H_P is expressed as

$$H_p = -GS^{+}S = -G \sum_{\mu\nu} b_{\mu}^{+} b_{\bar{\mu}}^{+} b_{\bar{\nu}} b_{\nu}, \tag{4}$$

where μ (ν) denotes the single particle state with a fixed signature r and parity π, and $\bar{\mu}$ ($\bar{\nu}$) is the state with the same parity but opposite signature, i.e., the time-reversal state of μ (ν). The effect of the time-reversal operator T on a single particle state is equivalent to that of the operator $R_x(\pi)$ except for a possible phase factor.

When $\omega = 0$, there are two types of bands in the low-lying excited spectra, i.e., pair-excitation band and pair-broken band. The pair excitation band for a 2n-particle system is expressed as

$$\mathscr{A}^{+}_{n\beta}|0\rangle = \sum_{\rho_1\cdots\rho_n} v^{\beta}_{\rho_1\cdots\rho_n}|\rho_1\bar{\rho}_1\cdots\rho_n\bar{\rho}_n\rangle, \tag{5}$$

where

$$|\rho_1\bar{\rho}_2\cdots\rho_n\bar{\rho}_n\rangle = b^{+}_{\rho_1}b^{+}_{\bar{\rho}_1}\cdots b^{+}_{\rho_n}b^{+}_{\bar{\rho}_n}|0\rangle,$$

$\beta = 0$ (yrast band), 1, 2, ...(pair-excitation bands). For these bands,

$$\langle S^{+}S\rangle = \sum_{\mu\nu}\langle S^{+}_{\mu}S_{\nu}\rangle = n + \sum_{\mu\neq\nu}\langle S^{+}_{\mu}S_{\nu}\rangle$$

$$= n + \sum_{\rho_1\cdots\rho_i\cdots\rho_n}\sum_{\rho'_i\neq\rho_i} v^{\beta}_{\rho_1\cdots\rho_i\cdots\rho_n}v^{\beta}_{\rho_1\cdots\rho'_i\cdots\rho_n}. \tag{6}$$

The first term on the right hand side is the number of particle pairs and the second term comes from the off-diagonal contributions of the pairing interaction. Due to the coherent effect, $<S^{+}S>$ will be much larger than n for the yrast band.

The pair-broken band ($r = +1, \pi = +$) is expressed as

$$\mathscr{A}^{+}_{n-1\beta}(\mu_0\bar{\nu}_0)|0\rangle = \sum_{\rho_1\cdots\rho_{n-1}} v^{\beta(\mu_0\bar{\nu}_0)}_{\rho_1\cdots\rho_{n-1}}|\mu_0\bar{\nu}_0\rho_1\bar{\rho}_1\cdots\rho_{n-1}\bar{\rho}_{n-1}\rangle,$$

μ_0 and $\bar{\nu}_0$ are the single particle orbits occupied by the unpaired particles, $\pi_{\mu 0} = \pi_{\nu 0}$. Similar to Eq.(6), we have

$$\langle S^{+}S\rangle = n - 1 + \sum_{\rho_1\cdots\rho_i\cdots\rho_{n-1}}\sum_{\rho'_i\neq\rho_i} v^{\beta(\mu_0\bar{\nu}_0)}_{\rho_1\cdots\rho_i\cdots\rho_{n-1}}v^{\beta(\mu_0\bar{\nu}_0)}_{\rho_1\cdots\rho'_i\cdots\rho_{n-1}}. \tag{7}$$

When $\omega \neq 0$, the eigenstates of H_{CSM} for the low-lying excited bands ($r = +1, \pi = +$) of an even-even rotational nucleus can be expressed as

$$|n\beta{+}{+}\rangle = \sum_{\rho_1\cdots\rho_n} v^{\beta}_{\rho_1\cdots\rho_n}|\rho_1\bar{\rho}_1\cdots\rho_n\bar{\rho}_n\rangle + \sum_{\mu_0\nu_0}\sum_{\rho_1\cdots\rho_{n-1}} v^{\beta(\mu_0\bar{\nu}_0)}_{\rho_1\cdots\rho_{n-1}}|\rho_1\bar{\rho}_1\cdots\rho_{n-1}\bar{\rho}_{n-1}\rangle, \tag{8}$$

$$\sum_{\rho_1\cdots\rho_n}|v^{\beta}_{\rho_1\cdots\rho_n}|^2 + \sum_{\mu_0\nu_0}\sum_{\rho_1\cdots\rho_{n-1}}|v^{\beta(\mu_0\bar{\nu}_0)}_{\rho_1\cdots\rho_{n-1}}|^2 = 1.$$

which consists of configurations with seniority $v = 0$ and $v = 2$. Calculation shows that the component of the configurations with more unpaired particles ($v \geq 4$) in the low-lying band is negligibly small[1]. The component of $K^{\pi} = 0^{+}$ fully-paired configurations in the band $|n\beta{+}{+}>$ is

$$P_{\beta} = \sum_{\rho_1\cdots\rho_n}|v^{\beta}_{\rho_1\cdots\rho_n}|^2, \tag{9}$$

and

$$\langle S^{+}S\rangle = n - (1 - P_{\beta}) + \sum_{\rho_1\cdots\rho_i\cdots\rho_n}\sum_{\rho'_i\neq\rho_i} v^{\beta}_{\rho_1\cdots\rho_i\cdots\rho_n}v^{\beta}_{\rho_1\cdots\rho'_i\cdots\rho_n}$$

1) The configurations of $v = 4$ are necessary for a more detailed calculation. But those of $v > 6$ are still negligible.

$$+\sum_{\mu_0\nu_0}\sum_{\rho_1\cdots\rho_i\cdots\rho_{n-1}}\sum_{\rho'_{i\neq\rho_i}} v^{\beta(\mu_0\bar{\nu}_0)}_{\rho_1\cdots\rho_i\cdots\rho_{n-1}} v^{\beta(\mu_0\bar{\nu}_0)}_{\rho_1\cdots\rho'_i\cdots\rho_{n-1}}. \tag{10}$$

The pair transfer matrix element between the adjacent even-even nuclei is

$$\langle n+1\beta'++|S^+|n\beta++\rangle = \sum_{\nu}\sum_{\rho_1\cdots\rho_n} v^{\beta'}_{\nu\rho_1\cdots\rho_n} v^{\beta}_{\rho_1\cdots\rho_n}$$

$$+\sum_{\mu_0\nu_0}\sum_{\nu}\sum_{\rho_1\cdots\rho_{n-1}} v^{\beta'(\mu_0\bar{\nu}_0)}_{\nu\rho_1\cdots\rho_{n-1}} v^{\beta(\mu_0\bar{\nu}_0)}_{\rho_1\cdots\rho_{n-1}}. \tag{11}$$

We are particularly interested in the pair transfer matrix element between the yrast bands and its variation with ω, let

$$R_0 = \frac{|\langle n+1\ 0++|S^+|n\ 0++\rangle|^2}{|\langle n+1\ 0++|S^+|n\ 0++\rangle|^2(\omega=0)}. \tag{12}$$

Obviously, due to the Coriolis interaction the ratio R_0 decreases with the increase of ω.

Usually the ground state of an even-even nucleus at $\omega = 0$, $|n0++(\omega=0)\rangle = \mathscr{A}^+_{n0}|0\rangle$ is regarded as the quasiparticle vacuum state. The projection of the band $|n\beta++>$ onto this state is

$$\langle n0++(\omega=0)|n\beta++(\omega)\rangle = \sum_{\rho_1\cdots\rho_n} v^{0}_{\rho_1\cdots\rho_n}(\omega=0)v^{\beta}_{\rho_1\cdots\rho_n}(\omega). \tag{13}$$

Especially, for $\beta = 0$, $|\langle n0++(\omega=0)|n0++(\omega)\rangle|^2$ represents the component of quasi-particle vacuum in the intrinsic state of a rotating nucleus. It is easy to prove

$$|\langle n0++(\omega=0)|n0++(\omega)\rangle| > P_0 > |\langle n0++(\omega=0)|n0++(\omega)\rangle|^2. \tag{14}$$

3. RESULTS AND DISCUSSIONS

As in Refs.[17,18], we analyze the low-lying eigen-states of H_{CSM} in a single-j model ($j^\pi = 13/2^+$). In this model the single particle levels take the form of

$$\varepsilon_{|\Omega|} = \kappa\frac{3\Omega^2 - j(j+1)}{j(j+1)} + e_0. \tag{15}$$

The truncated configuration energy is chosen as $E_c/\kappa = 2.5$ in the diagonalization of H_{CSM}. The calculations were done for $G/\kappa = 0.10$, 0.15, 0.20, 0.25 and 0.30. The results for $G/\kappa = 0.20$ are presented in the following, and those for other G values are similar.

First, we analyze the variation of pair transfer matrix elements. The ω–dependence of R_0, the relative strength of pair transfer reaction between the ground bands of 6- and 8-particle systems is plotted in Fig.1. It can be seen that when $\omega/\kappa \approx 0.18$, $R_0 \approx 0.5$, i.e., the pair transfer reaction strength decreases to about one half of that for $\omega = 0$. Calculations also show that the R_0-value decreases smoothly with the increase of ω and no sharp drop happens.

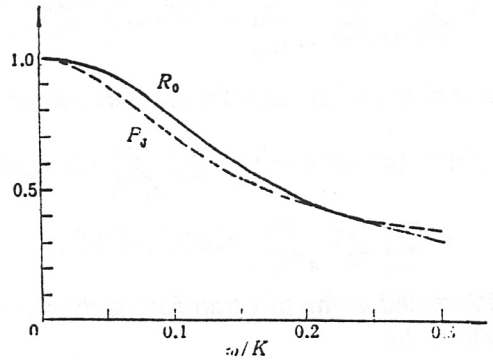

FIG. 1 Variations of R_0 and P_0 with ω . R_0 is the relative strength of the pair transfer reaction between the yrast bands of 6- and 8-particle systems and P_0 the component of the fully-paired $K^\pi = 0^+$ configurations in the yrast band wave function of 8-particle system.

It should be noted that the ω–dependence of the K-structure for the yrast band of HCSM, P_0, i.e., the component of the fully-paired configurations ($K^\pi = 0^+$, $v = 0$), is very similar to that of R_0. As ω increases, the pair-broken configurations ($K \neq 0$) are gradually mixed into the yrast band due to the Coriolis interaction. When P_0 decreases to 0.5, i.e., the weight of the pair-broken configurations is the same as that of the fully-paired con-

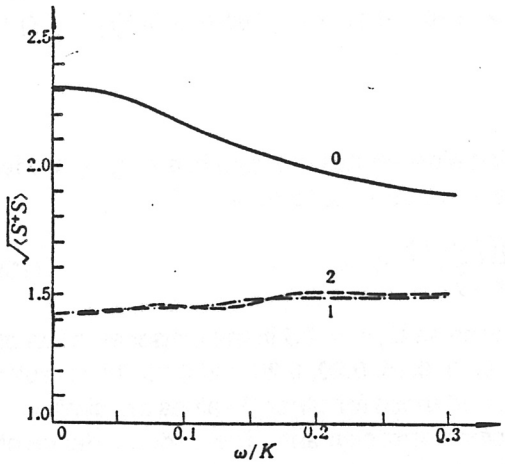

FIG. 2 Pairing parameter $\tilde{\Delta}$ /G for 6-particle system. 0, 1 and 2 correspond to the yrast band, the yrare band and the second excited band, respectively.

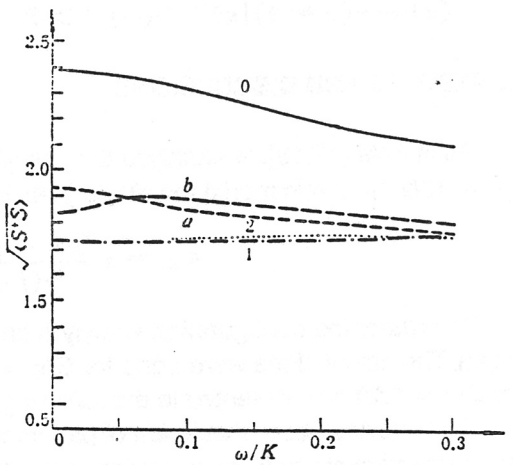

FIG. 3 Pairing parameter $\tilde{\Delta}$ /G for 8-particle system. 0, 1 and 2 correspond to the yrast band, the yrare band and the second excited band, respectively. a and b are referred to the lowest two bands in 7-particle system.

figurations, the overlap integral of the wave function onto that for the ground state at $\omega = 0$ (i.e., the so-called quasiparticle vacuum state) $|\langle n0++(\omega=0)|n0++(\omega)\rangle|^2 < P_0$ In this case, the intrinsic structure of the wave function becomes quite different from that of the quasiparticle vacuum state. In other words, the intrinsic structure of the wave function has undergone a great change. Perhaps we can say that in this case the nuclear superconductivity has disappeared in fact. Therefore, it seems reasonable to use P_0, the K-structure of the wave function for the yrast band, to describe the superconductivity of a deformed nucleus, as R_0 does.

The calculation had been carried out also for a two-j shell ($j^\pi = 13/2^+$ and $9/2^-$) model which should resemble the realistic case. The separation between the bottoms of these two shells, $\varepsilon_\Omega = 1/2(13/2^+) - \varepsilon_\Omega = 1/2(9/2^-)$, is fixed to be 1.0 κ. The calculated curves of R_0 and P_0 almost coincide with each other. A further calculation with the Nilsson levels is in progress. The preliminary result shows that the behaviour of the synchronistic variations of R_0 and P_0 remains unchanged. These facts reveal again that it is reasonable to use P_0 and the K-structure of the wave function to describe the superconductivity of a deformed nucleus.

Now we analyze the variation of the gap parameter $\tilde{\Delta} = G\sqrt{<S^+S>}$ with ω. The results are similar with the one deduced by P.Ring et al with the particle number projection before variational calculation,such as, the calculated $\tilde{\Delta}$-value for the yrast band varies with ω very slowly (Figs.2 and 3) and changes slightly even at very high rotational frequency. For instance, $\tilde{\Delta}(\omega/\kappa = 0.30)/\tilde{\Delta}(\omega = 0) \geq 80\%$. This fact can be understood as follows: $<S^+S>$ = number of particle pairs n for a pure fully-paired configuration and n - 1 for a pure one-pair-broken configuration. The $<S^+S>$-value for the yrast band will be enhanced to some extent due to the coherent effect of the pairing correlation. However, the yrast band and other low-lying bands are mainly composed of paired and pair-broken configurations. Usually, we have $<S^+S> \gtrsim n-1$. Hence, the variation of $\sqrt{<S^+S>}$ with ω becomes very slowly. However, according to the analyses given above, a great change of the K-structure of wave function and the pair transfer matrix element happens when $\omega/\kappa \gtrsim 0.20$, which implies that the status of nuclear pairing correlation has undergone a great change. Therefore, it seems not suitable to indicate the pairing phase transition by means of the variation of $\tilde{\Delta}$.

Contrary to the slow variation with ω, the gap parameter $\tilde{\Delta}$ is sensitive to the blocking effect. Systematic odd-even difference of $\tilde{\Delta}$ is found in the calculation. As shown in Figs.2 and 3, the $\sqrt{<S^+S>}$-values for the lowest two bands in 7- particle system (curves labelled by a and b) are smaller than those for the yrast bands in 6- and 8-particle systems. Furthermore, for the excited bands in even systems, the blocking effect is even stronger because their main components are pair-broken configurations, hence, the $\sqrt{<S^+S>}$-values (curves labelled by 1 and 2) become much smaller. Moreover, it can be seen that the gap parameter $\tilde{\Delta}$ for excited bands varies slightly with ω and almost remains a constant. For example, for the lowest two excited bands in 8-particle system, $\sqrt{<S^+S>} \approx 1.74 \pm 0.01$ for $\omega/\kappa < 0.30$ (Fig.2, curves 1 and 2). It is understandable because the excited bands are not superconductivity states even though at $\omega = 0$.

Finally, we discuss the variation of $\tilde{\Delta}$ with pair strength G. In the BCS or HFB approximation $\tilde{\Delta} = \Delta = 0$, when $G < G_c$. G_c depends on the location of the Fermi surface

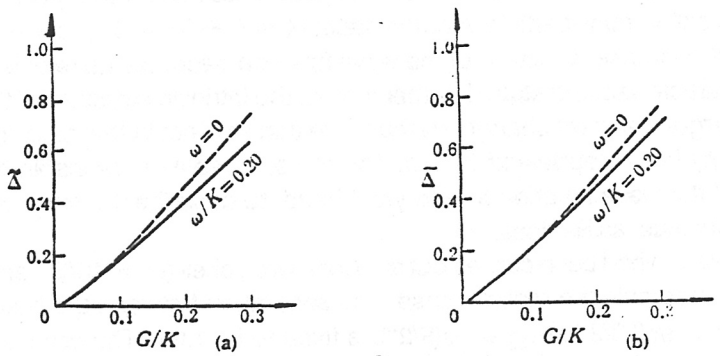

FIG.4 Variation of pairing parameter $\tilde{\Delta}$ with G. (a) 6-particle system (b) 8-particle system.

and the distribution of the single particle levels in the vicinity of the Fermi surface. When $G = G_c$, Δ displays a sharp discontinuity (c.f. Ref.[9], p.465). But unlike this "sharp transition", $\tilde{\Delta}$ approaches smoothly to zero as $G \to 0$ in the FBCS treatment (particle number projection before variation), which is similar to the result obtained with the PNC treatment in the present paper. Two examples are given in Fig.4. Fig.4(a) is the calculated results for 6-particle system and Fig.4(b) is that for 8-particle system. The dashed line and the solid line correspond to the $\omega/\kappa = 0$ and 0.20, respectively. Their behaviors are very similar.

4. CONCLUSION

The gap parameter $\tilde{\Delta} = G\sqrt{<S^+S>}$, which coincides with $\Delta = G<S>$ in the BCS or HFB approximation and is often used to characterize the nuclear superconductivity, varies very slowly with the nuclear rotational frequency ω. The change of $\tilde{\Delta}$ is very small even the intrinsic structure of the wave function has undergone a great change. Therefore, it seems not suitable to use $\tilde{\Delta}$ to indicate the nuclear phase transition from the superconductivity state to normal state. On the other hand, $\tilde{\Delta}$ depends sensitively on the blocking effect. Calculation displays a systematic odd-even Δ -difference.

The most direct evidence of the pairing phase transition can be found from the pair transfer matrix element, which is unfortunately very hard to be measured yet. According to the calculation in this paper, the variation of the pair transfer reaction strength (or R_0) with ω and that of P_0 (the component of fully-paired $K\pi = 0^+$ configurations in yrast bands) are almost synchronistic. Therefore, it is reasonable to use P_0, as well as R_0, as an indicator of the pairing phase transition in the deformed nuclei. Calculation displays that R_0 and P_0 decrease with the increase of ω but no sharp transition occurs.

Finally, it should be mentioned that the conclusions are drawn merely on the basis of the single-j model analyses. The calculations and analyses in more realistic single particle levels, such as the Nilsson level, are in progress.

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