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Alpha-Correlation in a Single-Level Model*

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The α -correlation in a single-level model with pairing forces independent of spin and isospin is discussed in this paper. The analytical solutions to this problem are obtained by using the Boson-Fermion representation. The paper also provides analytical expressions for the α -correlation energy and reduced rate of α -transfer and discusses the significance of the obtained results.

1. INTRODUCTION

The experimental facts have proved that α-clustering or α -correlation effects exist in heavy nuclei. Among them the spontaneous α-decay is perhaps the most convincing one. The α -transfer in direct reactions[1] and the pre-equilibrium emission of α -particles in compound nuclear reactions[2] have to be explained with preformation probabilities of α-particles. Recent studies pointed out that the dipole modes in heavy deformed nuclei are closely related to the excited states with α -clusters[3].

However, the theoretical study of α -clustering in heavy nuclei has not yet been fully developed. In this paper we consider a single-l-level model with pairing force of Wigner type independent of spin and isospin as a qualitative study of the main features of α-clustering. It is indeed an oversimplified model through which, however, we can get analytic solutions , thus we could further understand the essence of α -correlation and carry on further calculations. Section 2 introduces the boson-fermion composite representation of the model . Section 3 provides the eigensolutions as well as the results of α -correlation energies and α -transfer rates . The last section briefly discusses the results.

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2. THE BOSON-FERMION COMPOSITE REPRESENTATION OF THE SINGLE-l-LEVEL MODEL WITH PAIRING FORCES

The hamiltonian of the single-l-level model with pairing forces independent of spin and isospin is as follows,

$$H = \varepsilon \sum_{mm_sm_t} a^+_{mm_sm_t} a_{mm_sm_t} + 2\lambda \left[\sum_{\alpha} B^+_\alpha(\sigma) B_\alpha(\sigma) + \sum_{\mu} B^+_\mu(\tau) B_\mu(\tau) \right]. \tag{1}$$

where ε is the single particle energy, $B^+_\alpha(\sigma)$, $B_\alpha(\sigma)$, $B^+_\mu(\tau)$, $B_\mu(\tau)$ are creation and annihilation operators of nucleon pairs, respectively,

$$B^+_\alpha(\sigma) = \sqrt{\frac{1}{2}} [a^+a^+]^{L=0}_{M=0}\ {}^{S=1}_{M_S=\alpha}\ {}^{T=0}_{M_T=0},\ B_\alpha(\sigma) = (B^+_\alpha(\sigma))^+, \tag{2a}$$

$$B^+_\mu(\tau) = \sqrt{\frac{1}{2}} [a^+a^+]^{L=0}_{M=0}\ {}^{S=0}_{M_S=0}\ {}^{T=1}_{M_T=\mu},\ B_\mu(\tau) = (B^+_\mu(\tau))^+. \tag{2b}$$

2λ is the strength of the pairing force independent of spin and isospin. It is negative for attractive interaction.

The sixteen operators in bilinear forms of a^+ and a with orbital angular momentum L = 0 constitute the U(4) algebra. These sixteen operators together with the twelve operators given by Eq.(2) form the SO(8) algebra.

The problem can be solved directly with group-theoretic method. But we introduce in this paper the boson-fermion composite representation in order to show the α -correlation effect more clearly. In the composite representation, the nucleon pairs with zero orbital angular momentum are represented by bosons while the unpaired nucleons are still represented by fermions. The state vector of unpaired nucleons is denoted by $| v\gamma LM \rangle$, where v is the number of the unpaired nucleons, LM the orbital angular momentum and its z-component and γ set of other quantum numbers. This kind of state vectors has the following properties,

$$B_\alpha(\sigma) | v\gamma LM \rangle = 0, \tag{3a}$$

$$B_\mu(\tau) | v\gamma LM \rangle = 0. \tag{3b}$$

An arbitrary state vector in the composite space can then be expressed as

$$| F \rangle\rangle = F(b^+_\alpha(\sigma), b^+_\mu(\tau); (h_i)_F) | 0) | v\gamma LM \rangle, \tag{4}$$

where $(h_i)_F$ are operators of the U(4) subalgebra.

Obviously,

$$B_\alpha(\sigma) | F \rangle\rangle = 0,\ B_\mu(\tau) | F \rangle\rangle = 0. \tag{5}$$

The state vectors in the composite space are related to those in the original fermion space by the following transformation relation[4],

$$\left.\begin{aligned} &|\Psi\rangle = (0|U\mathscr{P}|F)\rangle, \\ &U = \exp\left[\sum_{\alpha} b_\alpha(\sigma)B_\alpha^+(\sigma) + \sum_{\mu} b_\mu(\tau)B_\mu^+(\tau)\right]. \end{aligned}\right\} \tag{6}$$

In the above expressions | o) is the boson vacuum state, $b_\alpha^+(\sigma)$, $b_\alpha(\sigma)$, $b_\mu^+(\tau)$, $b_\mu(\tau)$ are the creation and annihilation operators of bosons corresponding to those of nucleon pairs defined by Eq.(2) and $\mathscr{P}$ is the projection operator with which Eq.(6) always yields nonvanishing results.

The Dyson representation $\mathscr{G}^{(D)}$ in the composite space corresponding to the fermion operator G is defined by the following equations[4],

$$\langle\Psi_1|\Psi_2\rangle = \langle(F_1|\mathscr{P}\mathscr{N}\mathscr{P}|F_2)\rangle \tag{7}$$

$$\langle\Psi_1|G|\Psi_2\rangle = \langle(F_1|\mathscr{P}\mathscr{G}^{(D)}\mathscr{N}\mathscr{P}|F_2)\rangle \tag{8}$$

The operators

$$\left.\begin{aligned} &C^+ = \frac{1}{2}\{[B^+(\sigma)B^+(\sigma)]^{00} - [B^+(\tau)B^+(\tau)]^{00}\} \\ &C = (C^+)^+ \end{aligned}\right\} \tag{9}$$

are creation and annihilation operators of a four-nucleon cluster (α -cluster) which is totally symmetric in space coordinates of the four nucleons. Their boson-fermion composite representations obtained according to Eqs.(7) and (8) are as follows,

$$\mathscr{C}^{(D)} = \alpha$$

$$\mathscr{C}^{+(D)} = \alpha^+\left\{\left(1 - \frac{2n_B + n_F}{2(2l+1)}\right)\left(1 + \frac{4 - n_F}{2(2l+1)}\right) + \frac{3}{(2l+1)^2}\alpha^+\alpha\right\} \tag{10a}$$

$$+ \alpha^+\frac{(-1)}{2l+1}\sum_{h_i \in SU(4)}(h_i)_B^{(D)}(h_i^+)_F^{(D)}$$

$$+ \frac{1}{2}\{[(B^+(\sigma))_{BF}^{(D)}(B^+(\sigma))_{BF}^{(D)}]^{00} - [(B^+(\tau))_{BF}^{(D)}(B^+(\tau))_{BF}^{(D)}]^{00}\}, \tag{10b}$$

where

$$n_B = \sum_{\alpha} b_\alpha^+(\sigma)b_\alpha(\sigma) + \sum_{\mu} b_\mu^+(\tau)b_\mu(\tau) \tag{11a}$$

$$n_F = \sum_{mm_sm_t} a^+_{mm_sm_t}a_{mm_sm_t} \tag{11b}$$

$$\alpha^+ = \frac{1}{2}\{[b^+(\sigma)b^+(\sigma)]^{00} - [b^+(\tau)b^+(\tau)]^{00}\} \tag{12a}$$

$$\alpha = (\alpha^+)^+ \tag{12b}$$

$$\sum_{h_i \in SU(4)}(h_i)_B^{(D)}(h_i^+)_F^{(D)} = \sqrt{\frac{2}{2l+1}}\sum_{\alpha}[b^+(\sigma)\tilde{b}(\sigma)]^{10}_{\alpha 0}[a^+\tilde{a}]^{10}_{\bar{\alpha} 0}$$

$$+ \sqrt{\frac{2}{2l+1}}\sum_{\mu}[b^+(\tau)\tilde{b}(\tau)]^{01}_{0\mu}[a^+\tilde{a}]^{01}_{0\bar{\mu}}$$

$$-\sqrt{\frac{1}{2l+1}}\sum_{\alpha\mu}[b^+_{\tilde{\alpha}}(\sigma)b_{\tilde{\mu}}(\tau)+b_{\tilde{\alpha}}(\sigma)b^+_{\tilde{\mu}}(\tau)][a^+\tilde{a}]^{11}_{\tilde{\alpha}\tilde{\mu}} \quad (13)$$

$$(B^+_{\tilde{\alpha}}(\sigma))^{(D)}_{BF}=\sqrt{\frac{1}{2l+1}}\left\{(-\sqrt{2})[b^+(\sigma)[a^+\tilde{a}]^{10}]^{10}_{\tilde{\alpha}0}+\sum_{\mu}b^+_{\tilde{\mu}}(\tau)[a^+\tilde{a}]^{11}_{\tilde{\alpha}\tilde{\mu}}\right\}, \quad (14a)$$

$$(B^+_{\tilde{\mu}}(\tau))^{(D)}_{BF}=\sqrt{\frac{1}{2l+1}}\left\{(-\sqrt{2})[b^+(\tau)[a^+\tilde{a}]^{01}]^{01}_{0\tilde{\mu}}+\sum_{\alpha}b^+_{\tilde{\alpha}}(\sigma)[a^+\tilde{a}]^{11}_{\tilde{\alpha}\tilde{\mu}}\right\}. \quad (14b)$$

The boson-fermion composite representation of the hamiltonian given by Eq.(1) is as follows,

$$\mathscr{H}^{(D)}=2\left(\varepsilon+\lambda+\frac{\lambda}{2l+1}\right)n_B-\frac{2\lambda}{2l+1}n_B^2+\frac{12\lambda}{2l+1}\alpha^+\alpha+\varepsilon n_F$$
$$-2\lambda\left\{\sum_{h_i\in SU(4)}(h_i)^{(D)}_B(h_i^+)^{(D)}_F+\frac{n_Bn_F}{2(2l+1)}\right\} \quad (15)$$

As $\alpha^+\alpha$ is related to the operator representing the number of the four-nucleon cluster, the attractive pairing force favors the occurrence of such four-nucleon clusters.

In the Dyson representation, the Schrödinger equation

$$(H-E)|\Psi\rangle=0 \quad (16)$$

is reduced to

$$(\mathscr{H}^{(D)}\mathscr{P}-E)\mathscr{N}\mathscr{P}|F)\rangle=0,$$
$$(\mathscr{P}\mathscr{H}^{(D)+}-E)\mathscr{P}|F)\rangle=0. \quad (17)$$

The problem can be solved directly in this representation. Although there appears in the normalization an uncertain phase factor as well as an uncertain multiplicating factor, both of them have no influence on the calculated results [5].

3. EIGENSOLUTIONS OF THE HAMILTONIAN, α-CORRELATION ENERGIES AND α-TRANSFER RATES

In general, spin S and isospin T are contributed both by bosons and fermions. The situation becomes easier if fermion has no contribution and only boson does to S and T. The situation also becomes easier when the contributions of bosons and fermions are added. For these two cases, the basic vectors can be written in a general form as

$$|n_B;\ ASTM_SM_T\nu\gamma S(\nu)T(\nu)LM)\rangle$$
$$=(\alpha^+)^{n_\alpha}(\beta^+)^{n_\beta}(S_+)^{S+M_S}(T_+)^{T+M_T}(b^+_{-1}(\sigma))^{S-S(\nu)}(b^+_{-1}(\tau))^{T-T(\nu)}|0)|\nu\gamma S(\nu)T(\nu)$$
$$-S(\nu)-T(\nu)LM\rangle \quad (18)$$

where $|\nu\gamma S(\nu)T(\nu)-S(\nu)-T(\nu)LM\rangle$ denotes the state vector of the unpaired nucleons, ν is the number of such unpaired nucleons, $S(\nu)$ $-S(\nu)$ the spin and its z-component, $T(\nu)$,

$-T(\nu)$ the isospin and its third component, LM the orbital angular momentum and its z-component and γ set of other quantum numbers. In the above expression,

$$S_+ = S^{(D)}_{+B} + S^{(D)}_{+F} = -\sqrt{2l+1}\,(S^{(D)}_{1B} + S^{(D)}_{1F}), \tag{19}$$

$$T_+ = T^{(D)}_{+B} + T^{(D)}_{+F} = -\sqrt{2l+1}\,(T^{(D)}_{1B} + T^{(D)}_{1F}), \tag{20}$$

are the spin and isospin raising operators respectively, while

$$\beta^+ = \frac{1}{2}\{[b^+(\sigma)b^+(\sigma)]^{00} + [b^+(\tau)b^+(\tau)]^{00}\} \tag{21}$$

is the creation operator of another S = 0, T = 0 cluster independent from α^+. The quantum numbers in Eq.(18) satisfy the following relation,

$$4(n_\alpha + n_\beta) + 2(S - S(v) + T - T(v)) + v = A, \tag{22}$$

The nucleon number A is fixed for a given system. The non-orthogonal basic vectors given by Eq.(18) together with their conjugates $\overline{|n_\beta;\cdots)}\rangle$ form a biorthonormal set. Only general properties of the biorthonormal set will be used in the calculations and the explicit expression of $\overline{|n_\beta;\cdots)}\rangle$ is not needed.

According to the properties of the hamiltonian, STM $_S$ M_T $\nu\gamma$ $S(\nu)$ $T(\nu)$ LM are conserved quantities, hence eigensolutions of $H^{(D)}P$ and $PH^{(D)+}$ can be written as

$$\mathscr{N}\mathscr{P}|F(n;\, ASTM_SM_Tv\gamma S(v)T(v)LM)\rangle = \sum_{n_\beta=0}^{n} c^{(n)}_{n_\beta}|n_\beta;\, ASTM_SM_Tv\gamma S(v)T(v)LM)\rangle, \tag{23a}$$

$$\mathscr{P}\overline{|F(n;\, ASTM_SM_Tv\gamma S(v)T(v)LM)}\rangle = \sum_{n_\beta=n} \bar{c}^{(n)}_{n_\beta}\overline{|n_\beta;\, ASTM_SM_Tv\gamma S(v)T(v)LM)}\rangle, \tag{23b}$$

with the biorthonormal basic set, it is not difficult to evaluate the matrix elements $\langle(\overline{n'_\beta}|\mathscr{H}^{(D)}\mathscr{P}|n_\beta)\rangle$ They form a triangular matrix. The eigenenergies can easily be found as

$$\begin{aligned} E(n;ASTv\gamma S(v)T(v)L) = {} & \left(\varepsilon + \lambda + \frac{3\lambda}{2l+1}\right)(A-v) - \frac{\lambda}{4(2l+1)}(A-v)^2 + \varepsilon v \\ & - \frac{\lambda}{2l+1}[T - T(v) + S - S(v) + 2n][T - T(v) + S - S(v) + 2n + 4] \\ & - \frac{\lambda}{2(2l+1)}v(A-v) - \frac{2\lambda}{2l+1}[S(v)(S - S(v)) + T(v)(T - T(v)]. \end{aligned} \tag{24}$$

For a definite ν,

$$T + S + 2n = T(v) + S(v) + \begin{cases} 0, 2, 4, \cdots \left[\frac{A-v}{2}, 2(2l+1) - \frac{A-v}{2}\right]_<, & \frac{A-v}{2} \text{ is even} \\ 1, 3, 5, \cdots \left[\frac{A-v}{2}, 2(2l+1) - \frac{A-v}{2}\right]_<, & \frac{A-v}{2} \text{ is uneven} \end{cases} \tag{25a}$$

$$v = 0, 1, 2, \cdots, A \tag{25b}$$

$$T(v), S(v) \leqslant \frac{v}{2} \tag{25c}$$

For attractive pairing force, $\lambda < 0$, ν, n and T + S should take values as small as possible in ground states.

If $\nu = 0$, the excitation energies of a given system are determined by T + S + 2n which acts as a principal quantum number. If we consider systems of different nucleon numbers at the same time, the energy levels with the same value of T + S + 2n form a rotational band in the gauge space as shown in Fig.1.

If $\nu = 1$, we have two situations: (1) $(Z,N)_<$ is even, the ground state (T + S = 1) energies can be calculated from Eq.(24); (2) $(Z,N)_>$ is even, the ground state (T + S = 1) energies can be calculated from the corresponding formula for holes.

The separation energy of the last nucleon can be obtained from the ground state energies of $\nu = 0, 1$ shown in Fig.2. We can see the effect of α-correlation in addition to the odd-even effect.

The separation energy of a nucleon pair $S_2(A)$ can be obtained from the separation energies of two subsequent nucleons $S_1(A)$ and $S_1(A-1)$. The α-correlation energy can be obtained from the separation energies of two subsequent nucleon pairs,

$$C_\alpha(A) = \frac{1}{4}\{-S_2(A-2, T+S=1) + 2S_2(A, T+S=0) - S_2(A+2, T+S=1)\} = -\frac{5\lambda}{2(2l+1)} \tag{26}$$

Noticing that the spin and isospin of operators C^+ and C are zero, we obtain α-transfer rates for cases $\nu = 0, 1$ as

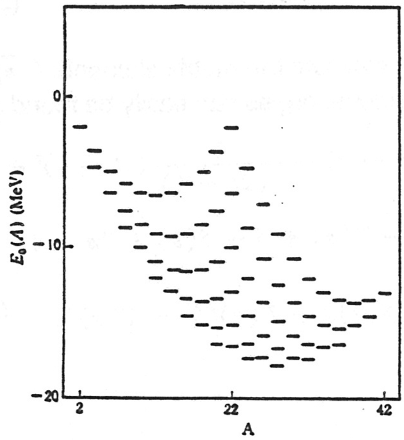

FIG. 1 Energy spectra for cases of $\nu = 0$. $\varepsilon = 0$, $\lambda = -1.0$ MeV, l = 5.

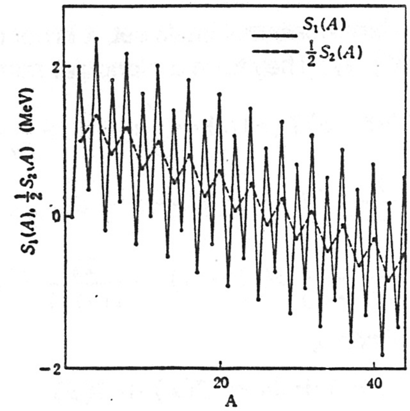

FIG. 2 Separation energies of the last nucleon $S_1(A)$ and the last nucleon pair $S_2(A)$. $\varepsilon = 0$, $\lambda = -1.0$ MeV l = 5.

$$B(A-4\rightarrow A,nST\upsilon\gamma L)=B(A\rightarrow A-4,nST\upsilon\gamma L)$$
$$=\langle C^{+}C\rangle_{AnST\upsilon\gamma L}=K(A;nST\upsilon)\cdot\left\{\left(1-\frac{A-4}{2(2l+1)}\right)\left(1-\frac{\upsilon-4}{2(2l+1)}\right)\right.$$
$$\left.-\frac{1}{(2l+1)^2}\left[\frac{\upsilon}{2}(S+T-\upsilon)+\left(\upsilon+\frac{\upsilon^2}{4}\right)\right]+\frac{3}{(2l+1)^2}K(A-4,nST\upsilon)\right\}. \quad (27)$$

where

$$K(A;nST\upsilon)=\frac{1}{12}\left[\left(\frac{A-\upsilon}{2}\right)^2+4\cdot\frac{A-\upsilon}{2}-(S+T-\upsilon+2n)\right.$$
$$\left.\cdot(S+T-\upsilon+2n+4)\right] \quad (28)$$

is the eigenvalue of the operator $\alpha^{+}\alpha$. The reduced α-transfer rates for the ground states are shown in Fig. 3. Curves corresponding to cases $\nu = 0$, $n = 0$, $T + S = 0, 1$ and $\nu = 1$, $n = 0$, $T + S = 1$ are indicated.

4. DISCUSSIONS

This paper points out that the α-correlation results from the existence of pairing forces between identical nucleons and different kinds of nucleons. The attractive pairing force favors the clustering of two protons with two neutrons. The α-transfer rates are directly related to the occurrence probability of α-clusters.

In the case of the existence of unpaired nucleons, the number of available states for pairing is reduced and hence the α-correlation effect is weakened owing to the Pauli principle. However, the main features remain the same as in the case of $\nu = 0$.

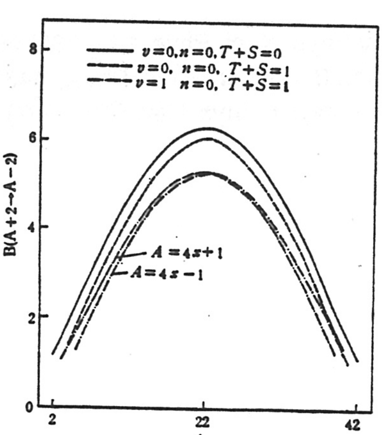

FIG. 3 Reduced α-transfer rates B(A + 2→A–2). $\varepsilon = 0$, $\lambda = -1.0$ MeV, $l = 5$.

In heavy nucleus, the fermi level of neutrons and protons belongs to different shells. It costs a definite amount of energy in the cross shell excitation of a proton or neutron pair. But, as pointed out in this paper, the α -clustering after the cross shell excitation of a nucleon pair will gain a certain amount of energy. Under the competition of these two factors it is possible to have a certain amount of excited configuration with α -cluster in the ground state of a heavy nucleus. Although such α -correlation effects may not be clearly exhibited in binding energies of heavy nuclei[6] owing to the small occurrence probabilities of α -clusters, it is significant in explaining the spontaneous α -decay and the α -transfer reaction. The occurrence probability of α -clusters increases with the excitation energy, therefore the rate of pre-equilibrium emission of α -particles in nuclear reactions increases with the incident energy. Moreover, if the α -cluster occurs at the nuclear surface in excited states, it is possible to have the associated dipole vibration.

What is considered in this paper is only the α -correlation in a single level model. But it is possible to use the analytic solutions obtained from this single level model to study the influence of excited configurations with α -clusters. This kind of investigation is in progress.[7]

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