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The One Pion Exchange Potential at Finite Temperature*

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By means of the imaginary time Green's function method, we extend the one pion exchange potential (OPEP) of the nucleon-nucleon interaction to the finite temperature. We sum up all the bubble diagrams and find that the temperature effect on OPEP is small at the low temperature region but large at the high temperature region. The effective mass of pion field $M\pi(T)$ will become imaginary at the critical temperature $T_c = 177$ MeV. It means that the attractive OPEP will disappear at T_c and the Phase transition takes place.

Thanks to the great development of the high energy heavy ion collision experiments, the study of phase transitions of liquid-gas and superconductivity [1-3] in nuclear matter and quark gluon plasma [4-5] has become one of the important areas in nuclear physics. Many authors have investigated various aspects of this area by using different methods. In this paper we try to relate the phase transition with the nucleon-nucleon interacting force. As is well known, *NN* interaction at long distance arises from one pion exchange, at intermediate distance from two-pion or σ-meson exchanges, while at short distance from vector-meson exchange [6,7]. The OPEP can be calculated in the quantum field theory from the second order perturbative Feynman diagrams, or by summing up all the bubble diagrams [8,9]. In order to extend OPEP to finite temperature, we use the technigue of generating functional at finite temperature in quantum field theory [10] and the Green's function method proposed by Matsubara [11]. The effective OPEP at finite temperature given by this method is a temperature dependent function. It is expected that with the

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in the above equation, the summation over n can be carried out by a contour integral in complex plane. Using the similar calculation of vacuum polarization in the quantum field theory, we have

$$\Pi^\beta(q^2) = \Pi_0(q^2) + \tilde{\Pi}^\beta(q^2) \tag{9b}$$

where

$$\Pi_0(q^2) = -4i(4\pi g^2)\int \frac{d^3k}{(2\pi)^3}\int_0^1 d\lambda \left\{\frac{1}{2(k^2 + m_n^2 - q^2(1-\lambda)\lambda)^{1/2}} + \frac{\lambda(1-\lambda)q^2}{2[k^2 + m_n^2 - q^2(1-\lambda)\lambda]^{3/2}}\right\}$$

$$\begin{aligned}\tilde{\Pi}^\beta(q^2) = {} & 4i(4\pi g^2)\int \frac{d^3k}{(2\pi)^3}\int_0^1 d\lambda \left\{\frac{1}{[k^2 + m_n^2 - q^2\lambda(1-\lambda)]^{1/2}}\right. \\ & \cdot \frac{1}{e^{\beta(k^2+m_n^2-q^2(1-\lambda)\lambda)^{1/2}} + 1} + \frac{q^2\lambda(1-\lambda)}{[k^2 + m_n^2 - q^2(1-\lambda)\lambda]^{3/2}} \\ & \cdot \frac{1}{e^{\beta(k^2+m_n^2-q^2(1-\lambda)\lambda)^{1/2}} + 1} - \frac{2\beta q^2\lambda(1-\lambda)}{[k^2 + m_n^2 - q^2\lambda(1-\lambda)]} \\ & \left. \cdot \frac{e^{\beta(k^2+m_n^2-q^2\lambda(1-\lambda))^{1/2}}}{(e^{\beta(k^2+m_n^2-q^2\lambda(1-\lambda))^{1/2}} + 1)^2}\right\}\end{aligned} \tag{10}$$

$$\cong \frac{8ig^2}{\pi}\left[\frac{1}{\beta^2}I_1(\beta) + \frac{q^2}{4}I_2(\beta) - q^2 I_3(\beta)\right] \tag{11}$$

and

$$\begin{aligned} I_1(\beta) &= \int_0^\infty \frac{x^2 dx}{(x^2 + a^2)^{1/2}[e^{(x^2+a^2)^{1/2}} + 1]}, \quad a = m_n\beta \\ I_2(\beta) &= \int_0^\infty \frac{x^2 dx}{(x^2 + a^2)^{3/2}[e^{(x^2+a^2)^{1/2}} + 1]}, \\ I_3(\beta) &= \int_0^\infty \frac{x^2 e^{(x^2+a^2)^{1/2}} dx}{(x^2 + a^2)[e^{(x^2+a^2)^{1/2}} + 1]^2} \end{aligned} \tag{12}$$

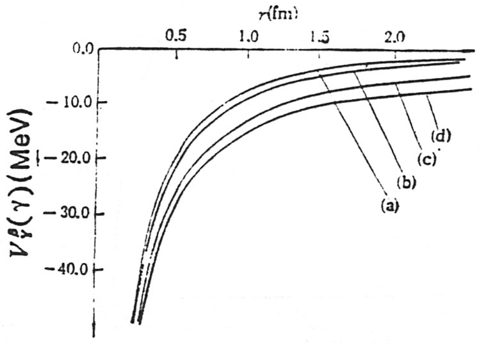

FIGURE 3 Effective OPEP at (a) T = 18.27 MeV, (b) T = 158.98 MeV, (c) T = 176.98 MeV, (d) T = 177.0 MeV (g^2 = 15.0).

As $T \to 0°K$, $\beta \to \infty$, it can be shown that $\bar{\Pi}^{\beta}(q^2) \to 0$, i.e. $\Pi_0(q^2)$ represents the contribution of zero temperature, and $\bar{\Pi}^{\beta}(q^2)$ is the finite temperature effect. Substituting Eqs.(9),(10), (11) and (12), into Eq.(6), in the non-relativity limit, we get

$$V_\beta(q) \cong -\frac{4\pi g^2(\sigma_2\cdot q)(\sigma_1\cdot q)}{4m_n^2}\frac{1}{q^2+m_\pi^2} - \frac{8g^2}{m_n^2}\frac{(\sigma_2\cdot q)(\sigma_1\cdot q)}{(q^2+m_\pi^2)^2} \cdot \left[\frac{1}{\beta^2}I_1(\beta) - \frac{q^2}{4}I_2(\beta) + q^2 I_3(\beta)\right] \quad (13)$$

in low temperature region. After taking the Fourier transformation, we return to the coordinate space

$$V_\beta(r) = -\frac{g^2}{m_n^2}\left[\frac{m_\pi}{4} - \frac{2m_\pi}{\pi}\left(I_3(\beta) - \frac{1}{4}I_1(\beta)\right)\right](\sigma_2\cdot\nabla)(\sigma_1\cdot\nabla)Y(m_\pi r) + \frac{g^2}{m_n^2}\left[\frac{I_1(\beta)}{\pi\beta^2} - m_\pi^2\left(I_3(\beta) - \frac{1}{4}I_1(\beta)\right)\right](\sigma_2\cdot\nabla)(\sigma_1\cdot\nabla)[rY(m_\pi r)] \quad (14)$$

where $Y(x) \equiv \frac{e^{-x}}{x}$. Obviously, Eq.(14) reduces to the ordinary OPEP at zero temperature. It can also be written in the form of spin-spin interaction or tensor interaction. We omitt the forms here.

In the general case, introducing

$$M_\pi^2(\beta) = (m_\pi^2 - 8g^2 I_1(\beta)/\pi\beta^2)/(1 - I_3(\beta) + I_2(\beta)/4) \quad (15)$$

as the effective mass of pion field in bubble diagram approximation at finite temperature, we rewrite Eq.(6) in coordinate representation as

$$V_\beta(r) = -\frac{g^2}{m_\pi^2(1 - I_3(\beta) + I_2(\beta)/4]}\cdot(\sigma_2\cdot\nabla)(\sigma_1\cdot\nabla)\cdot\left(\frac{e^{-M_\pi(\beta)r}}{r}\right) \quad (16)$$

Eq.(16) is the OPEP at finite temperature. Numerical calculations are shown in Fig.2 and Fig.3.

Fig.2 displays the relation between the effective mass $M_\pi(\beta)$ and temperature. It shows that in the region of $T<100$ MeV, the effect of temperature on OPEP can be neglected. However, when the temperature increases to the region of $T>100$ MeV, $M_\pi(\beta)$ decreases rapidly, at $T_c = 177$ MeV, it reduces to zero, as $T>T_c$, $M_\pi(\beta)$ becomes imaginary and OPEP disappears, the attractive part of *NN* interaction vanishes, and the phase transition of nuclei takes place. Fig.3 displays the variation of the Yukawa part of OPEP with temperature. For simplicity, the vertical axis is chosen as $V_Y^\beta(r) = -g^2\frac{e^{-M_\pi(\beta)r}}{r}$. In fact, the inclusion of spin-spin interaction and tensor interaction will not affect the qualitative results of Fig.3. We can see that the variation of OPEP is surely small in the low temperature region, but above 100 MeV the variation is considerable. With the increase of temperature, the curve becomes smoother, which means the decrease of the attractive nuclear force, and eventually the phase transition takes place.

Finally, we should like to point out that the isospin will not affect the qualitative results of this paper. Our result T_c = 177 MeV at which OPEP vanish is smaller than the temperature for quark deconfinement phase transition (usually about 200MeV--300MeV)[12]. This is reasonable, because with the increase of temperature, the interacting force between nucleons is destroyed first, and then the interacting force between quarks inside nucleons.

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