High Energy Physics and Nuclear Phyisics
Vol. 12, No. 2

Influence of the Interference of Odd and Even Waves on Spatial Correlation of Nucleon Pair

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The effect of the interference of odd and even waves on the spatial correlation of a pair of valent nucleons has been investigated.

The pairing of valent nucleon is a fundamental and important phenomenon in nuclear structure, which deserves further studies. The aim of the present paper is to study this phenomenon from an intuitive point of view for a better understanding of the spatial correlation of these valent nucleons. By doing so, ^{18}O is selected as a simple model to be studied in this paper.

The wavefunction of this system can be divided into two parts as P-space and Q-space, respectively,

$$\psi = \psi_P + \psi_Q \tag{1}$$

In ψ_P, the two valent nucleon of ^{18}O are restrained inside the S-D shell while all the other nucleons are restrained inside the core. The Q-space includes all the other excied configurations. Since all components in ψ_Q are much higher in energy, they are expected to be less important. Thus, as a basic approximation we suggest that the correlation of the valent nucleons be mainly determined by ψ_P. $\psi_P$fulfills the following schrödinger equation defined in the P-space as:

$$(H_0 + V_{eff})\varphi_P = E\varphi_P \tag{2}$$

Received August 25, 1987

TABLE 1

α	C_α			
	KUO	VY	CW	CS
$D_{5/2}$	.914	.897	.866	.909
$S_{1/2}$	.294	.324	.411	.362
$D_{3/2}$	.280	.301	.283	.206

where V_{eff} can be derived by rigorous theory of effective interaction [1] in principle. However, no rigorous expression of V_{eff} has ever been obtained. Approximate forms are usually used in practical calculations such as the KUO-force [2] originating from the Hamada-Johnson force and the VY-force [3] originating from the Reid soft core force.

Let the wavefunction in the P-space be denoted by

$$\psi_P = \sum_\alpha C_\alpha (a_\alpha^+ a_\alpha^+)_0 |\rangle \tag{3}$$

where $|\rangle$ denotes the core and α labels the single particle states. The calculated results of the structural coefficients C_α [2,3] of the 0_1^+ state (ground state) from the KUO-force and from the VY-force are listed in Table. 1.

The table lists out the results from a phenomenological effective interaction by Chung-Wildenthal [3] (CW-force) and from a simplified model (denoted by CS) where C_α is simply given by the following formula

$$C_\alpha^{[CS]} = (-)^{l_\alpha} \frac{\sqrt{2j_\alpha + 1}}{E_0 - 2\varepsilon_\alpha} \tag{4}$$

In Eq. (4) E_0 is taken as −3.9 (in MeV, the same is applied to later cases), ε_α is the zero-order energy of the α state. The wave function given by Eq.(4) originates from zero-range force, it represents a structure where the short range force is significant and thus can more effectively bind the pair of nucleons together. From Tab.1 we understand that the four types of wave functions are qualitatively close to each other. In our calculation we find that they give the same feature of the behavior of the nucleons. Hence, in the following we only give the calculated results of the KUO-force.

In order to reveal the pair correlation in a way where the collective rotation is decoupled with the internal motion, we change the variables from r_1 and r_2 (position vectors originating from the center) to **R** and **S** where R denotes the Euler-angles describing the orientation of the triangle formed by r_1 and r_2 ; **S** denotes ξ, β and θ_{12} with $\xi \equiv (r_1^2 + r_2^2)^{1/2}$, $\tan\beta \equiv r_1/r_2$, and θ_{12} = angle between r_1 and r_2, describing the internal motion. The following relation between these two sets of variables holds:

$$1 = \int |\psi_P|^2 d\mathbf{r}_1 d\mathbf{r}_2 = \int |\psi_P|^2 |J| \, d\mathbf{R} dS \tag{5}$$

where $|J|$ is the Jacobian and $dS = d\xi d\beta d\theta_{12}$ is an infinitesimal variation of shape. From Eq.(5) one can naturally define the shape-density as

$$\rho_S \equiv |\psi_P|^2 \cdot |J|$$

which is the probability density of the valent nucleon pair assuming a specific orientation and a specific shape. If we consider the two valent nucleons together with the center of the core as a three-body system, then the internal motion of this system (the motions inside the core are omitted) appears as a variation of shape and the geometric structure of this system relates closely with the maximum of the shape-density giving the most probable shape. Thus information on the structure and internal motions will be extracted from the shape-density.

Since the O^+ states are isotropic, ρ_S is only a function of ξ, β and θ_{12}. The one-body density can be calculated from the wave function and thereby we find that the most probable distance between a valent nucleon and the center in the O_1^+ state is 3.30fm. Then we fix r_1 at 3.30 fm along the $+z$-axis (this location is marked by a cross in Fig.1) and calculate ρ_s as a function of r_2 from the wave function of the O_1^+ state from the KUO-force.

The calculated results are plotted in the x-z plane in Fig.1a which is to be compared with those obtained from pure $(d_{5/2}, d_{5/2})_0$ configuration (in Fig.1b) and from pure $(s_{1/2}, s_{1/2})_0$ configuration (in Fig.1c).

There are three peaks in Fig.1a. A comparison with Fig.1b and 1c shows clearly that the appearance of these peaks originates from the interference of the D-and S-waves. The upper and lower peaks are of D-wave background while the intermediate peak is of S-wave background. It is evident that the shell structure imposes very strong restriction on the nucleon-nucleon correlation. One remarkable feature in Fig.1a is the

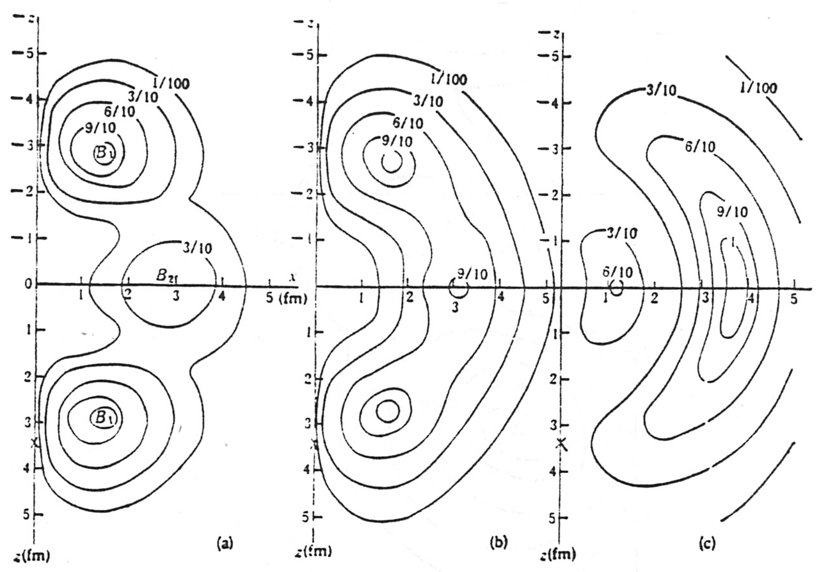

FIGURE.1 The $+z$ is in the vertical down direction. Since r s is axaiallysymmetric around the z axis, the direction of the z axis (the horizontal axis in the figure) can be arbitrary.

up-down symmetry with respect to the x-axis. However, this symmetry is not required by the physical reality. Instead, one should expect the two nucleons keep close to each other. This fact implies that our ψ_p (which contains only even-waves) may not be a good approximation to describe the actual behavior of pair-correlation. Hence it is necessary to enlarge the P-space to include both even and odd-waves to see the importance of the effect of even-odd interference on pair-correlation.

For this purpose we remove the P-F shell from the Q-space to P-space. The zero-order energies of all states of the enlarged P-space are listed in Table 2.

The structural coefficients C_α calculated from Eq.(4) are also listed there. The ρ_s of the 0_1^+ state from the enlarged ψ_p are plotted in Fig.2.

In Fig.2 we find that although there are still three peaks, the up-down symmetry with respect to the x-axis does not exist. The lowest peak (B_1) is now much higher than the other peaks, the pair of nucleons are now basically close to each other. From the location of B_1 we know that the most probable shape formed by the pair is $r_1 = r_2 =$ 3.30fm and $\theta_{12} = 28.9°$ (corresponding a most probable distance of 1.67 fm between

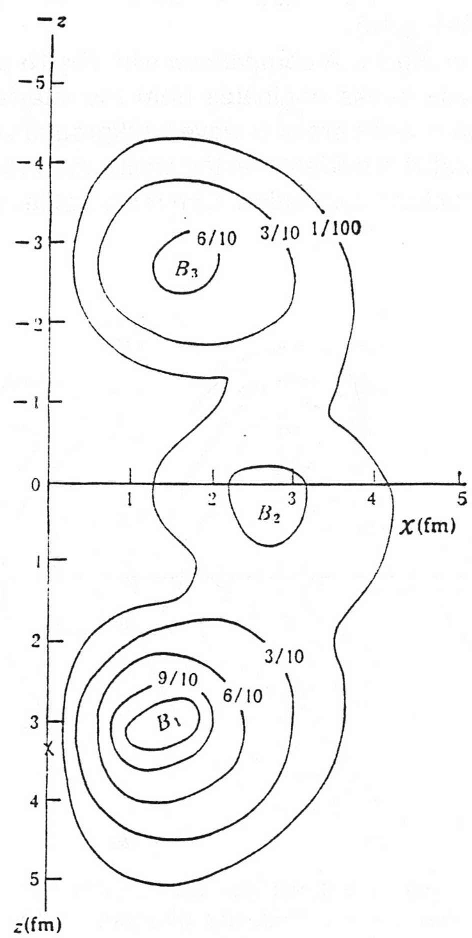

FIGURE. 2

TABLE 2

α	$D_{5/2}$	$S_{1/2}$	$D_{3/2}$	$F_{7/2}$	$P_{3/2}$	$P_{1/2}$	$F_{5/2}$
ε_α	0.	0.88	5.08	15.0	17.1	18.6	20.1
$C_\alpha^{[cs]}$	.896	.356	.202	−.119	−.075	−.049	−.079

the two nucleons). In this example the weight of the odd-wave component is only 2.85%, however the effect arising from this small component is remarkable.

We conclude that the interference of even and odd waves plays an important role in pair correlation. Though the amount of odd (or even) waves may be small, the effect of interference is remarkable. If only pure even (or odd) waves are included in the P-space, this model space is not sufficient in describing the spatial correlation of the nucleon pair. Since in most cases we work in a model space rather than in the full space, the above conclusion is noticeable. In particular there are some physical processes where two (or more) nucleons are simultaneously involved (e.g. double charge exchange), this kind of processes is usually sensitive to the nucleon-nucleon spatial correlation. Since the realization of the correct spatial correlation depends on the interference of odd and even waves, in this case both of them should be included in the *P*-space.

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