High Energy Physics and Nuclear Physics
Vol. 12, No. 2

Statistical Model for Partons in Nucleon, EMC Effect and Gluon Distribution Functions of Nucleon Inside Nuclei*

Li Zibang*, Peng Hongan**

(Liaoning Normal University*, Peking University**)

Assuming a nucleon as a thermodynamical system composed of partons (quarks and gluons), We discussed its temperature and chemical potential. We also derived the relations between the fractional momentum x distribution of the partons in nucleon and its corresponding statistical distribution. By analogy of the gluons in nucleon with the photons in the black-body, we calculated the effective temperature of the nucleon. Assuming the chemical potential of u, d quark to be the same and setting the reasonable values, we determined the variations of the temperature of nucleon in nucleus by using rescaling scheme. Thus, statistical model could explain the EMC effect quite well. If the chemical potential of gluons is small and negative, the statistical model can also explain the "largeness" of the ratio of the gluon distribution between bounded nucleon and free nucleon in the virtual photoproduction process of J/ψ in the small x region.

Received September 8, 1986
* The project supported by National Natural Science Foundation of China

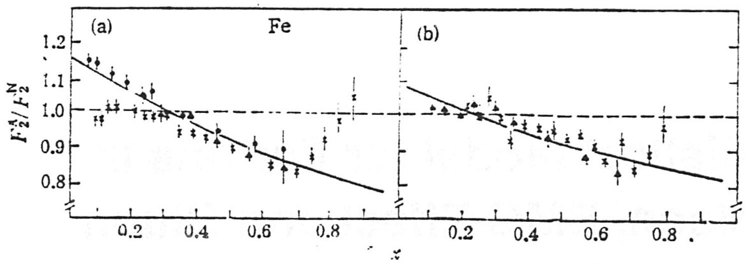

FIGURE1 The comparison of our results (solid line) with experimental data. a) The values of R from ^{56}Fe nucleus given by EMC(●), SLAC(×) and BCDMS(▲)(groups. b) The values of R from ^{12}C nucleus by SLAC and from ^{14}N nucleus by BCDMS groups, N_2 BCDMS(▲), C^2 SLAC(×).

1. INTRODUCTION

In recent years, there have been many studies about EMC effect in *l-A* electromagnetic deep inelastic scattering (DIS) process which show the softness of the *x*-distribution of the structure functions of bounded nucleons comparing with free nucleons. The data given by several experimental groups coincide with each other in the region $0.20 \lesssim x \lesssim 0.70$ and there are some theories which explain the experiments quite well. However, data in the region $x \lesssim 0.2$ given by the above groups are contradictory and clarified urgently. Several theoretical results are also in disagreement with each other and it is difficult to judge which one is correct [1]. Due to the complexity of the soft process of strong interaction in the small *x* region, it is rather difficult to clarify it.

Meanwhile, in 1984, the EMC group deduced the ratio of the gluon distribution between bounded and free nucleons $R_g(x, \hat{Q}^2, A) \equiv \frac{G^{(A)}(x, \hat{Q}^2)}{G^{(N)}(x, \hat{Q}^2)}$ from μ + Fe(or H.D)→$J/\psi + X$ process in the region

$$0.02 < x < 0.08$$

They found that R_g is not only larger than one in this small *x* region but also much larger than the ratio $R(x, Q^2, A) \equiv \frac{F_2^{(A)}(x, Q^2)}{F_2^{(N)}(x, Q^2)}$ in EMC effect (see Fig.1 and Fig.3). We pointed out [3] that it is impossible to explain this result by using those models which were used to account for the EMC effect. We guess that the "largeness" may be a reflection of the essential difference between the distribution of quarks and the distribution of gluons in the small *x* region. Based on this assumption, we attempt to discuss it with parton statistical model of nucleons in this paper. In statistical physics, many phenomena show that for Fermi and Bose distribution, there is only small difference at high energy but obvious difference at low energy, this feature may be useful for us to explain this problem. Therefore, we guess that it is possible to fit the experimental data both of $R(x,Q^2,A)$ and $R_g(x,\hat{Q}^2,A)$ by using a statistical model and it is also possible to explain why R_g is much larger than R in the small *x* region.

In order to treat a nucleon as a thermodynamical system, we have to specify the statistical distribution of its various subsystems and to define and resolve the temperature of the system and the chemical potential of its various subsystems. Because of color confinement, we can consider a nucleon as an ideal heat-reserved ball. By analogy of gluons in a nucleon with the photons in a black body, we estimate the temperature T of free nucleon and find that its value coincides with the temperature used in Ref.[4,5]. We also evaluated the chemical potential μ_q and $\mu_{\bar{q}}$ of quarks and the anti-quarks by assuming that quark u and d have the same chemical potential. Unlike the usual assumption that the chemical potential of gluon should be zero, we hold that it is not only permissible but also necessary to have a small and negative value for $\mu_g$at small x region. We use the rescaling scheme given in Ref.[6] to resolve the dependence of the chemical potential and the volume temperature of nucleons in nuclei on atom number A.

As mentioned above, by choosing reasonable values for the relevant parameters, we evaluate the ratio of R to R_g. For R (x,Q^2,A), our result fits the experiment very well in the region $0.20 \lesssim x \lesssim 0.70$. For R_g $(x,\hat{Q}^2,A)$, the result $(0 < x \lesssim 0.1)$ also coincides well with the data obtained from virtual photoproduction of J/ω. As x increases from zero, R_g decreases rapidly to almost the same values of R when $x \sim 0.1$. Since then, it decreases slowly as x increases. This shows that the difference of statistical features between quarks and gluons is unimportant for x larger than 0.1.

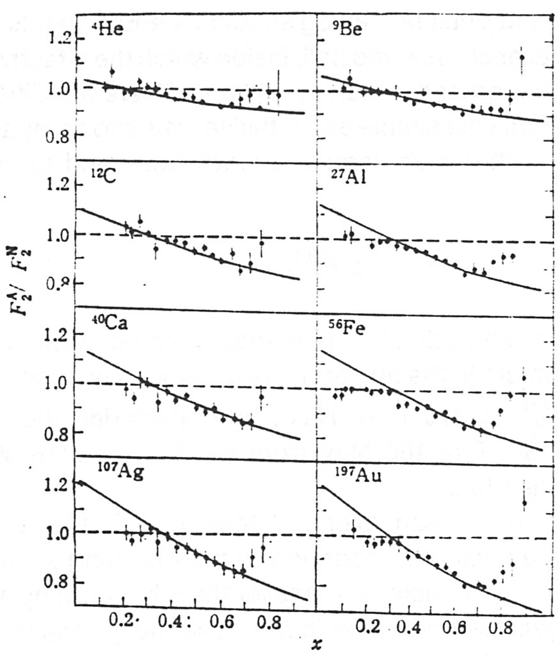

FIGURE2 Comparison of our results of R_g (solid line) with the data obtained from a serious of nuclei by SLAC group in the region $0 \leq Q^2 \leq 20$ (GeV/c)^2.

2. THERMODYNAMICAL FUNCTIONS OF THE STATISTICAL MODEL AND DISTRIBUTION FUNCTIONS OF PARTONS

We hold that the physical picture of the bag model is applicable to nucleons in nucleus just like the cases of free nucleons. Because we only study the variations of the structure of bounded nucleon relative to free nucleon, we can neglect the interactions between the partons inside the bag (therefore, the numbers of various kinds of partons conserve). When partons in nucleon are in thermodynamical equilibrium, quarks and gluons in the nucleon satisfy Fermi and Bose statistics respectively.

$$f_F = \frac{1}{e^{\beta(E-\mu)} + 1} \tag{1}$$

$$f_B = \frac{1}{e^{\beta(E-\mu)} - 1} \tag{2}$$

where E is the energy of the subsystem, $\beta = 1/kT$, k is the Bolzmann constant ($k = 1$ in natural unit), and μ is the chemical potential of the subsystem. We know from quantum statistics that μ is positive (negative) for fermion (anti-fermion) and must be negative or zero for boson. Otherwise it will lead to divergence.

Now let us discuss how to determine T and μ in a nucleon. From the point of view of the bag model and the color confinement, partons in a hadron couldn't exchange any energy with the physical vacuum outside the bag. Hence, it needs not to require that the temperature inside and outside the bag should be equal. In this sense, we can consider a hadron as an ideal heat-reserved ball, inside which the effective temperature for colored partons of mean momentum can be defined. There are different methods to evaluate T for a free nucleon. One simple and intuitive way is to compare the analogy of the gluons in a nucleon with the photons in a black body and to use the Stefan-Boltzmann law.

$$U = \frac{\pi^2}{15} V T^4. \tag{3}$$

Because DIS experiments show that the momentum carried by gluons in a nucleon is about half the total momentum in the nucleon, the energy is also about half the total. In Eq. (3), we set $U \simeq \frac{m_N}{2} \simeq 470$ MeV. Next, by considering the radius for free nucleon ~0.8 fm, we estimate $T \approx 160$ MeV from Eq.(3). It is consistent with T = 100--300 MeV adopted by Refs.[4,5].

Because T depends on the nucleon volume V, therefore, T of a bounded nucleon is different from that of a free nucleon. According to the rescaling scheme in Ref.[6], pressure born by the nucleons in a nucleus is smaller than that born by a free nucleon, therefore, its volume is larger. If we hold that in this case the gluons in a nucleon still carry half the total energy, then the effective temperature $T^{(A)}$ of a bounded nucleon is lower than that of a free nucleon.

In quantum statistics, when a "chemical" reaction reaches equilibrium, the following equation should be satisfied,

$$\sum_i \nu_i \mu_i = 0. \tag{4}$$

where the subscript i indicates the reactant or resultant, ν_i is a positive or negative integer and its absolute value represents the number of i-th reactant. For the "reaction in a nucleon, say, $q + \bar{q} \rightleftharpoons g$, we have

$$\mu_q + \mu_{\bar{q}} = \mu_g \tag{5}$$

It is known that the chemical potential μ_γ of photon is zero [1)] Ref.[7]. If by analogy of the gluons with photons we hold that μ_g is also zero. Thus, $\mu_q = -\mu_{\bar{q}}$.

There have been very few studies on the determination of μ_q and $\mu_{\bar{q}}$. One simple and approximate method is to use the normalization condition for quark distribution, i.e. to normalize the total distribution to the three valance quarks. By assuming that quarks u and d have the same chemical potential and the state density function is approximately a constant independent of the momentum, this condition is simplified to

$$3 = NV \frac{1}{(2\pi)^3} \int d^3p \left\{ \frac{1}{e^{\beta(E-\mu_q)} + 1} - \frac{1}{e^{\beta(E+\mu_q)} + 1} \right\} \tag{6}$$

where $N = 12$ is the degeneration factor of the flavor and color of the quarks, We have set $\mu_{\bar{q}} = -\mu_q$ in the above equation. We estimate $\mu_q \approx 60$ MeV for $T = 200$ MeV in Eq.(6). This value is consistent with $50 \lesssim \mu_q \lesssim 200$ MeV accepted in Refs.[4,5].

We know from statistical physics that the chemical potential μ is proportional to particle number density n for a fermion system in which the particle number is conserved. (for non-relativistic system, $\mu = an^{2/3}$; while for relativistic system $\mu = bn^{1/3}$, see Ref.[8]). Therefore, μ is inverse proportional to the volume of the system. Thus, after determining the volume variation $\Delta V = V^{(A)} - V$ for the bounded nucleon in the rescaling scheme, we could in the principle calculate the corresponding variation for the chemical potential $\Delta\mu = (\mu^{(A)} - \mu)$. we shall mention this point later in this paper.

There are scarcely any discussions about the chemical potential for gluons in nucleon except considering $\mu_g = 0$ by analogy of the gluon with photons as mentioned above. We hold that such analogy is reasonable in evaluating T, μ_q and $\mu_{\bar{q}}$, whereas it is doubtful in evaluating μ_g itself: the gluon field is a non-Abelian gauge field with self interaction, this character makes QCD to be different from QED in many aspects. On the other hand, low energy phenomenology shows that many non-perturbative effects could be explained effectively by ascribing a small effective mass to gluons.Hence, μ_g may not be rigorously zero. Maybe, it is reasonable to ascribe to it a small and negative

---

[1)] The photon is massless and without self-interaction. Thus, in the ground state of the system, the energy of the black body wouldn't be changed either by absorbing or by emitting a photon. Therefore, $\mu = 0$ [7].

value (at least, for soft process in the small x region). We take $\mu_g = -40$ to -20 MeV [2)] for the following calculation for free nucleon. As to $\mu_g^{(A)}$ for the bounded nucleons, we can only conjecture that its absolute value is less than μ_g.

Now, we discuss the relationship between the distribution function of the partdistribution and the statistical distribution in a statistical model. Since the distribution function of partons in DIS processes is described by the function momentum in infinite momentum system, it involves the problem of the covariant form of the statistical distribution. For this point, we can use the result derived from the guark-gluon plasma theory in heavy-ion collision [9] to rewrite Eqs.(1) and (2) into covariant forms,

$$f_F(p) = \frac{1}{e^{\beta(u_\nu p^\nu - \mu)} + 1} \tag{7}$$

$$f_B(p) = \frac{1}{e^{\beta(u_\nu p^\nu - \mu)} - 1} \tag{8}$$

where P^ν is the four-momentum of the system, u_ν is the "local" speed due to the pressure gradient inside the system. However, here we have no inner-pressure gradient, because it is in equilibrium state inside the nucleon. Therefore, u_ν is reduced to a global speed of a nucleon. In addition, we will not take into account the Fermi-motion of the nucleons in nuclei. Hence, Eqs.(7) and (8) return to Eqs.(1) and (2) respectively in the static system of the nuclei.

The expression of the DIS cross section in parton model is

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{2E'\alpha^2}{EQ^4} \sum_i e_i^2 \int (q_i(x) + \bar{q}_i(x)) \frac{Q^2}{2m_N} \delta\left(x - \frac{Q^2}{2m_N \nu}\right) dx \tag{9}$$

Eq.(9) is recast as follows in the statistical model

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{2E'\alpha^2}{EQ^4} \sum_i e_i^2 \int (f_{q_i}(x) + f_{\bar{q}_i}(x)) n_i(p) \frac{Q^2}{2m_N} \delta\left(x - \frac{Q^2}{2m_N \nu}\right) V d^3p \tag{10}$$

where $f_{q_i}(p)$ is given by Eq.(7). $n_i(p)$ is the state density which is relevant to quark flavor and color. When it is independent of P, it would reduce to the degeneration factor in Eq.(6). DIS process is described in infinite momentum system, thus, we have $p = xp$ in Eqs.(9) and (10). $u_\nu p^\nu = x u_\nu P^\nu = x M_N$. To discuss the DIS process, Eqs.(7) and (8) should be recast as

$$f_q(x) = \frac{1}{e^{\beta(x m_N - \mu_q)} + 1} \tag{11}$$

$$f_g(x) = \frac{1}{e^{\beta(x m_N - \mu_g)} - 1} \tag{12}$$

2) We estimated the influence on the effective temperature from Eq. (3) when $\mu_g \neq 0$.The correction is less than 5% for Eq. (3), even for $\mu_g = 50$ MeV. This shows that there is no essential influence in the present approach.

To be consistent with the approximation of parton model, we should let the state density of u quark and d quark to be equal and neglect their dependence on the transverse momentum $P_\perp$. Thus, it becomes $n_q(x)$. After integrating $P\perp$ on the right hand side of Eq.(10), we obtain

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{2E'\alpha^2}{EQ^4}\frac{5}{9}CV\int(f_q(x)+f_{\bar{q}}(x)xn_q(x)\frac{Q^2}{2m_N}\delta\left(x-\frac{Q^2}{2m_N\nu}\right)dx \tag{13}$$

where constant C depends on $(P_\perp^{Max})^2$. By comparing Eq.(13) with Eq.(9), we obtain the relation between the momentum distribution and the statistical distribution of quarks in the statistical model:

$$4(u(x)+\bar{u}(x))+(d(x)+\bar{d}(x)) = 5CVxn_q(x)(f_q(x)+f_{\bar{q}}(x)) \tag{14}$$

we obtain the gluon distribution function from the analysis of $\mu + N \rightarrow J/\psi + X$ process. Under the assumption of the semi-local duality and the approximation of narrow resonance the expression of the cross section of this process given by Ref.[10] is

$$\frac{d^2\sigma}{d\nu dQ^2} = f\frac{\alpha_s(Q^2)}{Q^2}\int\frac{xG(x,\hat{Q}^2)}{\hat{Q}^4}\left[\frac{\nu}{4E^2}-\frac{E-\nu}{2E\nu}-\frac{Q^2}{8E^2\nu}\right]\delta\left(x-\frac{\hat{Q}^2}{2m_N\nu}\right)dx \tag{15}$$

where f is a constant dependent on m_c and, $m_{J/\psi}$. It is the percentage of $c\bar{c}$ pair hadronizing into J/ψ. Meanwhile

$$x \equiv \frac{m_{J/\psi}^2+Q^2}{2m_N\nu} = \frac{\hat{Q}^2}{2m_n\nu}$$

is the x-variable of DIS with heavy quarks. In the statistical model, Eq.(15) becomes

$$\frac{d^2\sigma}{d\nu dQ^2} = f\frac{\alpha_s(Q^2)}{Q^2}\int f_g(p)n_g(p)\frac{x}{\hat{Q}^4}\left[\frac{\nu}{4E^2}-\frac{E-\nu}{2E\nu}-\frac{Q^2}{8E^2\nu}\right]\delta\left(x-\frac{Q^2}{2m_N\nu}\right)Vd^3p \tag{16}$$

Parallel to the discussion about the quark distribution function mentioned above, we obtain the relation between the momentum distribution and statistical distribution of the gluon under the parton model approximation.

$$G(x) = CVxn_g(x)f_g(x) \tag{17}$$

The difference between the parton distribution function in a bounded nucleon and that in a free nucleon is displayed in Eq.(14) and Eq.(17). The values of factor V and the parameters m_N, T and μ in f_q and f_g are different. The calculation procedure is to give the values to these parameters of free nucleons at first, then evaluate their corresponding values for bounded nucleons by using the rescaling scheme adopted in Ref. [6]. Because the state density $n(x)$ is independent of the thermodynamical quantities of the system under the approximation mentioned above, it can be canceled as it appears in the numerator and the denominator of R and R_g simultaneously.

TABLE 1 The values of $r^{(A)}$ $T^{(A)}$, $\mu_q^{(A)}$ for different nuclei A. r_0 = 0.8 fm, T = 200 MeV, μ = 100 MeV.

Nuclei	^{4}He	^{9}Be	^{12}C	^{27}Al	^{40}Ca	^{56}Fe	^{107}Ag	^{197}Au
r_A/r_0	1.02	1.04	1.05	1.07	1.07	1.08	1.10	1.11
$V^{(A)}/V$	1.06	1.13	1.16	1.22	1.22	1.26	1.33	1.37
$T^{(A)}$(MeV)	197	196	193	190	190	189	186	185
$\mu_q^{(A)}$(MeV)	80	70	65	60	58	55	50	40

3. RESULTS AND DISCUSSIONS

Based on the scheme mentioned above and the existing experimental data, we calculate the values of $R(x,Q^2,A)$ and $R_g(x,\hat{Q}^2$, Fe) for different values of Q^2 and different nuclei. First, from Eq.(14), we obtain

$$R(x,Q^2,A)\equiv\frac{F_2^{(A)}(x,Q^2)}{F_2^{(N)}(x,Q^2)}=\frac{V^{(A)}[f_q(x;m_N^{(A)},\mu_q^{(A)},T^{(A)})+f_{\bar{q}}(x;m_N^{(A)},\mu_{\bar{q}}^{(A)},T^{(A)})]}{V[f_q(x;m_N,\mu_q,T)+f_{\bar{q}}(x;m_N,\mu_{\bar{q}},T)]} \quad (18)$$

We display here the dependence of f_q, $\bar{q}$ on various parameters. Because we have neglected the dependence of q_i (x) and $\bar{q}_i$ (x) on Q^2 under the parton model approximation, the dependence of R or F_2 on Q^2 comes entirely from the rescaling parameters $V^{(A)}$, $m_N{}^{(A)}$, $T^{(A)}$ and $\mu_{q,\bar{q}}^{(A)}$ [6]. We take m_N = 940 MeV,r_N = 0.8 fm, T = 200 MeV, μ_q = $-\mu_{\bar{q}}$ = 100 MeV for free nucleons in our calculation, We evaluate $m_N^{(A)}$ and $V^{(A)}$ for different nuclei by using the rescaling scheme, and then evaluate $T^{(A)}$ from Eq.(3). So far there has been no any perfect method to evaluate the dependence of $\mu_q^{(A)}$ on A. One simple approach is to resolve μ_q by using the rescaling scheme combined with the normalization condition Eq.(6). Qualitatively speaking, because $V^{(A)}$ increases as A increases whereas the particle number density $n^{(A)}$ decreases, we conjecture that $\mu_q^{(A)}$ < $\mu_{\bar{q}}$. The results obtained from Eq.(18) are referred to Table 1, Fig.1 and Fig.2. For the gluon distribution, we obtain from Eq.(17)

$$R_g(x,\hat{Q}^2,A)\equiv\frac{G^{(A)}(x,\hat{Q}^2)}{G(x,\hat{Q}^2)}=\frac{V^{(A)}f_g(x;m_N^{(A)},\mu_g^{(A)},T^{(A)})}{Vf_g(x;m_N,\mu_g,T)} \quad (19)$$

Combining with EMC data, we calculate the values of $R_g(x,\hat{Q}^2$,Fe) for μ_q = −20, −30, −40 MeV. For μ_g^{Fe} all we know is that its absolute value should be smaller than μ_g. Hence, we only take μ_g^{Fe} = 0. The results are referred to Fig.3.

Our results show that it can consistently fit the experimental data of $R(x,Q^2, A)$ and $R_{cg}(x,\hat{Q}^2, A)$ by using the statistical model combining with the rescaling scheme and properly choosing the values of the parameters (especially, non-zero value for μ_{cg}). From Fig.1 and Fig.2, we see that our results R well fit the data given by different groups in the effective region of our model $0.20 \lesssim x \leqslant 0.70$. From Fig.(3) it can been seen that our results R_{cg} are satisfactorily consistent with the experimental data of the EMC group.

4. THE LAST REMARKS

(1) We hold that it is nothing new to explain the EMC effect by using the statistical model combining with the rescaling scheme, because the usual rescaling model starting from the momentum distribution of quarks was successful for the same purpose. This means that our results are the anticipated ones or our model is only expressed in statistical distribution language instead of in momentum distribution language. It is worthwhile to point out that our scheme has explained the "largeness" of R_{cg} in the small x region. The key points of doing this are : 1) to emphasize the statistical feature of gluons-Bose distribution and 2) to take a negative and non-zero value for μ_{cg}. The second point is clearly seen by comparing Eq.(11) with Eq.(12). Due to such consideration, the effect of $f_{cg}(x)$ in Eq.(19) is much larger than that in Eq.(18) in the small x region, and decreases rapidly as x increases untill it is about the same with $f_{g}(x)$.

(2) Refs.[4,5] also explained the EMC effect by using the statistical model. However,in Ref.[4] only the non-relativistic and classical Boltzmann statistics, for the distribution of valence quarks was considered and the temperature of different nuclei was obtained by fitting from the experimental data. Their results of the variations of the temperature and volume of nuclei depending on A coincide with ours. In Ref.[5], the authors claimed that there was a certain probability to deconfine the nucleons. They only discussed this part by using the statistical model (like the free electrons in a conductor), and they considered the rest as the same as that in a free nucleon. These two methods mentioned above explained the EMC effect quite well in the region $2.0 \lesssim x$

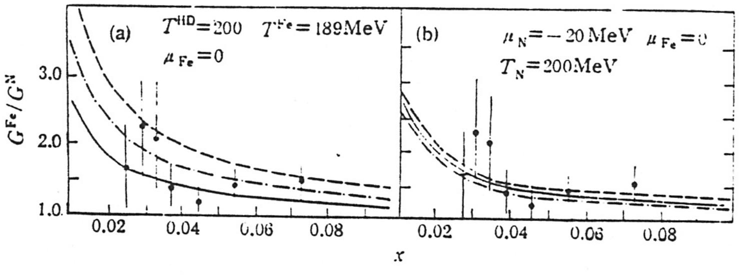

FIGURE 3 The comparison of our results of R_{cg} with that given by EMC group for different values of parameters. a) For different μ_{cg}, --- $\mu_N = -40$ MeV,··· $\mu N = -30$ MeV, $-\mu N = -20$ MeV. b) For different temperature of Fe, ---- $T_{Fe} = 194$ MeV, $-T_{Fe} = 189$MeV, -·-·- $T_{Fe} = 182$MeV.

$\lesssim$07. However, they didn't deal with how the gluon distribution function of the nucleons varies in the nuclei.

We acknowledge professor Zhang Zhaoxi for his helpful discussion.

REFERENCES

[1] R.G. Roberts, Bad Honnef Conference Report on Interaction of Electron and Photon at Middle Energy, 1984.
[2] J.J. Aubert et al., (EMC), Phys. Lett., 152B(1985), 433.
[3] Huang Weicheng, Li Zibang and Peng Hongan, High Energy Physics and Nuclear Physics, 11(1987), 175.
[4] C. Angelini et al., Phys. Lett., 154B(1985), 328.
[5] S. Gupta &K.V. Sarma, Zeit. Phys., C29(1985), 329.
[6] Liu Lianshou, Peng Hongan and Zao Weiqin, Chinese Sciences, AXXVIII(1985), 63.
[7] Landau, Statistical Physics (Chinese edition) 104.1.
[8] Wang Zhicheng, Theormodynamics and Statistical Physics (in Chinese).
[9] R.C.Hwa, preprint OITS-293(1986).
[10] R.J.N. Phillips, Selected Papers of 20th International Conference on High Energy Physics, Wisconsion(1980), 1040.