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The Mass Drift Fluctuation and the Shell Effect in Heavy Ion Collisions

Li Junqing*, Liu Janye*, Zhu Yongtai* and Zhu Jieding**

(Institute of Modern Physics, Academia Sinica, Lanzhou*)
(Lanzhou University**)

The master equation is solved numerically for the mass drift and fluctuation of three reaction channels. The driving potential is calculated by means of the D. Myers' mass formula plus shell and paring corrections. The results indicated that the lack of mass drift in the range of zero to a considerable energy loss in heavy ion collisions could be explained by the transport theory. Due to small mass mobility coefficient the mass does not drift considerably during a short time interval. The effect of the shell structure in the driving potential is obvious for the mass relaxation in the low energy loss region.

1. INTRODUCTION

The great interest and close attention have been paid to the phenomenon that the mass distribution of the projectile-like fragment broadens but does not drift in low energy deep inelastic heavy ion reactions in the energy loss region ranging from zero to a considerable large quantity. The transport theory developed since the seventies has been very successful for explaining the energy loss, mass, charge and angular momentum dissipations during the reactions [1]. However in explaining the above-mentioned phenomenon, the transport theory is facing a challenge. At present the explanations for this phenomenon are still controversial. It is thought that there might be some new mechanisms for the reaction besides the description of the statistical theory [2]. G. Moretto et al.[3] hold that the phenomenon for the lack of mass drift at the acting of the driving force of the system can be understood in terms of a

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temperature-dependent particle-exchange model. In that model the nucleus is considered to be a Fermi gas system. At the beginning of the reaction two nuclei contact each other and the numbers of nucleons flowing towards either the projectile or the target are the same. Therefore, the smaller nucleus achieves higher temperature. The temperature difference will readjust the direction of the total nucleon flux. There are more nucleons from the higher-temperature nucleus flowing to the lower-temperature nucleus. Since the driving potential of the system does the opposite effect, the mass number of the two nuclei thus remains constant at the beginning of the reaction. This is called the feed-back mechanism driven by the relative motion.

It is pointed out by H. Feldmeier [4] in the study of the particle exchange, energy and angular momentum dissipation in heavy ion collisions that the thermal feed-back has no significant effect on the final relaxation of the mass asymmetry. The mass mobility coefficient is small. So the mean charge value drifts very slowly. In a time interval of 1 to a few 10^{-21} sec the drifted mass number cannot be larger than a few units. A considerable mass drift appears after a long reaction time when a big window opens between the two nuclei.

A considerable success was achieved by treating the relaxation process of the observable in deep inelastic heavy ion reactions as a transport process and utilizing the Fokker-Planck equation. But technically the earlier solutions of the Fokker-Planck equation are mostly restricted to analytical ones with the parabola approximation for the driving potential, which differs significantly from the practical potential energy surface. The solutions give a better explanation for the fluctuations, but fail to explain the behaviour of the average values which are very sensitive to the details of the energy surface. To avoid this drawback, the master equation is solved numerically in this paper based on the transport theory founded by H. A. Weidenmuller and W. Nörenberg et al.. We are interested in the following questions: By taking into account the realistic driving potential, can the relaxation of the mass asymmetry coincide with the experimental results? Can the lack of the mass drift in the low energy loss region be explained? Further considering the nuclear structure, what effect will occur in the reaction process?

2. THE MODEL

Starting from the conventional transport theory [1,5] the master equation is coupled with the relative motion of the two ions via the kinetic energy loss and interaction time t. The relative motion is treated by means of a parametrized deflection function [6].

2.1. The Master Equation

Let the mass number of fragment 1 and 2 be A_1 and A_2 respectively, and the total mass number $A = A_1 + A_2$. $P(A_1,E_1,t)$, the distribution function of finding mass number A_1 with an excitation energy E_1 for fragment 1 at time t, which obeys the following master equation:

$$\frac{dP(A_1, E_1, t)}{dt} = \sum_{A_1'} W_{A_1 A_1'} [d_{A_1} P(A_1', E_1', t) - d_{A_1'} P(A_1, E_1, t)], \quad (1)$$

where $W_{A_1 A_1'}$ is the transition probability from the initial mass number A_1 and excitation energy E_1 to the final (A_1', E_1') and d_{A1} denotes the microscopic dimensions for the corresponding macroscopic variables. $\sum_{A_1'}$ is a sum over all possible masses that the fragment 1 may have.

2.2. The Transition Probability

$$W_{A_1 A_1'} = W_{A_1' A_1} = \frac{\tau_{mem}(A_1, E_1, A_1', E_1'; t)}{\hbar^2 d_{A_1} d_{A_1'}} \sum_{ii'} |\langle A_1', E_1', i' | V | A_1, E_1, i \rangle|^2, \quad (2)$$

where i represents the remaining quantum numbers orthogonal to mass and energy variables. The memory time is

$$\tau_{mem}(A_1, E_1, A_1', E_1', t) = (2\pi)^{1/2} \hbar \{\langle V^2(t) \rangle_{A_1, E_1} + \langle V^2(t) \rangle_{A_1', E_1'}\}^{-\frac{1}{2}}, \quad (3)$$

symbol $< >_{A_1, E_1}$ denotes the average expectation values with fixed (A_1, E_1).

The motion of the nucleons during the interaction of the two nuclei is described by the following single-particle Hamiltonian

$$H(t) = H_0(t) + V(t), \quad (4)$$

$$H_0(t) = \sum_K \sum_\nu \varepsilon_{\nu_K}(t) a^+_{\nu_K}(t) a_{\nu_K}(t), \quad (5)$$

$$V(t) = \sum_{K,K'} \sum_{\alpha_K \beta_{K'}} u_{\alpha_K \beta_{K'}}(t) a^+_{\alpha_K}(t) a_{\beta_{K'}}(t) = \sum_{K,K'} V_{KK'}(t), \quad (6)$$

$$u_{\alpha_K \beta_{K'}}(t) = U_{KK'} \left\{ \exp\left[-\frac{1}{2} \left(\frac{\varepsilon_{\alpha_K}(t) - \varepsilon_{\beta_{K'}}(t)}{\Delta_{KK'}} \right)^2 \right] - \delta_{\alpha_K \beta_{K'}} \right\}, \quad (7)$$

where $\varepsilon_{\nu K}(t)$ $(K = 1, 2)$ stands for single particle energies of fragment K. The time dependence is determined by the relative motion of colliding nuclei. The averages in Eqs. (2) and (3) are performed in a valence space $\Delta\varepsilon_K$ which lies symmetrically around the Fermi energy:

$$\Delta\varepsilon_K = \sqrt{\frac{4\varepsilon_K^*}{g_K}}, \quad \varepsilon_K^* = \varepsilon^* \frac{A_K}{A}, \quad g_K = \frac{A_K}{12} \quad (8)$$

where ε^* is the local excitation energy of the system. There are $N_K = \Delta\varepsilon_K . g_K =$ valence states and $m_K = N_K/2$ valence nucleons in $\Delta\varepsilon_K$. The dimension $d(m_1, m_2) = \binom{N_1}{m_1}\binom{N_2}{m_2}$ the interaction strength

$$U_{KK} = \frac{g_1^{1/3} g_2^{1/3}}{g_1^{1/3} + g_2^{1/3}} \cdot \frac{1}{g_K^{1/3} g_{K'}^{1/3}} 2\gamma_{KK'}$$

is taken to be the same for both proton and neutron. For the sake of convenience we take $\gamma_{11}=\gamma_{22}=\gamma_{12}=\gamma_{21}=3$ and $\Delta_{11}=\Delta_{22}=\Delta_{12}=\Delta_{21}=2$.

The averages in Eqs.(2) and (3) are carried out by using the method of spectral distributions. The results are

$$\langle V_{KK'}V^*_{KK'}\rangle = \frac{1}{4}U^2_{KK'}g_K g_{K'}\Delta_{KK'}\Delta\varepsilon_K\cdot\Delta\varepsilon_{K'}\left[\Delta^2_{KK'}+\frac{1}{6}(\Delta\varepsilon^2_K+\Delta\varepsilon^2_{K'})\right]^{1/2}, \quad (9)$$

$$\tau_{\mathrm{mem}}(A_K, E_K, t) = \hbar\left[\pi\Big/\sum_{KK'}\langle V_{KK'}V^*_{KK'}\rangle\right]^{1/2}. \quad (10)$$

According to Eq.(6) the transition probability can be written as

$$\begin{aligned} W(A_1, E_1, A'_1, E'_1) = {} & \frac{\tau_{\mathrm{mem}}(A_1, E_1, A'_1E'_1)}{d_{A_1}d_{A'_1}}\{[\omega_{11}(A_1, E_1, E'_1) \\ & + \omega_{22}(A_1, E_1, E'_1)]\delta_{A'_1A_1} + \omega_{12}(A_1, E_1, E'_1)\delta_{A'_1,A_1-1} \\ & + \omega_{21}(A_1, E_1, E'_1)\delta_{A'_1,A_1+1}\}, \end{aligned} \quad (11)$$

where

$$\omega_{KK'}(A_1, E_1, E'_1) = \sum_{K,K'A'_1}|\langle A_1, E_1, K|V_{KK'}|A'_1E'_1K'\rangle|^2 = d_{A_1}\langle V_{KK'}V^*_{KK'}\rangle. \quad (12)$$

2.3. The Local Excitation Energy and the Driving Potential of the System

The local excitation energy is defined as the following:

$$\varepsilon^* = E^* - [U(A_1) - U(A/2)] - \frac{(l-M)^2}{2J_{\mathrm{rel}}} - \frac{M^2}{2J_{\mathrm{int}}}, \quad (13)$$

where l, J_{rel} are the relative angular momentum and moment of inertia of the dinuclear system, respectively. M and J_{int} are the intrinsic angular momentum and moment of inertia of the system, respectively. The ground state energy is

$$U(A_1) = U_{LD}(A_1) + U_{LD}(A_2) - U_{LD}(A) + U_C(A_1) + U_N(A_1) \quad (14)$$

where the Coulomb energy is

$$U_C(A_1) = 1.44z_1z_2/R\mathrm{MeV}, \quad R = 0.5 + 1.36(A_1^{1/3}+A_2^{1/3})\mathrm{fm} \quad (15)$$

and the nuclear interaction energy is

$$U_N(A_1) = \alpha_n A_1^{1/3}A_2^{1/3}/(A_1^{1/3}+A_2^{1/3}); \quad \alpha_n = -15.2\mathrm{MeV} \quad (16)$$

The liquid drop energy $U_{LD}(A)$ is calculated by D. Myers' formula [7]. The shell and paring corrections are included. Here, only the axial deformation is taken into account. In Eq.(13), E^* denotes the intrinsic excitation energy of the composite system, E^* is provided by the relative kinetic energy loss and is a function of time t. M denotes the

corresponding intrinsic spin due to the relative angular momentum dissipation. They are all obtained by the classical treatment for the relative motion. The driving potential reads

$$U_{lM}(A_1) = U(A_1) + \frac{(l-M)^2}{2J_{rel}} + \frac{M^2}{2J_{int}}.$$

2.4. The Numerical Solution of the Master Equation

If it is assumed in Eq.(1) that $W_{A_1A_1'}$ are sharply and symmetrically peaked at A_1, then only the contribution from $W_{A_1,(A_1-1)}$ and $W_{A_1,(A_1+1)}$ are important. This assumption is based on the fact that during the collision the successive exchange of single nucleons is the dominant mechanism for the mass rearrangement, and the simultaneous transfer of two or more nucleons has a negligible probability. Thus, in the summation over A_1' in Eq.(1) only $A' = A_1 \pm 1$ remain, and Eq.(1) is largely simplified. The difference equations corresponding to Eq.(1) then become tridiagonal coupled algebraic equations. For the boundaries of the distribution function $P(A_1,E_1,t)$ we assume $P(A_1 < A_P/2,E_1,t) = 0$ and $P(A_1 > A_P + A_T/2,E_1,t) = 0$, where A_P and A_T are the masses of the projectile and the target respectively. In practical numerical calculation we often reduce the boundaries as long as the normalization of the distribution function is preserved. The calculation begins at the initial condition $P(A_1,E_1,t = 0) = \delta_{A_1,A_P}$. The time step $\Delta\tau$ is taken to be from 0.05 to 0.1×10^{-22} sec. In the slow varying kinetic energy loss region the transition probability does not change much and larger $\Delta\tau$ can be taken.

The calculations are carried out for different partial waves. By summing up the results of various partial waves we compare our calculation with the experimental data.

3. CALCULATED RESULTS AND ANALYSES

The numerical calculations are carried out for the ^{86}Kr (705 MeV) + ^{166}Er, ^{154}Sm (970 MeV) + ^{154}Sm and ^{144}Sm (1000 MeV) + ^{144}Sm reactions. The mean mass value of the projectile-like product is

$$\langle A_1 \rangle = \sum_{A_1} A_1 P(A_1, E^*, t) \Big/ \sum_{A_1} P(A_1, E^*, t),$$

and its fluctuation $\sigma_{A_1}^2 = \sum_{A_1} A_1^2 P(A_1, E^*, t) - \langle A_1 \rangle^2$ are calculated. Throughout the calculations the normalization factor is kept to be unity, only for very small-l partial waves the normalization factor is a little bit smaller, but is still above 0.97. During the reactions only one-nucleon-transfer process is considered. In each step whether proton or neutron is transferred depends on the lowest potential surface. Thus the neutrons and protons can be distinguished, and we can calculate:

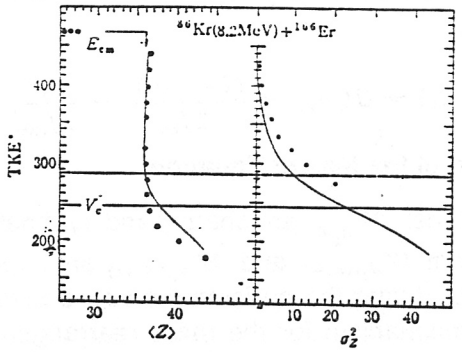

Fig.1 The mean charge value $<Z>$ and the corresponding variance σ_Z^2 for the ^{86}Kr + ^{166}Er reaction at incident energy of 8.2 MeV/u are represented as a function of the total kinetic energy of the products, TKE* (Data are from Ref.[8]).

$$\langle z_1\rangle = \sum_{A_1} z_1 P(A_1(z_1, N_1), E^*, t), \langle N_1\rangle = \sum_{A_1} N_1 P(A_1(z_1, N_1), E^*, t),$$

$$\sigma_z^2 = \sum_{A_1} z_1^2 P(A_1(z_1, N_1), E^*, t) - \langle z_1\rangle^2, \sigma_N^2 = \sum_{A_1} N_1^2 P(A_1(z_1, N_1), E^*, t) - \langle N_1\rangle^2.$$

During the calculation there are no free adjustable parameters. All the parameters are given in Section 2.

In Fig.1 the mean charge value $<Z>$ and the corresponding fluctuation σ_Z^2 of the projectile-like fragment are shown as a function of the total kinetic energy TKE* for reaction ^{86}Kr (705 MeV) + ^{166}Er[7]. The experimental data show that $<Z>$ remains a constant value of 36 for the total kinetic energy losses from zero to 180 MeV. The elementary distribution broadens but its average value does not shift. Due to a very rapid radial kinetic energy loss, the energy of 180 MeV is dissipated in a very short time interval (about 1.3×10^{-21} sec). It is not long enough for $<Z>$ to drift significantly along the direction of the driving force of the system. Since the shell and even-odd corrections are taken into account and the driving potential is not so smooth, which would block the drift of the charge and the nucleon to some extent at low excitation energy. On the left of the injection point of the driving potential there is a dip. It makes the mean value $<Z>$ has a tendency of moving first to a smaller value and then increasing. This vacillation keeps $<Z>$ in the vicinity of the incident identity for a relatively longer time interval. Because the nuclear temperature is increased due to the dissipated energy, the intrinsic states of the nucleus become sufficiently complex. The fluctuation which is related to the irreversibility of the process increases exponentially twice as fast as the mean value $<Z>$, and σ_z^2 can reach about 10 units. When the dissipated energy is fully relaxed, $\Delta E \approx 280$ MeV, i.e. TKE* $\approx$ 180MeV. This corresponds to a very long interaction time (above 20×10^{-21} sec). During such a long period of time, though the charge drift velocity is not so big, however, up to 8 units of charge shift can still be accumulated gradually. In Fig.1 we divide the processes into

two groups: above the dashed line is the process accomplished within 1.3×10^{-21} sec; below the dashed line it takes longer than 20×10^{-21} sec to finish the process.

In Fig.2 the relation between $<Z>$ and evolution time τ is given for the partial wave $l = 35.5$ case. The slope of the curve indicates the drift velocity V_Z of $<Z>$. At the beginning of the reaction the dissipated energy is small, the fine structure of the driving potential plays a role and $<Z>$ oscillates around the injection point $<Z> = 36$. Then $<Z>$ increases gradually. One can see that the slope for the increase of $<Z>$ is larger at smaller τ and becomes smaller at larger τ. In spite of this phenomenon $<Z>$ still increases considerably after a long drift time.

In our calculation we assume that the temperature of the system reaches thermal equilibrium very fast, namely, the excitation energies of fragments are distributed according to the mass, $E_K^* = E^* \frac{A_K}{A}$. We may also assume that the excitation energy is equally distributed at the beginning and thermal equilibrium is reached when the energy is relaxed, i.e.

$$E_K^* = E^* \left[A_1 + \left(\frac{A}{2} - A_1 \right) \exp(-\tau/\tau_{rad}) \right] \Big/ A ,$$

Here τ_{rad} is the relaxation time of the radial kinetic energy. We find that the transition probability is not so sensitive to the choice of the energy partition and the calculated results are not essentially affected.

The diffusion process is expected to reflect the shell structure effect since the shell correction is now included in the driving potential. Full kinematical measurements for symmetrical systems ^{154}Sm (970 MeV) + ^{154}Sm and ^{144}Sm (1000 MeV) + ^{144}Sm were carried out by E. C. Wu et al.[9]. ^{144}Sm has a spherical ground state with 82 neutrons in a closed neutron shell. ^{154}Sm is a strongly deformed nucleus with 10 valence neutrons. In the low energy loss region, the width of the charge distribution is much bigger for ^{144}Sm system than that for ^{154}Sm system. The ratio of σ_A^2/σ_Z^2 for these two systems is displayed in the upper part of Fig.3 as a function of TKEL. The discrete

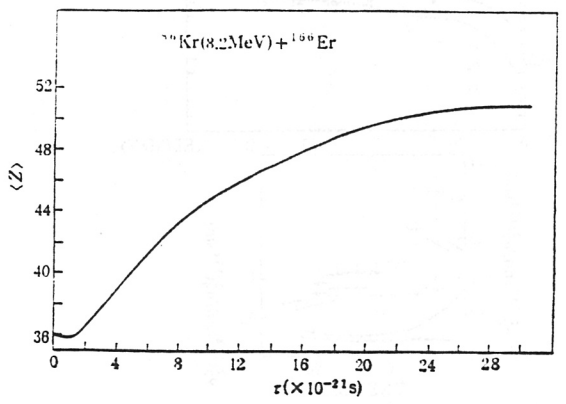

Fig.2 Mean charge value $<Z>$ for the ^{86}Kr (705 MeV) + ^{166}Er reaction. Results for the $l = 35.5$ partial wave are shown as a function of the evolution time.

points with error bars (dots and triangles) are the measured results. In the measured range of TKEL this ratio slightly decreases for the ^{154}Sm-system. Whereas the ratio increases with TKEL for ^{144}Sm-system and approaches a constant when TKEL is larger than 100 MeV. This ratio is smaller for the ^{144}Sm-system than that for the ^{154}Sm-system. This indicates that the charge drift for the ^{144}Sm-system is more important for the ^{144}Sm-system in the low TKEL region. This is due to the fact that the neutron shell is closed for ^{144}Sm. The dashed and solid lines represent the calculated results for these two systems respectively. The theory reproduces the trends of the experimental results, but the absolute value is smaller. The numerical calculation here is so time-consuming that it is impossible for us to adjust any parameters. For both systems the calculated values are smaller than the experimental ones. This indicates that the changing of the parameter for the symmetric term in the liquid drop formula will reduce the speed of the proton diffusion. In the bottom of Fig.3, the effect of the closed neutron shell can be further understood by the σ_A^2 to σ_Z^2 ratios of the two systems. The measured ratio σ_A^2 (^{144}Sm)/σ_A^2(^{154}Sm) is about 0.8 and is independent of TKEL. It indicates a slightly enhanced nucleon exchange for the heavier system. However, σ_Z^2(^{144}Sm)/σ_Z^2(^{154}Sm) is about 2 for the smallest TKEL measured and decreases to 1 with the increase of TKEL, i.e. in the ^{144}Sm-system the number of exchanged protons at small energy loss is twice as large as that in the ^{154}Sm-system. In the case of equal

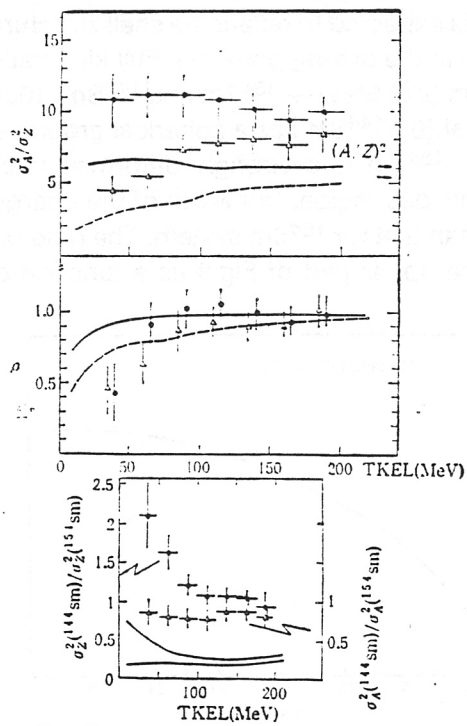

Fig.3 Upper part: σ_A^2/σ_Z^2 as a function of the total kinetic energy loss (TKEL) for the two systems. Middle part: correlation coefficient ρ as a function of TKEL. Lower part: Ratio of the Z-variances and the A-variances for the two systems as a function of TKEL. $\blacklozenge$ ^{154}Sm + ^{154}Sm (970 MeV), $\triangle$ ^{144}Sm + ^{144}Sm (100 MeV).

mass transfer, the proton transfer dominates and the neutron transfer is blocked by the closed neutron-shell. This effect gradually disappears with the increase of TKEL because the shell effect is smeared out by the high excitation energy. The effect gradually disappears with the increase of TKEL since the high excitation energy has washed out the shell effect. Again the calculated values give the trends of the experimental data. The absolute values are smaller compared with the data. In our calculation we find that the potential-energy valley for the ^{144}Sm-system is much narrower than that for the ^{154}Sm-system and the mass- and charge-diffusion for the ^{154}Sm-system is smaller. The theoretical value for the transition probability is not so sensitive to the excitation energy. In our calculation we use an over-simplified form for the interaction potential parameters, i.e., $\gamma_{11} = \gamma_{22} = \gamma_{12} = \gamma_{21}$, $\Delta_{11} = \Delta_{22} = \Delta_{12} = \Delta_{21}$. Actually, the parameters for excitation and transition processes are different. However, due to the limitation of our computation capability we are not able to perform more precise computations. The result presented here is only a qualitative description. In Fig.3 the TKEL dependence of the correlation coefficient ρ is given. Here we define $\sigma_A^2 = \sigma_N^2 + \sigma_Z^2 + 2\sigma_N\sigma_Z\rho$. The agreement between our theory and the experiment is rather good. ρ approaches 1 when TKEL > 100 MeV. This indicates that the neutron- and proton-exchange processes are fully correlated when TKEL > 100 MeV.

4. SUMMARY AND CONCLUSION

The microscopic mechanism of the nucleon diffusion relating to energy dissipation is of experimental and theoretical interest in heavy ion collisions. Especially the lack of mass drift in the low TKEL region is thought to be a new mechanism by many people. The numerical solution of the master-equation shows that this phenomenon can still be understood by the transport theory. Thus the shortcoming that the analytical solution of the Fokker-Planck equation cannot describe the details of the first moment of observerables is overcome. The mass (or charge) transport process is a slow one as compared with the energy dissipation, and the mass drift velocity is small. In coincidence with H. Feldmeier's conclusion we find that the mass number cannot drift by more than a few units within typical reaction time. The calculation shows that the feedback mechanism seems not to be so obvious and important. The shell correction in the potential energy surface of the system affects the local excitation and the temperature of the system. The second moment of the Z-distribution is very sensitive to the temperature and reflects the effect of the shell correction to the reaction process. When the energy loss dominates in the local excitation energy the effect of the shell correction becomes less important.

Numerical solution of the master-equation is a tremendous work. By a proper simplification it can still be used to describe the coupling of different mechanical variables, for example, the coupling between mass and angular momentum, which has not yet been calculated theoretically so far.

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