High Energy Physics and Nuclear Physics
Vol. 12, No. 2

# Angular Distributions of Fission-fragments in Interactions of $^{12}C$ Ions with Various Targets

**Liu Guoxing, Chen Keliang, Wang Sufang and Cai Wei**

(Institute of Modern Physics, Academia Sinica, Lanzhou)

Angular distributions of the fission-fragments were measured for a series of compound nuclei formed in the bombardments of $^{169}Tm$, $^{175}Lu$, $^{181}Ta$, W, Re, Pt, $^{197}Au$, Pb and $^{209}Bi$ by $^{12}C$ ions with mica nuclear track detectors and gold surface barrier silicon detector. All the measured angular distributions can be fitted satisfactorily by the theoretical formula based on the statistical saddle point model. The trend of variation for $K_0^2$ with increasing excitation energy $E^*$ was given at various ranges of fissility parameter $Z^2/A$ in this work.

## 1. INTRODUCTION

It is very important to measure the angular distribution of fission-fragments in the experiment of fission reaction induced by heavy ions because the experiment of this type has become an important means for nuclear reaction mechanism research. For many years the experimental and theoretical investigations of the fission-fragment angular distribution have been performed extensively [1]. It was shown that most of the experimental results on the fission-fragment angular distribution in fission reactions induced by light particles and lighter heavy-ions ($A$ < 20) can be described by the standard theory of angular distribution of fission-fragments on the basis of statistical saddle point model when the excitation energy ($E^*$ < 100 MeV) and angular momen-

Received December 9, 1986

tum ($l < 80\ \hbar$) of the compound nucleus are not too high. The saddle deformations derived from the fission anisotropies as a function of the fissility parameter $\chi$ are found to be in agreement with the results of calculation given by the liquid drop model. However, when heavier ions are introduced into the experiment, the measured angular distribution of the fission-fragments exhibits unusual large anisotropies for some reaction systems such as the system $^{32}S + ^{244}Cm$, which cannot be explained by the transition state theory of the compound nucleus. This unusual large anisotropy relates possibly to the fast fission process. The final state distribution of the fission-fragments is determined by the statistical distribution of the reaction intermediate stage (not the saddle point state). Thus the shape of the fissioning nucleus at this intermediate stage can be extracted from the measured angular distribution of the fission-fragments. For the fission reactions induced by $^{19}F$, $^{24}Mg$, $^{28}Si$ and $^{32}S$ ions, the derived value of the effective moment of inertia denoted by $\mathscr{I}_0/\mathscr{I}_{eff}$ decreases with increasing angular momentum at low angular momenta. It is consistent qualitatively with the prediction of the rotating liquid drop model. However the value of $\mathscr{I}_0/\mathscr{I}_{eff}$ cannot be obtained to quantitatively fit that given by the statistical saddle point model for any reaction system and bombardment energy. The heavier the mass of the projectile, the larger the experimental value of the effective moment of inertia deviates from the theoretical one. B. Back et al [2,3]. considered that the anomalous behavior of the angular distribution of fission-fragments reflects the breakdown of the assumption of $K$-distribution conservation from the saddle point to the scission point. The value of $K$ is readjusted adiabatically from the saddle point to the scission point. Therefore the measured final $K$-distribution reflects the thermodynamical equilibrium at the scission point. It was shown that if the angular distribution theory based on the statistical scission point model was used to fit the experimental data of the fission-fragment angular distribution, this model could not reproduce the variation tendency of the value of $K_0^2$ with the excitation energy $E^*$ of the fissioning nucleus.

It is very interesting to study the relationship between the projection of the total angular momentum on the nuclear symmetry axis $K$ and the total angular momentum I. R. Vanderbosch et al.[4] pointed out that the $K$-distribution was established at the saddle point in region $A$ of the low angular momentum $I$. When the angular momentum $I$ increases to region $B$ the $K$-distribution is determined at the place with a larger deformation than that at the saddle point. And in the region $C$ with larger angular momentum $I$ the barrier is not sufficiently well developed. The fission-fragments are emitted in the reaction plane perpendicular to the angular momentum $\vec{I}$. The fission fragment angular distribution shows approximately a form of $1/\sin\theta$. It is essential to clarify the relationship between the angular momentum $I$ and $K$-distribution if we want to have a better understanding of the properties of the static state of the fission saddle point and the dynamic process from the saddle point to the scission point.

In the present work the angular distributions of the fission-fragments were measured for a series of the compound nucleus formed in the reactions induced by 72.7, 69.6, 65.4, 64.3, 63.4 and 61.4 MeV $^{12}C$ ions on $^{169}Tm$, $^{175}Lu$, $^{181}Ta$, W, Re, Pt, $^{197}Au$, Pb and $^{209}Bi$ with mica nuclear track detectors and the solid state detector. All the measured angular distributions of the fission-fragments can be fitted satisfactorily by the theoretical saddle point model. The variation tendency of the value $K_0$ with the excitation energy $E^*$ of the fissioning nucleus is given. The values of the effective moment of inertia, denoted by $\mathcal{I}_0/\mathcal{I}_{eff}$, derived from the fission anisotropy are compared with those calculated with the rotating liquid drop model. The possibility to reproduce the variation tendency of the value $K_0$, extracted from the fission anisotropy, with the excitation energy $E^*$ is also discussed.

## 2. EXPERIMENTAL PROCEDURE

The experimental method of the measurement of fission-fragment angular distribution with mica nuclear track detectors was described in details in Ref. [5]. The results to be reported were obtained by using heavy ions beams from the 1.5 m heavy ions cyclotron at Lanzhou Institute of ModernPhysics. The energy of $^{12}C$ ions was 72.7 MeV. The beams passed through an installation to degrade the beam energy to the desired energies and then entered a scattering chamber with a diameter of 50 cm. Two apertures with a diameter of 3 mm each were mounted in the collimator 25 cm in length. The targets used in this experiment were prepared with vacuum evaporation, electron bombardment and ion spattering. The target thickness was about 200--500 $\mu g/cm^2$. The target was placed at the center of the scattering chamber and at 45° with respect to the beam direction. Faraday cup placed at the scattering chamber terminal was connected with a current integrator to measure beam intensities passing through the target. $^{12}C$ ion elastic scattering was measured to monitor the beam intensity with an Au-Si surface barrier detector which was placed at 25° with respect to the beam direction. The pre-etched mica track detectors with a size of $4 \times 12$ cm$^2$ recorded the fission-fragments leaving the target at 70°--172° region with respect to the beam direction. The distance from the mica detectors to the center of the target was 10 cm. After irradiation, the mica detectors were etched with 40% hydrofluoric acid at 50°C for 50 minutes. The fission-fragment tracks exhibited the regular diamond shape with the diagonal length of 15 $\mu$m. The scanning of the fission-fragment tracks was carried out with an optical microscope at a total magnification of $40\times12.5$. Thus the fission-fragment angular distributions for the above reactions as a function of the laboratory angle in degrees were obtained.

## 3. EXPERIMENTAL RESULTS AND DISCUSSION

### 3.1 The Experimental Results of the Fission-Fragment Angular Distribution.

The angular distributions of the fission-fragments as a function of the laboratory angle in degrees were obtained for the fission reactions induced by 72.7, 69.6, 65.4, 63.4 and 61.4 MeV $^{12}C$ ions on $^{169}Tm$, $^{175}Lu$, $^{181}Ta$, W, Re, Pt, $^{197}Au$, Pb and $^{209}Bi$. Because the fission-fragment angular distribution at the center-of-mass system for the fission reactions induced by low energy heavy ions is symmetric about 90°, the angular distributions of the fission-fragments as a function of the laboratory angle in degrees were only measured at the region of 70°--172° with respect to the beam direction. In order to analyze these angular distributions by using the theoretical model, they were transferred into the center-of-mass system with an assumption of full momentum-transfer of heavy ions to the compound nucleus. As an example, the fission-fragment angular distributions as a function of the center-of-mass angle in degrees for the reaction induced in the bombardment of $^{197}Au$ target with different energy $^{12}C$ ions are plotted in Fig.1. The experimental error bars correspond to the statistical error only. It can be seen from Fig.1 that the fission anisotropy increases with the increase of the bombardment energy for the same fission reaction.

### 3.2 The Theoretical Analysis of the Angular Distribution of the Fission-Fragments According to Various Models

(1) The statistical saddle point model. According to the statistical saddle point model, i.e. the transition state statistical model of the compound nucleus, the angular distribution depends on two quantities: the total angular momentum $I$ brought in by the projectile and the projection of the total angular momentum $I$ on the nuclear

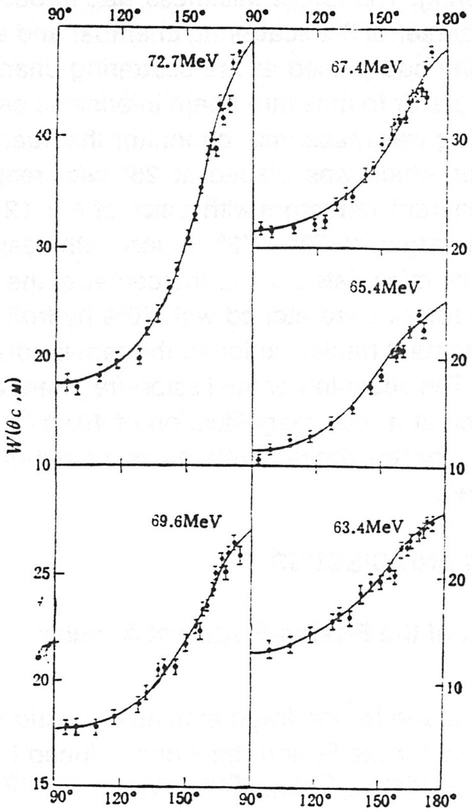


**FIGURE 1** The angular distributions of the fission fragments as a function of the center-of-mass angle for the reactions induced by 72.7, 69.6, 67.4, 65.4 and 63.4 MeV $^{12}C$ ions on $^{197}Au$.

symmetry axis $K$. Besides, an additional assumption is necessary to give a quantitative description of the angular distribution of the fission-fragments. The assumption is that the fission-fragments are emitted along the nuclear symmetry axis. Then the $K$-distribution established at the saddle point is not altered by the Coriolis forces on the path from the saddle point to the scission point. If the target and projectile spins are neglected and no particle-emission from the initial compound nucleus occurs before fission, i.e. the projection of the total angular momenta $I$ on the beam direction is zero ($M = 0$), the $K$-distribution based on the statistical model of normal temperature level density is given by I. Harlpern and V. Strutinsky [6],

$$\rho(K) = \frac{\exp\left(-\frac{K^2}{2K_0^2}\right)}{\sum_{K=-I}^{I}\exp\left(-\frac{K^2}{2K_0^2}\right)} \qquad K \leqslant I$$

$$= 0 \qquad K > I \tag{1}$$

$$K_0^2 = \frac{\mathscr{I}_{\rm eff}t}{\hbar^2}, \quad \mathscr{I}_{\rm eff}^{-1} = \mathscr{I}_{/\!/}^{-1} - \mathscr{I}_{\perp}^{-1} \tag{2}$$

where the $K$-distribution is Gaussian, $K_0$ is the standard deviation of the projection of the angular momentum of the fissioning nucleus on the nuclear symmetry axis, $\mathscr{I}_{\rm eff}$ and $t$ are the effective moment of inertia and the temperature of the fissioning nucleus at the saddle point respectively, $\mathscr{I}_{/\!/}$ is the moments of inertia about the nuclear symmetry axis and $\mathscr{I}_{\perp}$ is the moment of inertia about the axis perpendicular to the nuclear symmetry axis.

The expression for the angular distribution of the fragments is

$$W(\theta) \propto \sum_{I=0}^{\infty}(2I+1)T_I \times \frac{\sum_{K=-I}^{I}(2I+1)|d_{M,K}^{I}(\theta)|^2\exp\left(-\frac{K^2}{2K_0^2}\right)}{\sum_{K=-I}^{I}\exp\left(-\frac{K^2}{2K_0^2}\right)}, \tag{3}$$

The rotation wave function $d_{M,K}^{I}(\theta)$ in Eq.(3) is approximated by the following relation[7],

$$|d_{M,K}^{I}(\theta)|^2 \approx \frac{1}{\pi\left[\left(I+\frac{1}{2}\right)^2\sin^2\theta - M^2 - K^2 + 2MK\cos\theta\right]^{1/2}}, \tag{4}$$

Thus the fission-fragment angular distribution in the approximation (when $M = 0$) is given by [8],

$$W(\theta) = \sum_{I=0}^{\infty}\frac{(2I+1)^2T_I\exp(-I^2\sin^2\theta/4K_0^2)B_0(iI^2\sin^2\theta/4K_0^2)}{\mathrm{erf}[I/(2K_0^2)^{1/2}]}, \tag{5}$$

where $\mathrm{erf}[I/(2K_0^2)^{1/2}]$ is the error function defined by $\mathrm{erf}(x) = 2/\pi^{1/2}\int_0^x \exp(-t^2)dt$, $B_0$ is the zero order Bessel function with imaginary argument.

The solid lines in Fig.1 are the theoretical fission-fragment angular distributions calculated with Eq.(5), which give the best fit to the measured angular distributions. In the calculations of the theoretical fission-fragment angular distributions with Eq.(5) for the above fission reactions, a parabolic approximation to the real part of the optical model potential is made to calculate the transmission coefficient $T_I$.

The change of the value of $K_0^2$ derived from the measured fission anisotropy, denoted by $W(180°)/W(90°)$, with the excitation energy $E^*$ of the fissioning nucleus is given in Fig.2. The indicated error bars are attributed to the fission anisotropy error. It includes mainly that (a) the statistical error is about ±1--3%; (b) the error of the angle of the fission-fragment leaving the target with respect to the beam direction is about ± 2%; (c) the error induced from the conversion of the coordinate system is about ± 3%. The other uncertainties produced in the data analysis, originating from the uncertainties of the parameters chosen for calculation are not shown. The solid lines in Fig.2 are calculated with Eq.(2). The nuclear temperature $t$ is determined in terms of the effective excitation energy $E^*_{\mathrm{eff}}$ through an equation of the state: $at^2 - t = E^*_{\mathrm{eff}}$. The effective excitation energy at the saddle point is given by $E^*_{\mathrm{eff}} = E^* - E_f^1 - E_R^f$ where $E^*$ is the excitation energy of the fissioning nucleus, $E_f'$ is the effective fission potential barrier height given by the formula $E_f' = E_f + \Delta_f$, where $\Delta_f$ is the energy gap of the fissioning nucleus at the saddle point configuration, $\Delta_f \approx 0.7$ and 0 MeV for even-even and even-odd fissioning nuclei respectively. $E_R^f$ is the rotating energy of the fissioning nucleus at the saddle point calculated by the formula $E_R^f = \hbar^2 I^2/2\mathscr{I}_\perp + t_R \simeq \hbar^2 I^2/2\mathscr{I}_\perp \simeq \hbar^2 \bar{I}^2/2\mathscr{I}_\perp$, where $\mathscr{I}_\perp \sim 2\mathscr{I}_0$, Here $\mathscr{I}_0$ is the moment of inertia of rigid body for the spherical nucleus and is given by $\mathscr{I}_0 = (2/5)mr_0A^{5/3}$, where m is one

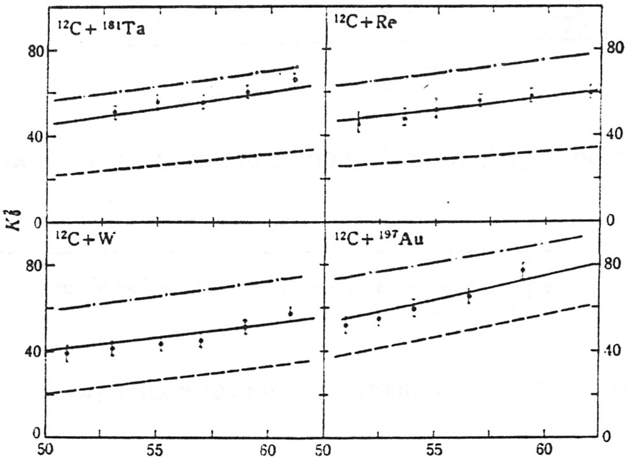


**FIGURE 2** The variation of the $K_0^2$ value with the excitation energy $E^*$ of the fissioning nuclei for the reactions $^{12}C + ^{181}Ta$, W, Re and $^{197}Au$. The dot-dashed lines are calculated with Eq.(9). The dashed lines are based on the statistical scission point model developed by H. Rossner et al..

atomic mass unit and $r_0$ = 1.2249 fm is the nuclear radius parameter. For the sake of these calculations, $\overline{I^2}$ was estimated with the optical model. The value of the level density parameter is given by $a$ =$A$/8 MeV$^{-1}$, with $A$ being the mass number of the compound nucleus. The effective moment of inertia at the saddle point is taken from Ref.[9], the fissility parameter $\chi$ is given as follows,

$$x = \frac{1}{50.833(1 - 1.7826 I_0^2)} \frac{Z^2}{A}, \quad (6)$$

where $I_0 = (N - Z)/A$ is an isotopic spin factor of the compound nucleus. It can be seen from Fig.2 that for the fission reactions induced by lighter heavy ions ($A < 20$) the change of the value of $K_0^2$ extracted from the measured fission anisotropy $W(180°)/W(90°)$ with the excitation energy $E^*$ can be reproduced satisfactorily by the standard theory of the fission-fragment angular distribution on the basis of the statistical saddle point model. The various tendency of the value for $K_0^2$ with the excitation energy $E^*$ for arbitrary given region of fissility parameter $Z^2/A$ is given in Fig.3. As a comparison, some results of the analysis of Ref.[1] are also plotted in Fig.3. Sometimes it is conventional to plot the value of $K_0^2$ as a function of the square root of the excitation energy $E^*$. For the Fermi gas the value of the nuclear temperature $t$ is proportional to $E_{eff}^{1/2}$. At low angular momentum it is assumed that the fission barrier and the effective moment of inertia do not change with the angular momentum $I$, the variation of the value of $K_0^2$ with the effective excitation energy $E^*_{eff}$ is expected to be a straight line.

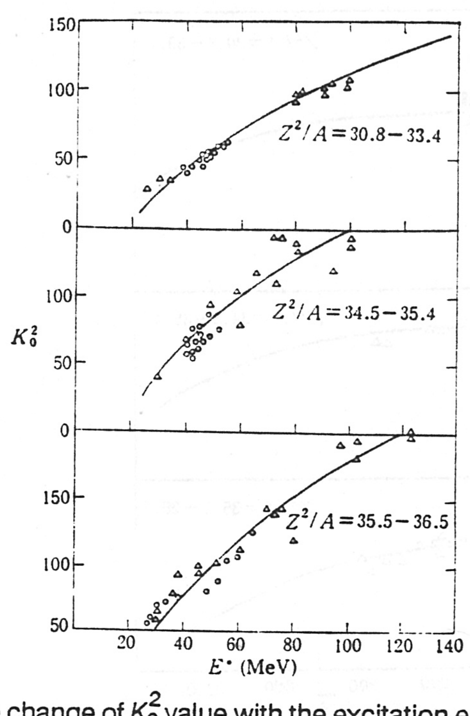


**FIGURE 3** The change of $K_0^2$ value with the excitation energy $E^*$ for given fissility parameter $Z^2/A$. o- this work. Δ– data taken from Ref.[1].

However the value of $K_0^2$ can probably be considered to be a smooth function of the excitation energy and the fissility parameter $Z^2/A$ from the overall pattern of Fig.3. In some region of $Z^2/A$ there may be some deviation due to the specific structure effects and the reaction mechanism, the information of the fission reaction mechanism can probably be extracted from these deviations. But the variation of the value for $K_0^2$ with the excitation energy given in Fig.3 does not reveal the angular momentum effect.

The change of the value for the effective moment of inertia denoted by $\mathcal{J}_0/\mathcal{J}_{eff}$ with the square of the angular momentum for certain given bins of $Z^2/A$ is shown in Fig.4 and some results from Ref.[1] are also plotted in this figure. The solid lines are drawn on the basis of rotating liquid drop model [10]. Our experimental results are laid in the region of the low angular momentum. In the range of the $Z^2/A \approx$ 33.0--34.0 the results of the present work are lower than those predicted by the rotating liquid drop model, but most of the experimental results are found to be in agreement with the theoretical values given by the rotating liquid drop model. It can be seen from Fig.4 that the value of $\mathcal{J}_0/\mathcal{J}_{eff}$ does not change essentially with the angular momentum $I$ in the region of $\overline{I^2} < 800$. It is well known that the value of $\mathcal{J}_0/\mathcal{J}_{eff}$ is a good expression for the shape of the fissioning nucleus at the saddle point.

(2) The statistical scission point model. Recently, the statistical scission point model has been developed independently by P. Bond [11] and by H. Rossner et al.[12]

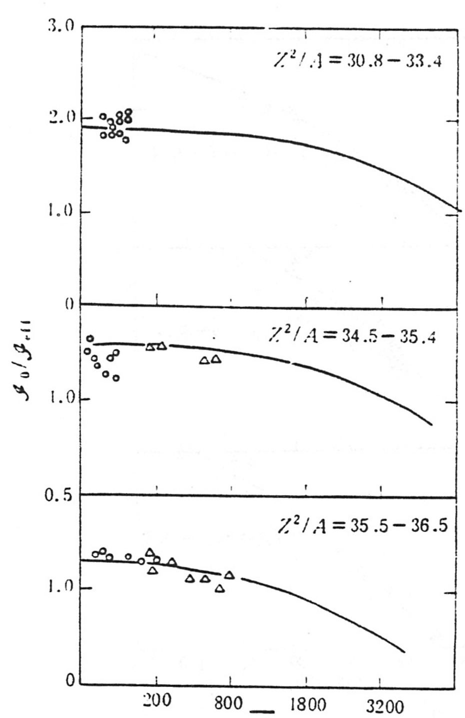


**FIGURE 4** The variation of the value $\mathcal{J}_0/\mathcal{J}_{eff}$ with the angular momentum $I^2$ for given fissility parameter $Z^2/A$. o- this work, Δ– data taken from Ref.[1].

to explain the fission-fragment angular distribution. The assumption of this model is that the $K$-distribution established at the saddle point is readjusted adiabatically during the descent from the saddle point to the scission point thus the fission-fragment angular distribution reflects a statistical distribution of $K$ values at the scission point. Assuming that the system at the scission point rotates like a rigid body, one finds that the distribution of $K$ values is still a Gaussian distribution with a variance,

$$K_0^2 = \frac{\mathscr{I}_{eff} t_s}{\hbar^2}, \quad \mathscr{I}_{eff}^{-1} = \mathscr{I}_{\parallel}^{-1} - \mathscr{I}_{\perp}^{-1}, \tag{7}$$

where $t_s$ is the nuclear temperature at the scission point. It is necessary to calculate the shape and the temperature of the fissioning nucleus at the scission point when the variation of the value of $K_0^2$ with the excitation energy $E^*$ is investigated under the statistical scission point model. It is assumed that the fission has experienced a completely damping motion from the saddle point to the scission point. The observed kinetic energy arises solely from the Coulomb repulsion between the fragments at the scission point, which limits the extent of compression of the nucleus. The total kinetic energy of the fission system is accurately reproduced by assuming that it represents the Coulomb repulsion between two coaxial ellipsoids of equal volume which separate each other by a distance of $d = 2$ fm and have a ratio of minor to major of $a/c = 0.58$, thus,

$$\frac{\left(\frac{Z_e}{2}\right)^2}{2c + d} = E_K, \tag{8}$$

where $Z$ is the charge number of the fissioning nucleus. If the fission-fragments are emitted along the nuclear symmetry axis, the parallel and perpendicular components, with respect to the nuclear symmetry axis, of the sum of the rotation moments of inertia of two fragments are $\mathscr{I}_{\perp} = (1/5)A(a+c)^2$ and $\mathscr{I}_{\parallel} = (2/5)Aa^2$, respectively.

The nuclear temperature at the scission point is given by [2],

$$t_s^2 = \frac{E_{cm} + Q_s - E_K - E_d - E_R^f}{a}, \tag{9}$$

where $E_{cm}$ is the center-of-mass energy of the system of the reaction. $Q_s$ is defined as the reaction $Q$ value when the nucleus splits into two symmetric mass fragments. $E_d$ is the energy bound in the fragment deformation taken from Ref. [2]. $E_k$ is estimated on the basis of the systematology of the total kinetic energy release in fission given by V. Viola et al. [13].

The theoretical angular distribution of the fission-fragments given by H. Rossner et al. on the basis of the statistical scission point model assumed that the initial angular momentum $I$ of the fission nucleus takes partitions into orbital momentum and channel spin of two primary fragments statistically, i.e. $\vec{I} = \vec{l} + \vec{s}$. The quantities $c$ and $a$ are the principal semi-axis of an ellipsoid, which represents the shape of each primary fission-fragment. A constant ratio of $c/a = 1.85$ is chosen. The nuclear temperature at the scission point is given by

$$t_s^2 = \frac{0.5(E_{cm} + Q_s - E_K - E_d - E_R^f)}{a}. \tag{10}$$

The variation of the value of $K_0^2$ with the excitation energy $E^*$ was also calculated on the basis of the statistical scission point model. The results of the calculation are plotted in Fig.2. The dashed lines are based on the statistical scission point model given by H. Rossner et al.. The dot-dashed lines are calculated with Eq.(9). It can be seen from Fig.2 that the change of the value of $K_0^2$ with the excitation energy $E^*$ of the fissioning nucleus cannot be reproduced quantitatively by the statistical scission point model. The experimental results obtained by B. Back et al. also show that the same statement holds even for heavy reaction systems formed in the bombardment of target with projectiles heavier than $^{20}Ne$ ions. Therefor we consider that the theory of the fission-fragment angular distribution based on the statistical scission point model should be improved.


## ACKNOWLEDGMENT

The authors would like to thank Dr. H. F. Ma for his participation in part of the experiment of this work. We would also like to thank Prof. J. W. Zheng for his help and the useful discussion.


## REFERENCES


[1] L. C. Vaz and M. Alexander, Phys. Rep. 97(1983),1.
[2] B. B. Back, et al.,'Phys. Rev C31(1985),2104. Phys. Rev. Lett. 46(1985),1068.
[3] B. B. Back et al., Phys. Rev. C32(1985),195.
[4] R. Vanderbosch, Conference on Nuclear Physics with Heavy Ions. State University of New York. Stong Brook. April 14-16(1883).
[5] Proceeding of the Scientific and Technical Reports of Institute of Modern Physics, Academia Sinica 5(1981),120.
[6] I. Harlpern and V. M. Strutinski, Proceeding of Second United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva, (1957),p.408.
[7] G. A. Wheeler, Fast Neutron Physics, New.York, (1963) pt. II P.2015.
[8] J. R. Huzenga et al., Phys. Rev. 177(1976),1826.
[9] S. Cohen and W. J. Swiatacki, Ann. Phys. 22(1963),406.
[10] S. Cohen, F. Plasil and W. J. Swiatecki, Ann. Phys. 82(1974),577.
[11] P. D. Bond, Phys. Rev. Lett., 52(1984),414.
[12] H. H. Rossner et al., Phys. Rev. Lett., 53(1984),38.
[13] V. E. Viola et al., Phys. Rev. C31(1985),1550.