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Quasi-Adiabatic Approximation for Slowly-Changing Quantum System and Berry's Phase Factors

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By analyzing the symmetry of a quantum system in terms of the method of the group theory, a quasi-adiabatic approximate method for solving a Schrödinger equation is presented. The method is to study the transition problem of the quantum system with the Hamiltonian that changes slowly but finitely. As a result of zeroth-order approximate, the quantum adiabatic theorem for the degenerate case is proved strictly, and the topological Berry's phase factors are introduced. A geometrical interpretation of the violation in the adiabatic condition is given, and it is demonstrated that the Berry's phase factors exist generally in the quantum processes with the time scale which is comparable with the period of the Hamiltonian. Finally, a possible observable effect is pointed out of the Berry's phase factor in a slowly changing process where the adiabatic condition is violated.

1. INTRODUCTION

Berry's phase factor [1] is a physical concept with obvious nonintegrable topological properties, which is rare in quantum mechanics. It can be used to explain the Aharonov-Bohm effect [2] and Aharonov-Susskind effect [3]. Moreover, it has been verified in new experiments.

Recently, G. Delacretaz et al. observed the existence of the Berry's phase factors in Na molecular system [4]. A. Tomita and R. Y. Chiao succeeded in observing the Berry's phase factor of boson (photon) in an experiment of optical fibers, which was suggested by Chiao himself and Y. S. Wu [5].

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In theoretical aspects P. Nelson, L. Alvarez-Gaume and H. Sonoda pointed out that there is a close relation between the chiral anomalies in the gauge theories and the Berry's phase factors [6]. G. M. Semenoff and P. Sodano discussed the quantum Hall effect in terms of this concept [7]. Berry and some other authors also studied the classical and semi-classical correspondence of the quantum Berry's phase factor [8]. Obviously, the concept of the phase factor has attracted the attention of many physicists.

The important concept on Berry's phase factors was presented by M. V. Berry in the investigation of quantum quasi-adiabatic processes [1]. Consider a quantum system with the Hamiltonian

$$\hat{H} = \hat{H}[R_1(t), R_2(t), \cdots, R_N(t)] \equiv \hat{H}[R(t)] \tag{1}$$

that depends on time t by parameters $R(t)$:$\{R_1(t), R_2(t), \ldots, R_N(t)\}$ which span the parameter manifold M. The quantum adiabatic theorem [9] states as follows. When the Hamiltonian change from an initial value $\hat{H}[R(t_0)]$ at time t_0 to a final value $\hat{H}[R(t_1)]$ at t_1 within the infinite time interval $T = t_1 - t_0 \to \infty$, i.e. the variation of $\hat{H}[R(t)]$ is infinitely slow, if initially the system is in an eigenstate ϕ_n $[R(t_0)]$ of $\hat{H}[R(t_0)]$, then it will pass into the eigenstate ϕ_n $[(R(t_1)]$ of $\hat{H}[R(t_1)]$ at time t_1. From this theorem, the solution of the Schrödinger equation is [9]

$$\Psi(x, t) = \exp\left[\frac{1}{i\hbar}\int_{t_0}^{t} \varepsilon_n[R(t')]dt'\right] \cdot \phi_n[R(t)] \tag{2}$$

where $\varepsilon_n[R(t)]$ is an eigenvalue of $\hat{H}[R(t)]$ corresponding to the eigenstate $\phi_n[R(t)]$.

In 1984 M. V. Berry of Bristol University pointed out a mistake of Eq.(2) and proposed a correct geometric phase should be added to the dynamical phase $\frac{1}{i\hbar}\int_0^t \varepsilon_n[R(t')]dt'$ of $\Psi(x,t)$.

$$\nu_n(t) = \int_{t_0}^{t} \left\langle \phi_n[R(t)] \middle| \frac{\partial}{\partial t}\phi_n[R(t)] \right\rangle dt.$$

When $\hat{H}[R(t)]$ changes along a closed path C: $\{R(t)\,|\,R(t_0) = R(t_1)\}$ on the parameter manifold, this phase becomes the Berry's phase $\nu_n(c)$. B. Simon interpreted $\nu_n(c)$ as a holonomy of an Hermitian linear bundle $\{(R,\phi)\,|\,\hat{H}[R]\phi = \varepsilon\,(R)\phi\}$ in a fiber bundle over basis manifold M: $\{R\}$, which is locally isomorphism with $M \times \mathscr{H}$ where $\mathscr{H}$ is the Hilbert space. Then people began to pay attention to the Berry's phase factor and its relevant problems.

However, it is necessary to re-consider the proof of the quantum adiabatic theorem because the old adiabatic theorem has the wrong corollary; at the same time, it should be discussed that the problem in the existence of the Berry's phase factor when the adiabatic condition does not hold. For this purpose, we suggest the quasi-adiabatic approximate method, the zeroth-order of which is just the adiabatic approximation.

2.SYMMETRY ANALYSIS

Suppose that in the Schrödinger picture the Hamiltonian $\hat{H}[R(t)]$ of the system has a symmetry described by a group G, which is independent of time t, and no accident degeneracy will appear. As the parameters change. A set of eigenstates. $\phi^{[p]}_{n\alpha}[R(t)](\alpha = 1,2,\cdots,d_p)$ corresponding to the same eigenvalue $\varepsilon^{[p]}_n[R(t)]$ can be chosen as the standard basis of an irreducible representation $\Gamma^{[p]}$ of the group G. According to the orthogonality theorem in the group representation theory, we have

$$\langle\phi^{[p]}_{m\alpha}[R(t)]\,|\,\phi^{[q]}_{n\beta}[R(t')]\rangle = \delta_{pq}\delta_{\alpha\beta}\cdot\langle\phi^{[p]}_{m1}[R(t)]\,|\,\phi^{[p]}_{n1}[R(t')]\rangle \tag{3}$$

where the double indices (α,m) indicate that different energy levels $\varepsilon^{[p]}_n[R(t)]$ for an index p may correspond to the same irreducible representation $\Gamma^{[p]}$. Let $C(m)$ denote the set of all the values p for a definite m. In order to describe the evolution of the system, we introduce the process projection operators

$$\begin{aligned} P^{mp\alpha}_{nq\beta}[t_0,t] &= |\phi^{[q]}_{n\beta}[R(t)]\rangle\langle\phi^{[p]}_{m\alpha}[R(t_0)]| \\ P_{mp\alpha}[t_0,t] &= P^{mp\alpha}_{mp\alpha}[t_0,t], \qquad P_{mp\alpha}[t] = P_{mp\alpha}[t,t]; \end{aligned} \tag{4}$$

where $P_{mp\alpha}[t]$ is the usual projection operator of the state $\phi^{[p]}_{m\alpha}[R(t)]$.

Let $|\psi(t)\rangle = U(t_0,t)\,|\psi(t_0)\rangle$ be a formal solution of the Schrödinger equation $i\hbar\frac{\partial}{\partial t}\psi(t) = \hat{H}[R(t)]\psi(t)$.

$$|\psi(t)\rangle = U(t_0,t)\,|\psi(t_0)\rangle,$$

Then, the defined evolution operator $U(t_0,t)$ satisfies

$$i\hbar\frac{\partial}{\partial t}U(t_0,t) = H[R(t)]U(t_0,t),\ U(t_0,t_0) = 1 \tag{5}$$

$U(t_0,t)$ plays double roles: by making use of $U(t_0,t)$, we can either describe the evolution of the quantum system in the Heisenberg picture or give the wave function $\psi(t)$ from $\psi(0)$ in the Schrödinger picture. The operator $U(t_0,t)$ is expressed in terms of (4) as

$$U(t_0,t) = \sum_{mp\alpha}\sum_{nq\beta}\exp\left\{\frac{1}{i\hbar}\int_{t_0}^{t}\varepsilon^{[p]}_m[R(t')]dt'\right\}C^{mp\alpha}_{nq\beta}[t]\cdot P^{nq\beta}_{mp\alpha}[t_0,t] \tag{6}$$

where $= \sum_{mp\alpha} = \sum_{m}\sum_{p\in C(m)}\sum_{\alpha=1}^{d_p}$.

Substituting (6) into(5) and using the properties of the projection operators

$$\hat{H}[R(t)]P^{nq\beta}_{mp\alpha}[t_0,t] = \varepsilon_{mp}[R(t)]P^{nq\beta}_{mp\alpha}[t_0,t] \tag{7}$$

we obtain

$$\begin{aligned} \frac{d}{dt}C^{mp\alpha}_{ls\gamma}[t] &= \sum_{nq\beta}\langle\phi^{[p]}_{m\alpha}[R(t)]\,|\,\nabla_R\phi^{[q]}_{n\beta}[R(t)]\rangle\cdot\frac{dR}{dt} \\ &\times\exp\left\{\frac{1}{i\hbar}\int_{t_0}^{t}[\varepsilon^{[p]}_m[R(t')] - \varepsilon^{[q]}_n[R(t')]]dt'\right\}\cdot C^{nq\beta}_{ls\gamma}(t) \\ C^{mp\alpha}_{nq\beta}[0] &= \delta_{ml}\delta_{ps}\delta_{\alpha\gamma} \end{aligned} \tag{8}$$

where $dR = (dR_1, dR_2, \cdots, dR_N)$ and $\nabla_R = \left(\frac{\partial}{\partial R_1}, \frac{\partial}{\partial R_2}, \cdots, \frac{\partial}{\partial R_N}\right)$ are the 1-form and 1-vector over the manifold M respectively.

Owing to the orthogonality relations (5) obtained from the symmetry of $\hat{H}[R]$, the matrix $\tilde{M}$ defined by

$$M^{nq\beta}_{mp\alpha}(t) = \langle \phi^{[p]}_{m\alpha}[R(t)] | \nabla_R \phi^{[q]}_{n\beta}[R(t)] \rangle \tag{9}$$

is quasi-diagonal. In fact, because an irreducible representation $\Gamma^{[p]}$ cannot change into another one $\Gamma^{[p']}$ by changing the parameters continuously, we have

$$\begin{aligned} M^{nq\beta}_{mp\alpha}(t) &= \lim_{\Delta R \to 0} \frac{1}{\Delta R} [\langle \phi^{[p]}_{m\alpha}[R] | \phi^{[q]}_{n\beta}[R + \Delta R] \rangle \\ &- \langle \phi^{[p]}_{m\alpha}[R] | \phi^{[q]}_{m\beta}[R] \rangle] = \delta_{pq}\delta_{\alpha\beta} M^{nq1}_{mp1}. \end{aligned} \tag{10}$$

Then, (8) is greatly simplified by the symmetry analysis:

$$\begin{aligned} \frac{d}{dt} C^{mp\alpha}_{l s\gamma}[t] &= -\sum_n \langle \phi^{[p]}_{m\alpha}[R(t)] | \nabla_R \phi^{[p]}_{n\alpha}[R(t)] \rangle \cdot \frac{dR}{dt} \\ &\times \exp\left\{\frac{i}{\hbar} \int_{t_0}^{t} [\varepsilon^{[p]}_m[R(t')] - \varepsilon^{[p]}_n[R(t')]] dt'\right\} C^{np\alpha}_{l s\gamma}(t). \end{aligned} \tag{11}$$

3. INTEGRAL EQUATIONS OF EVOLUTION MATRIX

According to the above discussion, it will not lead to confusion to write: $C^{np\alpha}_{l s\gamma}(t) = C^m_l(t)$, $\varepsilon^p_m[R(t)] = \varepsilon_m(t)$. For certain $\hat{H}[R(t_0)]$ and $\hat{H}[R(t_1)] (t_1 - t_0 = T)$, we study the influence of the changing rate $\dot{R}(t)$ of $\hat{H}[R(t)]$ on the behavior of the solutions of (11). Introducing the relative time $\tau = t/T$ $(0 \le \tau \le 1)$ and letting

$$C^m_l(t) = C^l_m(T \cdot \tau) = b^m_l(\tau), \quad R^\alpha(t) = R^\alpha(T \cdot \tau) = S^\alpha(\tau)$$

we may rewrite (11) as

$$\begin{aligned} \frac{d}{d\tau} b^m_l(\tau) &= -\sum_n S_{mn} b^n_l(\tau) \\ b^m_l(0) &= \delta^m_l, \end{aligned} \tag{12}$$

where the elements of matrix $\tilde{S}(\tau)$ are defined as

$$\begin{aligned} S_{mn}(\tau) &= \beta_{mn}(\tau) e^{iT\alpha_{mn}(\tau)} \\ \alpha_{mn}(\tau) &= \exp \frac{1}{\hbar} \int_{\tau_0}^{\tau} \left[\varepsilon_m[S(\tau')] - \varepsilon_n[S(\tau')] \right] d\tau' \\ \beta_{mn}(\tau) &= \langle \phi_m[S(\tau)] | \nabla_s \phi_n[S(\tau)] \rangle \cdot dS/d\tau \end{aligned} \tag{13}$$

The integral equation of $b^m_l(\tau)$ in $U(t_0, t)$ is

$$b_l^m(\tau) = \delta_l^m - \sum_n \int_{\tau_0}^{\tau} S_{mn}(\tau')b_l^n(\tau')d\tau' \tag{14}$$

that is a set of Volterra integral equations. So we reduced the calculation of $U(t_0, t)$ to that of an infinite-dimensional matrix $\tilde{b}(\tau) = (b_l^m(\tau))$. The matrix form of (14) is

$$\tilde{b}(\tau) = E - \int_{\tau_0}^{\tau} \tilde{S}(\tau)\tilde{b}(\tau)d\tau \tag{14'}$$

which is the starting point of our discussion.

It is quite difficult to solve (12) or (14) exactly. By observing the asymptotic behaviors of (14) or (14') in a slowly changing process scaled by T, we propose the quasi-adiabatic approximate method.

4. QUASI-ADIABATIC APPROXIMATE METHOD

For simplicity we let $t_0 = 0$ in the following discussion.

Defining a matrix $F^l(T,\tau)$:

$$F_{mn}^l(T, \tau) = \int_0^{\tau} S_{mn}(\tau')b_l^n(\tau')d\tau' \tag{15}$$

and integrating it by parts when $m \neq n$, we have

$$F_{mn}^l(T, \tau) = \frac{1}{T} G_{mn}(\tau)b_l^n(\tau) + \frac{1}{T^2} P_{mn}(\tau) \frac{d}{d\tau} [K_{mn}(\tau)b_l^n(\tau)] + O\left(\frac{1}{T^3}\right) \tag{16}$$

where

$$G_{mn}(\tau) = \frac{\hbar\beta_{mn}(\tau)}{i[\varepsilon_m[S(\tau)] - \varepsilon_n[S(\tau)]]} \cdot e^{iT\alpha_{mn}(\tau)}$$

$$K_{mn}(\tau) = -\frac{\hbar\beta_{mn}(\tau)}{\varepsilon_m[S(\tau)] - \varepsilon_n[S(\tau)]}, \quad P_{mn}(\tau) = \frac{\hbar \exp[iT\alpha_{mn}(\tau)]}{\varepsilon_m(\tau) - \varepsilon_n(\tau)}$$

and they are all the continuous functions of τ. Because $\alpha_{mn}(\tau)$ is a continuous monotonic function, for the infinitely slowly-changing process, $T \to \infty$, $F^l(T,\tau)$ goes to a diagonal matrix. Since $1/T$ is very small when T is large enough, substituting (16) into (14), we obtain

$$\begin{aligned} b_l^m(\tau) = \delta_l^m - \int_0^{\tau} S_{mm}(\tau')b_l^m(\tau')d\tau' - \frac{1}{T}\sum_{m\neq n} G_{mn}(\tau)b_l^n(\tau) \\ - \frac{1}{T}\sum_{m\neq n} \frac{d}{d\tau}[K_{mn}(\tau)b_l^n(\tau)P_{mn}(\tau)] + O\left(\frac{1}{T^3}\right) \end{aligned} \tag{17}$$

and their coupled differential equations

$$\begin{aligned} \frac{d}{d\tau} b_l^m(\tau) + S_{mm}(\tau)b_l^m(\tau) = -\frac{1}{T}\sum_{n\neq m} \frac{d}{d\tau} [G_{mn}(\tau)b_l^n(\tau)] \\ - \frac{1}{T^2}\sum_{n\neq m} \left\{P_{mn} \frac{d}{d\tau} [K_{mn}(\tau)b_l^n(\tau)]\right\} \end{aligned}$$

$$+ O\left(\frac{1}{T^3}\right) \tag{18}$$

According to the powers of $1/T$, we discuss the quasi-adiabatic approximation for (18). Letting

$$b_l^m(\tau) = b_l^{[0]m}(\tau) + \frac{1}{T} b_l^{[1]m}(\tau) + \frac{1}{T^2} b_l^{[2]m}(\tau) + \cdots \tag{19}$$

and substituting it into (18), we obtain the equality of coefficients with the same powers in both sides of (18)

$$\begin{cases} \frac{d}{d\tau} b_l^{[0]m}(\tau) + S_{mm}(\tau) b_l^{[0]m}(\tau) = 0 & (20\text{-}1) \\ \frac{d}{d\tau} b_l^{[1]m}(\tau) + S_{mm}(\tau) b_l^{[1]m}(\tau) = -\sum_{n \neq m} \frac{d}{d\tau} [G_{mn}(\tau) b_l^{[0]n}(\tau)] & (20\text{-}2) \\ \frac{d}{d\tau} b_l^{[2]m}(\tau) + S_{mm}(\tau) b_l^{[2]m}(\tau) = -\sum_{n \neq m} \frac{d}{d\tau} [G_{mn}(\tau) b_l^{[1]n}(\tau)] & \\ \quad + \sum_{n \neq m} \frac{d}{d\tau} \left[P_{mn}(\tau) \frac{d}{d\tau} [K_{mn}(\tau) b_l^{[0]n}(\tau)] \right] & (20\text{-}3) \\ \cdots\cdots\cdots\cdots & \end{cases}$$

where the initial conditions are $b_l^{[0]n}(0) = \delta_l^n$, $b_l^{[i]n}(0) = 0$, $i = 1, 2, 3, \cdots\cdots$. The above equations can be solved by iteration

$$\begin{cases} b_l^{[0]m}(\tau) = \delta_{ml} \exp\left[-\int_0^\tau S_{mn}(\tau') d\tau' \right] \\ b_l^{[1]m}(\tau) = \exp\left[-\int_0^\tau S_{mm}(\tau') d\tau' \right] \int_0^\tau f_{ml}^{[1]}(\tau') \exp\left[\int_0^{\tau'} S_{mm}(\tau'') d\tau'' \right] d\tau' \\ b_l^{[2]m}(\tau) = \exp\left[-\int_0^\tau S_{mm}(\tau') d\tau' \right] \int_0^\tau f_{ml}^{[2]}(\tau') \exp\left[\int_0^{\tau'} S_{mm}(\tau'') d\tau'' \right] d\tau' \\ \cdots\cdots\cdots\cdots \end{cases} \tag{21}$$

where

$$f_{ml}^{[1]}(\tau) = \sum_{n \neq m} \frac{d}{d\tau} [G_{mn}(\tau) b_l^{[0]n}(\tau)]$$

$$f_{ml}^{[2]}(\tau) = \sum_{m \neq n} \left[\frac{d}{d\tau} G_{mn}(\tau) b_l^{[1]n}(\tau) + P_{mn}(\tau) \frac{d}{d\tau} [K_{mn}(\tau) b_l^{[0]n}(\tau)] \right].$$

From this discussion we can calculate the solutions to any order if needed.

5. OBSERVABLE EFFECTS AND GEOMETRICAL INTERPRETATIONS OF BERRY'S PHASE FACTORS IN NON-ADIABATIC CASES

Having obtained the approximate solutions of $U(t_0,t)$, we are able to discuss the problem in the Schrödinger picture. By analyzing the approximate solutions in each order, it is easy to see the universality of the Berry's phase factors.

Under the adiabatic limit $T\to\infty$, the evolution matrix of zeroth-order approximation is

$$U^{[0]}(t_0, t) = \sum_m \exp\left[-\int_{\tau_0}^{\tau} S_{mm}(\tau')d\tau'\right] \cdot \exp\left[\frac{T}{i\hbar}\int_{\tau_0}^{\tau} \varepsilon_m(S(\tau'))d\tau'\right] \cdot P_m(t_0, t) \quad (22)$$

which is just consistent with the results obtained by M. V. Berry and B. Simon: when $\hat{H}[R(t)]$ changes along a closed path C on the parameter manifold, there is a geometrical phase

$$\nu_m(c) = i\oint \langle\phi_m(R)|d_R\phi_m(R)\rangle = \iint_{S_c} \langle d_R\phi_m(R)|\wedge|d_R\phi_m(R)\rangle \quad (23)$$

added to the dynamic phase $\frac{T}{i\hbar}\int_{\tau_0}^{\tau} \varepsilon_m(S(\tau'))d\tau'$, where d_R is the exterior differential operator on the parameter manifold and S_C is a submanifold of M spanned by the curve C, $C = \delta S_C$. It is easy to see that the phase of 1-from $\langle\phi_m(R)|d_R\phi_m(R)\rangle$ is nonintegrable and the phase of 2-form $\langle d_R\phi_m(R)|\wedge|d_R\phi_m(R)\rangle$ has a local gauge invariant, i.e. this 2-form is independent of the choice of the phase of $\phi_m(R)$. Considering

$$P_m(t)U^{[0]}(t_0, t)|\phi_K[R(0)]\rangle = \delta_{mK}U^{(0)}(t_0, t)|\phi_K[R(0)]\rangle \quad (24)$$

we proved the quantum adiabatic theorem strictly. Under the adiabatic limit $T\to\infty$, there are no transitions: the system with the initial state $\phi_K[R(0)]$ will be in the state $\phi_K[R(t)]$ at time t, therefore, Eq.(22) is right. (24) can be regarded as the mathematical expression of the quantum adiabatic theorem.

For the slowly changing processes with finite time scale T, the adiabatic condition does not hold, we need to consider the 1-order approximation. Because $b_l^{[0]n} = \delta_{nl}b_n^{[0]n}$, then $b_l^{[1]l} = 0$, $b_n^{[1]l} \neq 0\ (n \neq l)$.

The 1-order evolution matrix is

$$U^{[1]}(t_0, t) = U^{[0]}(t_0, t) + \frac{1}{T}\sum_{m\neq n} b_n^{[1]m}(\tau)\exp\left[\frac{T}{i\hbar}\int_{\tau_0}^{\tau} \varepsilon_m(\tau')d\tau'\right] P_m^n(t_0, t) \quad (25)$$

that has the following properties

$$P_l(t)U^{[1]}(t_0, t)|\phi_K[R(0)]\rangle = \begin{cases} U^{[0]}(t_0, t)|\phi_K[R(t_0)]\rangle, & l = K \\ \frac{1}{T}\, b_K^{[1]l}(\tau)\exp\left[\frac{T}{i\hbar}\int_{\tau}^{\tau_0} \varepsilon[S(\tau')]d\tau'|\phi_l[R(t)]\rangle\right], & l \neq K \end{cases} \quad (26)$$

Eq.(26) shows that the transitions or mixtures of states will appear when the adiabatic condition does not hold, but we can abstract the term with the Berry's phase in the time scale T.

We can still describe the cases when adiabatic condition is violated in the language of the fiber bundle. Because the eigenvalue $\varepsilon(R)$ and the eigenfunction $\phi(R)$ of $\hat{H}(R)$ are continuous functions, $\{(R, \phi)|\hat{H}(R)\phi(R) = \varepsilon(R)\phi(R)\}$ determines an Hermitian linear bundle over the parameter manifold. Let $\phi(R)$ be a fiber over a point R on M. When

$\hat{H}(R)$ changes along a closed path C on the basis manifold M and comes back to its starting point, in the non-adiabatic cases, the state of the system will not completely come back to the fiber F_{R_0} over the starting point. If the system comes back to F_{R_0} with a probability P, then it is scattered over the other fibers with the probability $\propto P/T^2$. The part coming back to F_{R0} has to move to a point A_a which deviates off its starting point A_s. The difference between A_a and A_s is indicated by the Berry's phase, which is just the holonomy in the fiber bundle. When $T \to \infty$, we naturally obtain the geometrical interpretation of the Berry's phase factor given by B. Simon.

The Berry's phase factors in a slowly changing process of $\hat{H}[R]$ also have observable effects even if the adiabatic condition does not hold. In a neutron interference experiment, a polarized beam of neutrons along a magnetic field $B'(t_0)$ splits into two beams, one passes through a constant magnetic field B, while the other passes through a varying magnetic field $B'(t)$ ($|B'(t)| = |B|$) that rotate slowly with an angular velocity ω and an precession angle θ about B. In the adiabatic condition, Berry predicted that when two beam interfere in the center point of a screen, the interfere intensity (contrasting with the counting rates) is

$$I(\theta) = \cos^2[\pi(1 - \cos\theta)] \tag{27}$$

According to the discussion in this paper, the 1-order approximate correction of $I(\theta)$ for a non-adiabatic process with a finite T is

$$I'(\theta) = \cos^2[\pi(1 - \cos\theta)] + \frac{1}{4}\lambda^2\left[\sin\theta\sin\left[\pi\left(\cos\theta - \frac{2}{\lambda}\right)\right]\frac{1 + \frac{1}{4}\lambda[1 - \cos\theta]}{1 - \frac{1}{2}\lambda\cos\theta}\right]^2 \tag{28}$$

where $\lambda = \frac{\omega}{\omega_0} = \omega \Big/ \frac{1}{2}\mu_B|B|$.

The conclusions in this paper may be verified through the experimental measurement of $I'(\theta)$. Of course, if T is smaller, it is necessary to consider higher order approximations.

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