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Particle-Number-Conserving Approach for Treating Cranking Hamiltonian*

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The particle-number-conserving approach for treating nuclear pairing correlation is extended to treat the eigenvalue problem of the cranking Hamiltonian. Detailed information on the seniority structure and the *K*-structure of the low-lying bands, and their spin alignments, signature splittings, and pair-transfer matrix elements can be provided in this formalism.

1. INTRODUCTION

In the past decade the cranking shell model (CSM) was widely used for the description of the nuclear high-spin states and met with great success [1-4]. Usually, the eigenvalue problem of CSM Hamiltonian is solved by means of the generalized Bogolyubov transformation [5]. Some serious shortcomings (irrespective of whether taking into account the self-consistency or not), however, were pointed out by I. Hamamoto [6,7], i.e., the usual CSM calculation is not reliable at the bandcrossing regions and at small rotational frequency (near bandhead). While some defects (e.g. angular momentum non-conservation) are inherent in this model [7], some others are brought about by the approximation method. For example, P. Ring pointed out [8] that the calculated result using HFB method for nuclear pairing phase transition is unreliable.

In this paper the particle-number-conserving (PNC) approach for treating the nuclear pairing correlation [9,10] is extended to treating the eigenvalue problem of CSM Hamil-

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tonian[1]. Because the number of valence particles, which dominate the behavior of the low-lying excited states, is not very large(~10) and the average pairing strength G is not very strong ($G \lesssim d/2$, d being the average single-particle level space near Fermi surface), the number of important configurations (say, the configurations with weight >1%) in the low-lying excited states is not large, therefore, it is not difficult to obtain an accurate PNC solution. In the PNC treatment of pairing correlation some serious difficulties encountered in the BCS method (blocking effect and residual quasiparticle interaction, the gap parameter Δ being sensitively configuration-dependent, the occurrence of excessive spurious states, etc.) disappear. It is expected that some defects brought about by the HFB approximation may be overcome in the PNC treatment for the CSM Hamiltonian.

In Sect.2 the general formalism of the PNC treatment for the eigenvalues and eigenfunctions of the CSM Hamiltonian is presented. Obviously, once the PNC wave functions are obtained, various nuclear properties, e.g. the K-structure, the seniority structure, the spin-alignment, $\langle J_x \rangle$ and its fluctuation, $\sqrt{\langle J_x^2 \rangle - \langle J_x \rangle^2}$ the band crossing frequency, signature splitting, the pair transfer matrix element and the pairing phase transition, etc., can be investigated in detail. The calculated results and discussions will be published elsewhere.

2. FORMALISM

2.1 Hamiltonian

The CSM Hamiltonian for an axially symmetric deformed nucleus is chosen as usual,

$$H_{\mathrm{CSM}} = H_{\mathrm{intr}} - \omega J_x, \tag{1}$$

where $-\omega J_x$ is the Coriolis interaction. The collective rotation is along the x-axis, which is perpendicular to the symmetry axis (z-axis).

$$J_x = \sum_{\mu\nu} \langle \mu | j_x | \nu \rangle a_\mu^+ a_\nu, \tag{2}$$

where μ,ν denote the single-particle states. H_{intr} is the intrinsic Hamiltonian

$$\begin{aligned} H_{\mathrm{intr}} &= H_{\mathrm{SP}} + H_{\mathrm{P}}, \\ H_{\mathrm{SP}} &= \sum_\nu \varepsilon_\nu (a_\nu^+ a_\nu + a_{\bar{\nu}}^+ a_{\bar{\nu}}), \\ H_{\mathrm{P}} &= -G \sum_{\mu,\nu>0} a_\mu^+ a_{\bar{\mu}}^+ a_{\bar{\nu}} a_\nu. \end{aligned} \tag{3}$$

1) PNC treatment is essentially a shell model calculation. However, instead of a truncation of the single-particle level which is adopted in the usual shell model, BCS and HFB calculations, a truncation of many-body configuration energy is adopted in PNC treatment. Its reasonableness has been demonstrated in the appendix of Ref.[10]. This kind of truncation is very convenient and feasible for treating the low-lying excited states.

For $\omega = 0$ the component of angular momentum along the z-axis, J_z, is a constant of motion, i.e., $K = \sum_i \Omega_i$ is a good quantum number (Ω_i is the projection of the angular momentum of the i-th nucleon). In this case, H_{CSM} can be diagonalized in separate subspace with fixed K, parity π, and seniority V (the number of unpaired particles). For example, the pair-excitational states for 2n-particle system ($V = 0, K^\pi = 0^+$) are expressed as

$$\mathscr{A}^+_{n\beta}|0\rangle = \sum_{\rho_1\cdots\rho_n} v^{\beta}_{\rho_1\cdots\rho_n} S^+_{\rho_1\cdots\rho_n}|0\rangle, \qquad (4)$$
$$S^+_{\rho_1\cdots\rho_n} = S^+_{\rho_1}\cdots S^+_{\rho_n}, \quad S^+_{\rho_i} = a^+_{\rho_i}a^+_{\bar{\rho}_i}, \quad i = 1, 2, \cdots$$

with β = o (ground band),1,2,...(pair-excitational bands).
A pair-broken state of seniority 2($V = 2$) is expressed as

$$\mathscr{A}^+_{n-1,\beta}(st)|0\rangle = a^+_s a^+_t \sum_{\rho_1\cdots\rho_{n-1}} v^{\beta(st)}_{\rho_1\cdots\rho_{n-1}} S^+_{\rho_1\cdots\rho_{n-1}}|0\rangle, \qquad (5)$$
$$K = \Omega_s + \Omega_t, \quad \pi = \pi_s \cdot \pi_t$$

where s and t denote the single-particle levels occupied by two unpaired particles. The states with more unpaired particles ($V = 4,6,...$) can be expressed in a similar way.

For $\omega \neq 0$, K and V are no longer good quantum numbers, but parity π still remains. The configurations with different K, V, but with the same parity π can mix with each other. For simplicity it is usually assumed that H_{SP} remains invariant with respect to a rotation of 180 about the x-axis, $R_x(\pi)$. Nilsson's Hamiltonian with quadrupole and hexadecapole deformation satisfies such a requirement. In this case, the signature r (eigenvalue of $R_x(\pi)$) is a good quantum number. Thus, H_{CSM} can be diagonalized in each subspace with fixed parity and signature (π, r).

2.2 Choice of Single-Particle Bases

As $\omega \neq 0$, j_z (the z-component of the single particle angular momentum) is no longer a conservative quantity, therefore, it is not suitable to use j_z to label the single-particle states. However, if H_{SP} possesses the invariance with respect to $R_x(\pi)$, we may use $r = i$ (or an equivalent additive conservative quantity $\alpha = \pm 1/2$, $r = e^{-i\pi\alpha}$) to label the single-particle state. Because $[j_z^2, R_x(\pi)] = 0$, the eigenstates of $R_x(\pi)$ can be expressed as a linear superposition of the doubly-degenerate eigenstates of j_z. For definiteness, let χ_Ω denote the eigenstate of j_z with the eigenvalue $\Omega > 0$, and define

$$\chi_{\bar{\Omega}} = R_x(\pi)\chi_\Omega = e^{-i\pi j_x}\chi_\Omega, \qquad (6)$$

which is also the eigenstate of j_z with eigenvalue $-\Omega$ ($\chi_{\bar{\Omega}}$ turns out to be the time-reversal state of χ_Ω apart from a possible phase factor). Both χ_Ω and $\chi_{\bar{\Omega}}$ are the eigenstates of j_z^2, corresponding to the eigenvalue Ω^2. Let

$$\varphi_{\Omega\alpha} = \frac{1}{\sqrt{2}}[1 + e^{i\pi\alpha}R_x(\pi)]\chi_\Omega = \frac{1}{\sqrt{2}}(\chi_\Omega + e^{i\pi\alpha}\chi_{\bar{\Omega}}), \quad \alpha = \frac{1}{2} \qquad (7)$$
$$\varphi_{\Omega\bar{\alpha}} = \frac{1}{\sqrt{2}}[e^{i\pi\alpha} + R_x(\pi)]\chi_\Omega = \frac{1}{\sqrt{2}}(e^{i\pi\alpha}\chi_\Omega + \chi_{\bar{\Omega}}), \quad \bar{\alpha} = -\frac{1}{2}$$

it is easy to prove that both of them are the eigenstates of $R_x(\pi)$,

$$\begin{aligned} R_x(\pi)\varphi_{\Omega\alpha} &= e^{-i\pi\alpha}\varphi_{\Omega\alpha} = -i\varphi_{\Omega\alpha}, \\ R_x(\pi)\varphi_{\Omega\bar{\alpha}} &= e^{i\pi\alpha}\varphi_{\Omega\bar{\alpha}} = i\varphi_{\Omega\bar{\alpha}}. \end{aligned} \tag{8}$$

In the second quantization formalism, the canonical transformation (7) may be expressed as

$$\begin{aligned} b_\nu^+ &= \frac{1}{\sqrt{2}}(a_\nu^+ + e^{i\pi\alpha}\alpha_{\bar{\nu}}^+), \\ b_{\bar{\nu}}^+ &= \frac{1}{\sqrt{2}}(e^{i\pi\alpha}a_\nu^+ + a_{\bar{\nu}}^+). \end{aligned} \tag{9}$$

It is easy to show that

$$\begin{aligned} \{b_\nu, b_{\nu'}^+\} &= \delta_{\nu\nu'}, \\ \{b_{\bar{\nu}}, b_{\bar{\nu}'}^+\} &= \delta_{\nu\nu'}, \end{aligned} \tag{10}$$

and all the other anticommutators vanish.

Under this transformation, the form of the expression (1) of H_{CSM} remains unchanged except a is replaced by b. Particularly, it may be verified that

$$a_\nu^+ a_{\bar{\nu}}^+ = b_\nu^+ b_{\bar{\nu}}^+ \tag{11}$$

In the new representation, the matrix elements of the single particle angular momentum j_x are as follows:

(1) The matrix element of j_x between the states with different signatures vanishes,

$$\langle\varphi_{\Omega_1\alpha}|j_x|\varphi_{\Omega_2\bar{\alpha}}\rangle = 0. \tag{12}$$

(2) The matrix element of j_x between the states of the same signature is

$$\begin{aligned} \langle\varphi_{\Omega_1\alpha}|j_x|\varphi_{\Omega_2\alpha}\rangle &= \langle\chi_{\Omega_1}|j_x|\chi_{\Omega_2}\rangle \\ &+ e^{i\pi\alpha}\langle\chi_{\bar{\Omega}_1}|j_x|\chi_{\Omega_2}\rangle\delta_{\Omega_1,\frac{1}{2}}\delta_{\Omega_2,\frac{1}{2}}, \quad \alpha = \pm\frac{1}{2} \end{aligned} \tag{13}$$

where $\langle\chi_{\Omega_1}|j_x|\chi_{\Omega_2}\rangle$ and $\langle\chi_{\bar{\Omega}_1}|j_x|\chi_{\Omega_2}\rangle$ can be calculated with the well-known Nilsson wave function. It should be noted that in our phase convention (see Eq.(6))

$$\begin{aligned} \langle\chi_{\Omega_1}|j_x|\chi_{\bar{\Omega}_2}\rangle &= -\langle\chi_{\bar{\Omega}_1}|j_x|\chi_{\Omega_2}\rangle, \\ \langle\chi_{\bar{\Omega}_1}|j_x|\chi_{\bar{\Omega}_2}\rangle &= \langle\chi_{\Omega_1}|j_x|\chi_{\Omega_2}\rangle. \end{aligned} \tag{14}$$

(3) It is worthwhile to mention that for $\Omega_1 = \Omega_2 = 1/2$,

$$\langle\varphi_{(\frac{1}{2})_1\alpha}|j_x|\varphi_{(\frac{1}{2})_2\bar{\alpha}}\rangle = e^{-i\pi\alpha}\langle\chi_{(\frac{1}{2})_1}|j_x|\chi_{(\frac{1}{2})_2}\rangle, \quad \alpha = \pm\frac{1}{2} \tag{15}$$

which is the only signature-dependent matrix element and is responsible for all the signature splittings.

2.3 Many-Particle Configurations and Relevant Matrix Elements

For simplicity, we restrict ourselves to the ground band($\pi = +, r = +1$) and the relevant low-lying excited bands of even-even nuclei (the other bands in even-even nuclei or odd-A nuclei can be treated similarly). The configurations needed to be considered are as follows:

(a) Fully-paired configurations ($V = 0$)

$$|\rho_1\bar{\rho}_1\cdots\rho_n\bar{\rho}_n\rangle = b^+_{\rho_1}b^+_{\bar{\rho}_1}\cdots b^+_{\rho_n}b^+_{\bar{\rho}_n}|0\rangle, \quad K^\pi = 0^+, \quad r = +1 \tag{16}$$

(b) One-pair-broken configurations ($V = 2$)

$$\begin{aligned}|\nu_1\bar{\nu}_2\rho_1\bar{\rho}_1\cdots\rho_{n-1}\bar{\rho}_{n-1}\rangle &= b^+_{\nu_1}b^+_{\bar{\nu}_2}b^+_{\rho_1}b^+_{\bar{\rho}_1}\cdots b^+_{\rho_{n-1}}b^+_{\bar{\rho}_{n-1}}|0\rangle, \\ K &= \pm|\Omega_{\nu_1} - \Omega_{\nu_2}|, \pm(\Omega_{\nu_1} + \Omega_{\nu_2}), \\ \pi &= \pi_{\nu_1}\cdot\pi_{\nu_2} = +, \quad r = +1,\end{aligned} \tag{17}$$

It should be noted that for given blocked levels ($\nu_1\ \nu_2$) there are two kinds of $r = +1$ configurations, i.e.,

$$|\nu_1\bar{\nu}_2\rho_1\bar{\rho}_1\cdots\rho_{n-1}\bar{\rho}_{n-1}\rangle \quad \text{and} \quad |\nu_2\bar{\nu}_1\rho_1\bar{\rho}_1\cdots\rho_{n-1}\bar{\rho}_{n-1}\rangle \tag{18}$$

and both of them should be considered.

(c) Two-pair-broken configurations ($V = 4$)

For $r = +1$, the configurations can be expressed as

$$|\nu_1\nu_2\bar{\nu}_3\bar{\nu}_4\rho_1\bar{\rho}_1\cdots\rho_{n-2}\bar{\rho}_{n-2}\rangle, \quad \pi_{\nu_1}\pi_{\nu_2}\pi_{\nu_3}\pi_{\nu_4} = + \tag{19}$$

It should be noted also that for given blocked levels ($\nu_1\nu_2\nu_3\nu_4$), there are eight $r = +1$ configurations, six of which are of $\alpha = 0$, and the other two are of $\alpha = \pm 2$. For example, for ($\nu_1\nu_2\nu_3\nu_4$) = (1234), the configurations of $\alpha = 0$ are

$$|12\bar{3}\bar{4}\rangle, |13\bar{2}\bar{4}\rangle, |14\bar{2}\bar{3}\rangle, |23\bar{1}\bar{4}\rangle, |24\bar{1}\bar{3}\rangle, |34\bar{1}\bar{2}\rangle, \tag{20}$$

and those of $\alpha = \pm 2$ are

$$|1234\rangle \quad \text{and} \quad |\bar{1}\bar{2}\bar{3}\bar{4}\rangle \tag{21}$$

(d) The configurations with six or more unpaired particles ($V = 6,8,...$) may be treated similarly. However, the calculation shows that for the ground band and the low-lying excited bands in even-even nuclei the component of the configurations with $V \geq 6$ is very small for not too high rotational frequency ($\hbar\omega \lesssim 0.50\mathrm{MeV}$). This is consistent with the conclusion drawn in Ref.[11] that the component of the configurations with $V \geq 6$ is less

than 10^{-5} and can be neglected.

The matrix elements of J_x between many-particle configurations are as follows:

(a) Considering the selection rule for J_x, $\Delta K = \pm 1$, the matrix elements of J_x between the fully-paired configurations ($V = 0$) vanish,

$$\langle \bar{\rho}_{n'}\rho_{n'}\cdots\bar{\rho}_{1'}\rho_{1'}|J_x|\rho_1\bar{\rho}_1\cdots\rho_n\bar{\rho}_n\rangle = 0 \tag{22}$$

(b) Because J_x is a one-body operator, the matrix element does not vanish only when the fully-paired configuration ($V = 0$) and a one-pair-broken one ($V = 2$) differ by one single-particle state. For example,

$$\begin{aligned}\langle \bar{\rho}_{n-1}\rho_{n-1}\cdots\bar{\rho}_1\rho_1\bar{2}1|J_x|\rho\bar{\rho}\rho_1\bar{\rho}_1\cdots\rho_{n-1}\bar{\rho}_{n-1}\rangle \\ \equiv \langle\cdots\bar{2}1|J_x|\rho\bar{\rho}\cdots\rangle \\ = \langle\chi_{\Omega_1}|j_x|\chi_{\Omega_1}\rangle(\delta_{\rho 1}+\delta_{\rho 2}) \\ + e^{-i\pi\alpha}\langle\chi_{\bar{\Omega}_1}|j_x|\chi_{\Omega_1}\rangle\delta_{\Omega_1,\frac{1}{2}}\delta_{\Omega_1,\frac{1}{2}}(\delta_{\rho 1}-\delta_{\rho 2}),\end{aligned} \tag{23}$$

here (and in the following) the "..." represents the fully-paired particles not involved in the transition. It should be noted that only when $\Omega_1 = \Omega_2 = 1/2$, $\pi_1 = \pi_2$ and all the other quantum number are not the same, can the second term in (23) be nonzero.

(c) Matrix elements between two one-pair-broken configurations ($V = 2$)

The diagonal matrix element is

$$\langle\cdots\bar{2}1|J_x|1\bar{2}\cdots\rangle = (j_x)_{11} + (j_x)_{\bar{2}\bar{2}}; \tag{24}$$

The off-diagonal elements are

$$\begin{aligned}\langle\cdots\bar{2}'1|J_x|1\bar{2}\cdots\rangle &= (j_x)_{\bar{2}'\bar{2}}, \\ \langle\cdots\bar{2}1'|J_x|1\bar{2}\cdots\rangle &= (j_x)_{1'1},\end{aligned} \tag{25}$$

$$\begin{aligned}\langle\cdots\bar{2}2\bar{2}'1|J_x|1\bar{2}2'\bar{2}'\cdots\rangle &= -(j_x)_{22'}, \\ \langle\cdots\bar{1}1\bar{2}1'|J_x|1\bar{2}1'\bar{1}'\cdots\rangle &= -(j_x)_{\bar{1}\bar{1}'}.\end{aligned} \tag{26}$$

where

$$\begin{aligned}(j_x)_{ij} &= \langle\varphi_{\Omega_i a}|j_x|\varphi_{\Omega_j a}\rangle, \\ (j_x)_{\bar{i}\bar{j}} &= \langle\varphi_{\Omega_i a}|j_x|\varphi_{\Omega_j a}\rangle.\end{aligned}$$

(d) Matrix elements between two two-pair-broken configurations ($V = 4$)

Because H_c (or J_x) is a one-body operator, it is easy to show that the matrix elements of J_x between $\alpha = 0$ and $\alpha = \pm 2$ configurations vanish, though these configurations are of $r = +1$; namely, only when $\Delta\alpha = 0$, the matrix elements of J_x may not vanish. This selection rule is stronger than that for r (i.e., Δr, no). Therefore, for the ground band and the relevant low-lying excited bands in even-even nuclei, only the six configurations ($\alpha = 0$) of $V = 4$ need to be considered. Hence the diagonal matrix element is

$$\langle \cdots \bar{4}\bar{3}21 | J_x | 12\bar{3}\bar{4} \cdots \rangle = (j_x)_{11} + (j_x)_{22} + (j_x)_{\bar{3}\bar{3}} + (j_x)_{\bar{4}\bar{4}}, \tag{27}$$

The off-diagonal matrix elements may not vanish only when two configurations ($V = 4$) differ by one single-particle state

$$\begin{aligned}
\langle \cdots \bar{4}\bar{3}21' | J_x | 12\bar{3}\bar{4} \cdots \rangle &= (j_x)_{1'1}, \\
\langle \cdots \bar{4}\bar{3}2'1 | J_x | 12\bar{3}\bar{4} \cdots \rangle &= (j_x)_{2'2}, \\
\langle \cdots \bar{4}\bar{3}'21 | J_x | 12\bar{3}\bar{4} \cdots \rangle &= (j_x)_{\bar{3}'\bar{3}}, \\
\langle \cdots \bar{4}'\bar{3}21 | J_x | 12\bar{3}\bar{4} \cdots \rangle &= (j_x)_{\bar{4}'\bar{4}},
\end{aligned} \tag{28}$$

$$\begin{aligned}
\langle \cdots \bar{1}1\bar{4}\bar{3}21' | J_x | 12\bar{3}\bar{4}1'\bar{1}' \cdots \rangle &= -(j_x)_{\bar{1}\bar{1}'}, \\
\langle \cdots \bar{2}2\bar{4}\bar{3}2'1 | J_x | 12\bar{3}\bar{4}2'\bar{2}' \cdots \rangle &= -(j_x)_{\bar{2}\bar{2}'}, \\
\langle \cdots \bar{3}3\bar{4}\bar{3}'21 | J_x | 12\bar{3}\bar{4}3'\bar{3}' \cdots \rangle &= -(j_x)_{\bar{3}3'}, \\
\langle \cdots \bar{4}4\bar{4}'\bar{3}21 | J_x | 12\bar{3}\bar{4}4'\bar{4}' \cdots \rangle &= -(j_x)_{\bar{4}4'}.
\end{aligned} \tag{29}$$

(e) Matrix elements between $V = 4$ and $V = 2$ configurations

$$\begin{aligned}
\langle \cdots \bar{4}\bar{3}21 | J_x | 1\bar{3}\rho\bar{\rho} \cdots \rangle &= -(j_x)_{42}(\delta_{\rho 3} + \delta_{\rho 4}) \\
\langle \cdots \bar{4}\bar{3}21 | J_x | 1\bar{4}\rho\bar{\rho} \cdots \rangle &= +(j_x)_{32}(\delta_{\rho 3} + \delta_{\rho 2}) \\
\langle \cdots \bar{4}\bar{3}21 | J_x | 2\bar{3}\rho\bar{\rho} \cdots \rangle &= (j_x)_{41}(\delta_{\rho 4} + \delta_{\rho 1}) \\
\langle \cdots \bar{4}\bar{3}21 | J_x | 2\bar{4}\rho\bar{\rho} \cdots \rangle &= -(j_x)_{31}(\delta_{\rho 3} + \delta_{\rho 1})
\end{aligned} \tag{30}$$

Matrix elements between the configurations with seniority $V \geq 6$ can be treated similarly.

2.4 General Form of the Eigenstates of H_{CSM}

For example, the general form of the eigenstates ($\pi = +$, signature $r = +1$) of H_{CSM} for a $2n$-particle system can be expressed as $|2n, \beta, \pi = +, r = +1\rangle$, or simply labelled by $|2n, \beta{+}{+}\rangle$, $\beta = 0$ (yrast band), 1,2,3,...(excited bands).

$$\begin{aligned}
|2n, \beta{+}{+}\rangle = &\sum_{\rho_1 \cdots \rho_n} v^{\beta}_{\rho_1 \cdots \rho_n} | \rho_1 \bar{\rho}_1 \cdots \rho_n \bar{\rho}_n \rangle \\
&+ \sum_{\nu_1 \nu_2} \sum_{\rho_1 \cdots \rho_{n-1}} v^{\beta(\nu_1 \nu_2)}_{\rho_1 \cdots \rho_{n-1}} | \nu_1 \bar{\nu}_2 \rho_1 \bar{\rho}_1 \cdots \rho_{n-1} \bar{\rho}_{n-1} \rangle \\
&+ \sum_{\nu_1 \nu_2 \nu_3 \nu_4} \sum_{\rho_1 \cdots \rho_{n-2}} v^{\beta(\nu_1 \nu_2 \nu_3 \nu_4)}_{\rho_1 \cdots \rho_{n-2}} | \nu_1 \bar{\nu}_2 \nu_3 \bar{\nu}_4 \rho_1 \bar{\rho}_1 \cdots \rho_{n-2} \bar{\rho}_{n-2} \rangle \\
&+ \cdots .
\end{aligned} \tag{31}$$

Once the eigenfunctions are obtained, all the relevant properties can be calculated, e.g. the seniority structure of the wave function. Let P_V denote the component of the configurations with seniority $= V$ in the wave function $|2n, \beta{+}{+}\rangle$, namely,

$$
\begin{aligned}
P_0^{(\beta)} &= \sum_{\rho_1 \cdots \rho_\pi} |\, v^{\beta}_{\rho_1 \cdots \rho_\pi} |^2 \\
P_2^{(\beta)} &= \sum_{\nu_1 \nu_2} \sum_{\rho_1 \cdots \rho_{\pi-1}} |\, v^{\beta(\nu_1\nu_2)}_{\rho_1 \cdots \rho_{\pi-1}} |^2 \\
P_4^{(\beta)} &= \sum_{\nu_1\nu_2\nu_3\nu_4} \sum_{\rho_1 \cdots \rho_{\pi-2}} |\, V^{\beta(\nu_1\nu_2\nu_3\nu_4)}_{\rho_1 \cdots \rho_{\pi-2}} |^2 \\
&\cdots\cdots
\end{aligned}
\tag{32}
$$

Obviously, the following normalization condition holds,

$$
P_0^{(\beta)} + P_2^{(\beta)} + P_4^{(\beta)} + \cdots\cdots = 1 \tag{33}
$$

It is very important to investigate the variation of seniority structure with angular frequency ω, which is helpful for investigating the pairing transition.

Similarly, using wave functions (31) the matrix elements of pair-transfer reaction can be calculated. Also it is easy to obtain the information on the angular momentum alignment, signature splitting, etc.

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