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Coulomb Correction of the p-^{4}He Low-Energy Scattering Phase Shifts

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A simple approach of the Coulomb correction to the p-^{4}He low-energy scattering phase shifts is proposed, in which the channel radius of each partial wave is taken as an adjustable parameter. By using the Coulomb correction the n-^{4}He scattering observable predicted from p-^{4}He scattering phase shifts are in excellent agreement with the experiment. Our calculations show that the neutron polarization depends sensitively on the channel radii of *P* waves and the obtained channel radii are adequate to all different sets of phase shifts.

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1. INTRODUCTION

Helium is the most widely used analyzer of polarized neutrons. The accurate data for n-^{4}He scattering are required. It is well known that, for reasons of the beam preparation and detection, scattering processes with charged constituents are more readily performed than those involving neutral particles. According to the charge symmetry of nuclear force the only difference between the Hamiltionians of the mirror systems p-^{4}He and n-^{4}He is the Coulomb interaction between the incident particle and the target nucleus. Then, the p-^{4}He scattering phase shifts after making Coulomb corrections may be used to predict the observable of the n-^{4}He scattering.

Because of the spin-orbit potential in strong hadronic interaction, the scattering phase shifts depend both on the orbital angular momentum l and the total angular momenta j. From the experimental data the resultant phase shifts Δ_{lj} can only be determined. The p-^{4}He phase shifts $\bar{\delta}_{lj}$, given in literatures and defined by the following relation.

$$\Delta_{lj} = \sigma_l + \bar{\delta}_{lj}. \tag{1}$$

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Where σ_l is the phase shifts due to the point-like charge Coulomb interaction.

Writing $\bar{\delta}_{l,l+\frac{1}{2}} = \bar{\delta}_{l+}$, $\bar{\delta}_{l,l-\frac{1}{2}} = \bar{\delta}_{l-}$, the differential cross section $d\sigma/d\Omega$ and the polarization $P(\theta)$ are expressed as follows [1]:

$$\frac{d\sigma}{d\Omega} = |f(k,\theta)|^2 + |g(k,\theta)|^2, \tag{2}$$

$$P(\theta) = \frac{2\mathrm{Re}[f^*(k,\theta)g(k,\theta)]}{|f(k,\theta)|^2 + |g(k,\theta)|^2}, \tag{3}$$

where

$$f(k,\theta) = f_C(\theta) + \frac{1}{2ik}\sum_l [(l+1)(e^{2i\bar{\delta}_{l+}} - 1) + l(e^{2i\bar{\delta}_{l-}} - 1)]e^{2i\sigma_l}P_l(\cos\theta), \tag{4}$$

$$g(k,\theta) = \frac{1}{2k}\sum_l [e^{2i\bar{\delta}_{l+}} - e^{2i\bar{\delta}_{l-}}]e^{2i\sigma_l}\sin\theta\frac{d}{d(\cos\theta)}P_l(\cos\theta) \tag{5}$$

and $f_C(\theta)$ is the Coulomb scattering amplitude.

The p-^{4}He phase shifts $\bar{\delta}_{lj}$ are determined from the experimental data of observable $d\sigma/d\Omega$ and $P(\theta)$ by using the above formulas. It should be noticed that $\bar{\delta}_{lj}$ are not real p-^{4}He phase shifts δ_{lj} due to the hadronic interaction. In order to obtain δ_{lj} the following Coulomb distortion effects have to be corrected.

(1) The real charge distributions of the proton and ^{4}He are not point-like. It is important for high energy scattering to take into account their form factors.

(2) $\bar{\delta}_{lj}$ are the phase shifts of Coulomb distorted wave caused by both the short range hadronic interaction and the residual electromagnetic interaction. It is different from δ_{lj} which are the phase shifts of plane wave caused only by the short range hadronic interaction. This Coulomb correction is vital for low energy scattering.

The mathematical details of the above arguments are derived in the Appendix.

2. CORRECTION OF THE COULOMB DISTORTION EFFECT

We compare the wave functions between the p-^{4}He and n-^{4}He scatterings in two regions. Inside the channel radius the electromagnetic interaction may be totally neglected as compared to the influence of the hadronic interaction for such few-body systems and then their wave functions are almost the same. It is reasonable to assume that the logarithmic derivatives of the radial wave functions of the p-^{4}He and n-^{4}He are equal at the channel radius R_C.

Outside the radius of the elastic scattering channel there is only a long-range electromagnetic interaction between p and ^{4}He. This interaction contains both the direct folding potential and the exchange potential as known from the resonance group method (RGM). The exchange potential is shortly ranged and may be neglected outside the channel radius. The residual Coulomb interaction is

$$V^R = \left\langle \Psi(1,2,3,4) \middle| \left(\frac{e^2}{|q - r_3|} + \frac{e^2}{|q - r_4|} - \frac{2e^2}{q}\right) \middle| \Psi(1,2,3,4) \right\rangle$$

where e denotes the charge of proton, labels 1 and 2 indicate neutrons in the ^{4}He nucleus and labels 3 and 4 indicate protons in the ^{4}He nucleus. q and r_i are the relative coordinate between the proton and ^{4}He and the coordinate of the i-th nucleon relative to the center of ^{4}He respectively.

In the region outside the channel radius, $q > r_i$, the multipole expansion of the potential is

$$\frac{1}{|\boldsymbol{q}-\boldsymbol{r}_i|} = \sum_L \frac{r_i^L}{q^{L+1}} P_L(\hat{q}\cdot\hat{r}_i)$$

Then,

$$V^R = \left\langle \Psi(1,2,3,4) \left| \left(\frac{e^2}{q} + \frac{e^2}{q} - \frac{2e^2}{q} \right) \right| \Psi(1,2,3,4) \right\rangle$$
$$+ \left\langle \Psi(1,2,3,4) \left| \frac{e^2}{q^2} \sum_{i=3}^{4} r_i P_1(\hat{r}_i\cdot\hat{q}) \right| \Psi(1,2,3,4) \right\rangle$$
$$+ \left\langle \Psi(1,2,3,4) \left| \frac{e^2}{q^3} \sum_{i=3}^{4} r_i^2 P_2(\hat{r}_i\cdot\hat{q}) \right| \Psi(1,2,3,4) \right\rangle$$
$$+ \cdots$$

where the first term is obviously equal to zero and the other terms depend on the charge distribution of the ^{4}He nucleus and they are also equal to zero by Winger-Eckart theorem, if $\Psi(1,2,3,4)$ is of a spherical symmetry.

It is true that outside the channel radius the residual Coulomb potential of the p-^{4}He system vanishes. Then the radial wave functions of the p-^{4}He scattering are described by the regular and irregular Coulomb wave functions $F_l(kq)$ and $G_l(kq)$. The radial wave functions of the n-^{4}He scattering are described by the spherical Bessel function $j_l(kq)$ and the spherical Neumann functions $n_l(kq)$.

Let $\overline{R}_{lj}(kq)$ and $R_{lj}(kq)$ denote the radial wave functions of the proton and neutron respectively. At the channel radius $q = R_c$, their logarithmic derivatives are $\overline{\xi}_{lj}$ and ξ_{lj} respectively and

$$\overline{\xi}_{lj} = \frac{1}{\overline{R}_{lj}(kq)} \frac{d}{dq} \overline{R}_{lj}(kq)\big|_{q=R_c}$$
$$= k\left[\frac{F_l'(\rho) + \operatorname{tg}\overline{\delta}_{lj} G_l'(\rho)}{F_l(\rho) + \operatorname{tg}\overline{\delta}_{lj} G_l(\rho)} - \frac{1}{\rho}\right]_{\rho=kR_c} \tag{6}$$

where the primes denote differentiation with respect to ρ, and k is the wave vector of the incident particle relative to the target nucleus. The following formulas have been used for obtaining Eq.(6)

$$\overline{R}_{lj}(kq) = \frac{F_l(kq) + \operatorname{tg}\overline{\delta}_{lj} G_l(kq)}{q}.$$

$$\xi_{lj} = \frac{1}{R_{lj}(kq)} \frac{d}{dq} R_{lj}(kq)\big|_{q=R_c}$$
$$= k\left[\frac{j_l'(\rho) - \operatorname{tg}\delta_{lj} n_l'(\rho)}{j_l(\rho) - \operatorname{tg}\delta_{lj} n_l(\rho)}\right]_{\rho=kR_c}, \tag{7}$$

As discussed above, it is reasonable to make the approximation,

$$\bar{\xi}_{lj} = \xi_{lj}. \tag{8}$$

This equation gives the relation between the phase shifts $\overline{\delta}_{lj}$ and δ_{lj}. If the pure strong hadronic phase shifts δ_{lj} are obtained, then, it is easy to calculate the n-^{4}He scattering observable by using δ_{lj} instead of $\overline{\delta}_{lj}$ and putting $\sigma_l = 0$ in Eq.(2)–(5).

3. RESULTS AND DISCUSSIONS

We can obtain δ_{lj} from $\overline{\delta}_{lj}$ by Eqs.(6)–(8) with an adjustable parameter R_C. We take the same value of $R_C = 2.0$ fm for each partial wave. The Calculated observable for four different incident neutron energies are shown in Figs. 1 and 2. As compared to those fitting to the experimental data by Morgans and Walter, the differential cross sections $d\sigma/d\Omega$ are in good agreement, while the neutron polarizations $P(\theta)$ have some deviations at lower incident energies. In order to improve the defects we take into account the different ranges of effective interactions for various partial waves. Thus, the channel radii for various partial waves should have different values. In addition, from the RGM calculations [2] we know

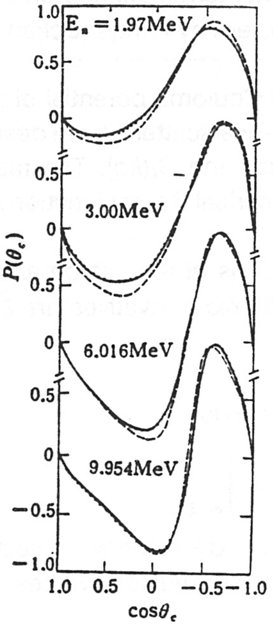

FIG. 1 Neutron polarizations of the n-^{4}He scattering. —— Morgan's fit to the experimental data. - - - our calculated results with $R_C = 2.0$ fm. ···· our calculated results with R_C values given in Eq.(9). For the latter two curves the Satchler's p-^{4}He phase shifts are taken as the input.

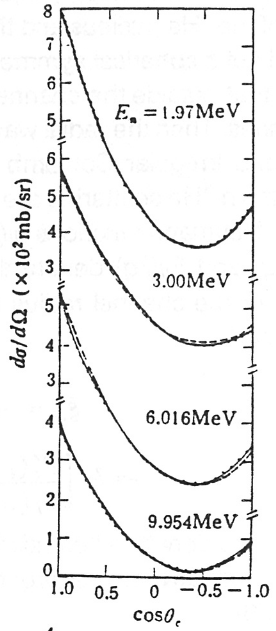

FIG. 2 n-^{4}He differential cross sections. Other explanations are the same as in Fig. 1.

that the effective potentials are weakly dependent on the incident energies. The energy dependence on R_c can be neglected.

Using R. G. Satchler's [3] p-^{4}He phase shifts as input and taking Morgan's results [4] of fitting to the experiment as a reference we adjust the channel radii R_c for various partial waves. It turns out that the neutron polarization is very sensitive to the channel radii of the P waves, especially for $P_{3/2}$ wave. We keep the channel radii of the D and F waves at 2.0 fm since their contributions are very small for low energy scatterings. The channel radii for the S and P waves are taken as follows,

$$R_c(S) = 2.52\,\mathrm{fm},\quad R_c(P_{3/2}) = 2.08\,\mathrm{fm},\quad R_c(P_{1/2}) = 2.51\,\mathrm{fm} \tag{9}$$

The calculated differential cross sections for the n-^{4}He scattering as well as the neutron polarizations are in excellent agreement with those obtained by Morgan and Watter [4] as shown in Fig. 1 and Fig. 2.

The results of the phase shift analyses are not unique. Besides Satchler's work, others have published independently their phase shifts for the p-^{4}He scattering, including those obtained from the effective interaction range theory by Schwandt [5] and Arndt [6], and from the parameter fitting of the R-matrix by Stammbach et al. [7]. In order to test the values of R_c in Eq.(9) we introduce the coulomb distortion corrections to other sets of phase shifts. The calculated polarizations are compared with the Morgan's results in Figs.4 and

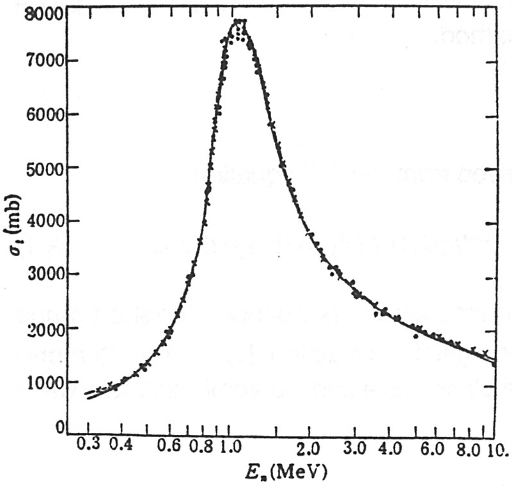

FIG. 3 Total cross sections vs. neutron energies. *** obtained by Morgan's fit to the experimental data. —— our calculated results with R_c values given in Eq.(9). The Stammbach's p-^{4}He phase shifts are used as the input.

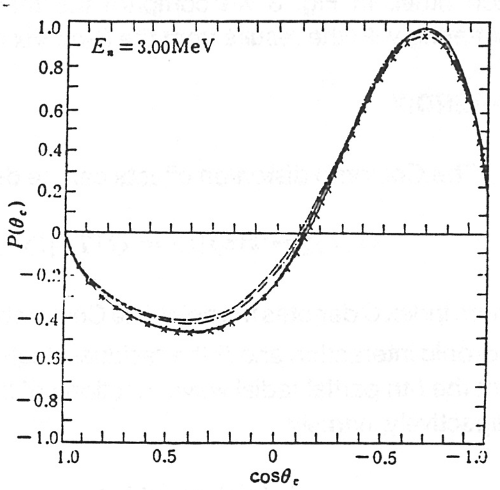

FIG. 4 Comparison of the neutron polarizations for the n-^{4}He scattering at E_n = 3.00 MeV. The calculated results correspond to various sets of p-^{4}He phase shifts with R_c values given in Eq.(9). *** Morgan's fit to the experimental data. Different phase shifts are used.—— Satchler's. $-\cdot-$ Schwandt's.--- Stammbach, Arndt's phase shifts.

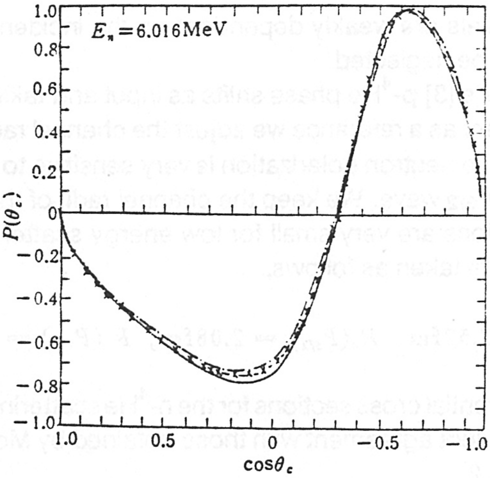

FIG. 5 Comparison of the neutron polarizations at $E_n = 6.016$ MeV. Other explanations are the same as in Fig. 4.

5. From the comparison one finds that this set of the channel radii given in Eq.(9) is adequate.

The calculated differential cross sections from different phase shifts almost overlap each other. In Fig. 3 we compare the theoretical total cross sections for the p-^{4}He scattering with the results from the R-matrix method.

APPENDIX

The Coulomb distortion effects can be derived from the *A-S* equation [8]

$$\langle l|T_j(p\text{-}^4\mathrm{He})|l\rangle = \langle l|T^C|l\rangle + e^{2i\sigma_l}\langle C,l|T_j^{SR}(p\text{-}^4\mathrm{He})|C,l\rangle, \tag{A.1}$$

where index *C* denotes the point-like Coulomb interaction, index *S* denotes the short-range hadronic interaction and *R* the residual electromagnetic interaction. $|l\rangle$ and $|C,l\rangle$ represent the *l*-th partial radial wave functions of the plane wave and Coulomb distorted wave respectively, namely

$$\langle r|l\rangle = j_l(kr),\ \langle r|C,l\rangle = \frac{1}{kr}F_l(kr),$$

The transition matrices expressed according to phase shifts are as follows

$$\langle l|T_j(p\text{-}^4\mathrm{He})|l\rangle = -\frac{\hbar^2}{2\mu}\frac{1}{2ik}(e^{2i\Delta_{lj}}-1) \tag{A.2}$$

$$\langle l|T^C|l\rangle = -\frac{\hbar^2}{2\mu}\frac{1}{2ik}(e^{2i\sigma_l}-1) \tag{A.3}$$

$$\langle C,l|T_j^{SR}(p\text{-}^4\mathrm{He})|C,l\rangle = -\frac{\hbar^2}{2\mu}\frac{1}{2ik}(e^{2i\bar{\delta}_{lj}}-1) \tag{A.4}$$

Substituting (A.2)–(A.4) into (A.1) we have

$$\Delta_{lj} = \sigma_l + \bar{\delta}_{lj}$$

The phase shifts δ_{lj} from the pure strong interaction should be the same for the p-^{4}He and n-^{4}He scatterings under the assumption of the charge symmetry of nuclear force, namely

$$\langle l|T_j^S(p\text{-}^4\mathrm{He})|l\rangle = \langle l|T_j(n\text{-}^4\mathrm{He})|l\rangle = -\frac{\hbar^2}{2\mu}\frac{1}{2ik}(e^{2i\delta_{lj}}-1) \qquad \text{(A.5)}$$

where

$$\langle l|T_j^S(p\text{-}^4\mathrm{He})|l\rangle = \int j_l(kr)T_j^S(p\text{-}^4\mathrm{He})j_l(kr)r^2dr \qquad \text{(A.6)}$$

$$\langle C, l|T_j^{SR}(p\text{-}^4\mathrm{He})|C, l\rangle = \frac{1}{k^2}\int F_l(kr)T_j^{SR}(p\text{-}^4\mathrm{He})F_l(kr)dr \qquad \text{(A.7)}$$

It is apparent from (4) and (5) that the differences between δ_{lj} and $\bar{\delta}_{lj}$ are due to the differences between T_j^S and T_j^{SR} as well as $|l\rangle$ and $|C, l\rangle$.

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