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The Mean-Field Analysis for Chiral Symmetry Phase Transition*

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A mean-field theory is used to study the chiral phase transition at finite temperature. The critical temperature and the phase diagram are obtained.

The chiral phase transition at finite temperature has recently been the object of intense study [1]. To analyze the nature and mechanism of the phase transition, very complex methods, such as Monte Carlo simulation in lattice gauge theory [2] and Dyson equation at finite temperature [3], are often used. Monte Carlo simulation can, in principle, provide some insight into the phase transition from the Lagrangian of QCD. Nevertheless, the connection between the lattice world and the continuum is tenuous at present because of limited computing power. The result obtained by this method is very rough. The order parameter as a function of temperature can also be obtained if Dyson equation is used. As is well known, it is very difficult to obtain the solution of Dyson equation. Therefore, it is worth developing a simple method by which the qualitative picture of the chiral phase transition can be obtained. Keeping this in mind, we will apply Walecka's mean-field theory [4] to analyze the chiral phase transition in this communication.

In the mean-field theory, the boson field operators in the equation of motion for fermion are replaced by the corresponding average values. Thus the field equation for fermion can be solved exactly. The fermion pair condensation $\langle \bar{\psi}\psi \rangle$ can be calculated and the phase structure of the system can be determined.

We consider the following Lagrangian with chiral invariance [5]:

$$\mathscr{L} = \mathscr{L}_0 + \mathscr{L}_I \tag{1}$$

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$$\mathscr{L}_0 = -\bar{\psi}\gamma\cdot\partial\psi - \frac{1}{2}\partial\phi_p\partial\phi_p - \frac{1}{2}m^2\phi_p^2 - \frac{1}{2}\partial\phi_s\partial\phi_s - \frac{1}{2}m^2\phi_s^2 \tag{2}$$

$$\mathscr{L}_I = ig\bar{\psi}\gamma_5\psi\phi_p + g\bar{\psi}\psi\phi_s \tag{3}$$

The order parameter which describes the phases of the system is

$$\varphi = (\varphi_p^2 + \varphi_s^2)^{1/2} \tag{4}$$

where

$$\varphi_p = \langle\phi_p\rangle = \mathrm{Tr}e^{-\beta H}\phi_p/\mathrm{Tr}e^{-\beta H},\quad \varphi_s = \langle\phi_s\rangle = \mathrm{Tr}e^{-\beta H}\phi_s/\mathrm{Tr}e^{-\beta H} \tag{5}$$

In the case of zero temperature, $\varphi = \varphi_S \neq 0$, the chiral symmetry is spontaneously broken. In a previous work [6], we obtained the order parameter as a function of temperature in the same model by a functional method and found that the chiral symmetry is restored ($\varphi = 0$) when $T \geq T_C = 340$ MeV. Now we study the behavior of the model at finite temperature by the mean-field theory.

According to the standard procedure, the equations of motion for scalar fields can be easily established:

$$(\Box - m^2)\phi_p = -ig\bar{\psi}\gamma_5\psi \tag{6}$$

$$(\Box - m^2)\phi_s = -g\bar{\psi}\psi \tag{7}$$

Taking the Gibbs average of Eq.(6) and Eq.(7) respectively, we obtain the following equations [6]:

$$m^2\varphi_p = ig\langle\bar{\psi}\gamma_5\psi\rangle \tag{8}$$

$$m^2\varphi_s = g\langle\bar{\psi}\psi\rangle \tag{9}$$

In order to calculate the Gibbs averages $\langle\bar{\psi}\gamma_5\psi\rangle$ and $\langle\bar{\psi}\psi\rangle$, it is necessary to solve the following equation of motion for fermion.

$$(\gamma\cdot\partial + ig\gamma_5\phi_p + g\phi_s)\psi = 0 \tag{10}$$

Obviously, it is extremely difficult to acquire an exact solution. However, Walecka showed [4] that, because the quantum correction is not large, if the density is high enough, it is reasonable to replace the scalar field operator with corresponding classical field operator in the equation of motion for fermion. We will extend his method to the case in which temperature is finite.

Replacing the boson field operators in Eq.[10] with the corresponding Gibbs averages, we obtain

$$(\gamma \cdot \partial + ig\gamma_5\varphi_p + g\varphi_s)\psi = 0 \tag{11}$$

This equation can be solved exactly. Applying the operator $(\gamma \cdot \partial + ig\varphi_p\gamma_5 - g\varphi_s)$ on both sides of Eq. (11), we have

$$(\Box - g^2\varphi^2)\psi = 0 \tag{12}$$

So the effective mass of fermion is $g\varphi$. The general solution of Eq.(11) is

$$\psi(x) = \frac{1}{(2\pi)^{3/2}} \int dK \sqrt{\frac{g\varphi_s}{E_K}} \sum_{\sigma=1}^{2} (u_{K\sigma} C_{K\sigma} e^{iK\cdot x} + V_{K\sigma} d^{+}_{K\sigma} e^{-iK\cdot x}) \tag{13}$$

where

$$u_{K\sigma} = \left[\frac{E_K + g\varphi_s}{2g\varphi_s}\right]^{1/2} \begin{pmatrix} \xi_\sigma \\ \dfrac{\sigma \cdot K + ig\varphi_p}{E_K + g\varphi_s} \xi_\sigma \end{pmatrix} \tag{14}$$

$$V_{K\sigma} = \left[\frac{E_K + g\varphi_s}{2g\varphi_s}\right]^{1/2} \begin{pmatrix} \dfrac{\sigma \cdot K - ig\varphi_p}{E_K + g\varphi_s} \xi_\sigma \\ \xi_\sigma \end{pmatrix} \tag{15}$$

$$E_K = (K^2 + g^2\varphi^2)^{1/2} \tag{16}$$

We now calculate the fermion pair condensation $\langle\bar{\psi}\psi\rangle$ in the case that chemical potential is zero. According to Eq.(13) and relations [7]

$$\langle C^{+}_{K\sigma} C_{K\sigma}\rangle = \frac{V}{(2\pi)^3} n_K = \frac{V}{(2\pi)^3} \frac{1}{1 + \exp(\beta E_K)} \tag{17}$$

$$\langle d^{+}_{K\sigma} d_{K\sigma}\rangle = \frac{V}{(2\pi)^3} \bar{n}_K = \frac{V}{(2\pi)^3} \frac{1}{1 + \exp(\beta E_K)} \tag{18}$$

we obtained

$$\begin{aligned} \langle\bar{\psi}\psi\rangle &= \frac{1}{V}\left\langle \int dx \bar{\psi}(x,0)\psi(x,0)\right\rangle \\ &= \frac{1}{V}\int dK \frac{g\varphi_s}{E_K} \sum_{\sigma=1}^{2} [\langle C^{+}_{K\sigma} C_{K\sigma}\rangle + \langle d^{+}_{K\sigma} d_{K\sigma}\rangle] \\ &= \int \frac{dK}{(2\pi)^3} \frac{\gamma g\varphi_s}{E_K} \frac{2}{1 + \exp(\beta E_K)} \end{aligned} \tag{19}$$

where we have used the transnational invariance and neglected the zero point oscillation. In the same way, we get

$$\langle\bar{\psi}\gamma_5\psi\rangle = -i\int \frac{dK}{(2\pi)^3} \frac{\gamma g\varphi_p}{E_K} \frac{2}{1 + \exp(\beta E_K)} \tag{20}$$

Substituting Eqs.(19) and (20) into Eqs.(9) and (8) respectively, we have

$$\varphi_p F(\varphi, \beta) = 0 \tag{21}$$

$$\varphi_s F(\varphi, \beta) = 0 \tag{22}$$

where

$$F(\varphi, \beta) = m^2 - \int \frac{dK}{(2\pi)^3} \frac{\gamma g}{E_K} \frac{2}{1 + \exp(\beta E_K)} \tag{23}$$

and $\gamma(=2)$ is the spin degeneracy factor. φ as a function of temperature is determined by the equation

$$F(\varphi, \beta) = 0 \tag{24}$$

Setting $\varphi = 0$ in Eq.(24), we obtain the critical temperature based on which the chiral symmetry is restored

$$T_C = \sqrt{\frac{12}{\gamma}} \frac{m}{g} = 326\text{MeV} \tag{25}$$

where we have taken $g = 15$ and $m = 2000$ MeV.

We now consider the effect of nonzero chemical potential. In this case

$$n_K = \frac{1}{1 + \exp[\beta(E_K - \mu)]} \tag{26}$$

$$\bar{n}_K = \frac{1}{1 + \exp[\beta(E_K + \mu)]} \tag{27}$$

The equations for φ_P and φ_S can be written as

$$\varphi_p F(\varphi, \beta, \mu) = 0 \tag{28}$$

$$\varphi_s F(\varphi, \beta, \mu) = 0 \tag{29}$$

where

$$F(\varphi, \beta, \mu) = m^2 - \int \frac{dK}{(2\pi)^3} \frac{\gamma g^2}{E_K} \left(\frac{1}{1 + \exp[\beta(E_K - \mu)]} + \frac{1}{1 + \exp[\beta(E_K + \mu)]} \right) \tag{30}$$

Setting $\varphi = 0$ in the equation $F(\varphi, \beta, \mu) = 0$, we obtain an equation for the phase boundary curve ($T_C - \mu_C$ curve)

$$m^2 = \gamma g^2 \int_0^\infty dyy \left(\frac{1}{1 + \exp[\beta_c(y - \mu_c)]} + \frac{1}{1 + \exp[\beta_c(y + \mu_c)]} \right) \tag{31}$$

The resultant phase diagram is shown in Fig. 1.

In conclusion, we have studied the chiral phase transition with the mean-field theory.

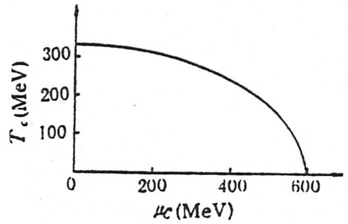

FIG. 1 Critical temperature T_C vs critical chemical potential μ_C.

Although the calculation is very simple, the result is in good agreement with those obtained from much more complex calculation. We, therefore, hold that it is instructive to acquire some qualitative picture of the chiral phase transition by the mean-field theory before more complex tools are used.

REFERENCES

[1] J. Cleymans, R. V. Gavai and E. Schonen. *Phys. Rep.* 130(1986), 217; G, Baym, in: Quark Matter Formatiqn and Heavy Ion Collisions, edited by M. Jacob and H. Staz (World Scientific, Singapore, 1982).
[2] J. Kogut, M. Stone, H. W. Wyld, S. H. Shenker. J. Shigemitsu and D. K. Sinclair, *Nucl. Phys.* 225(FS9)(1983), 326.
[3] A. Kocic, *Phys. Rev.* D33(1986), 1785.
[4] J. D. Walecka, *Ann. Phys.* (N.Y.) 83(1974), 491.
[5] D. Lurie, Particles and Fields (Interscience, New York, 1968), 453.
[6] Liu Baohua and Li Jiarong, *Phys. Rev.* D37(1988), 190.
[7] H. Umezawa and H. Matsumoto, Thermo Field Dynamics and Condensed States, (North-Holland Publishing Company, New York, 1982), 123.