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Variational Calculation of the Glueball Mass in 2 + 1 Dimensional SU(2) LGT

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A variational calculation is made on the glueball mass in 2 + 1 dimensional $SU(2)$ lattice gauge theory by using a Hamiltonian which possesses exact ground state and correct continuum limit. In the range of $1.3 \leq 1/g^2 \leq 7$, a good scaling behavior $am = 2.28\, g^2$ is obtained, which is in agreement with the weak-coupling perturbation theory and the results obtained by another Hamiltonian which does not possess correct continuum limit.

Lattice gauge theory provides us with a renormalization scheme of calculating non-perturbative effects. Recently, we have proposed a lattice Hamiltonian H_1 with exact ground state [1]:

$$H_1 = \frac{g^2}{2a} \exp(-R_1) E_l^a \exp(2R_1) E_l^a \exp(-R_1) \tag{1}$$

where $R_1 = \frac{1}{2g^4 c_N} \sum_p \mathrm{Tr}(U_p + U_p^+)$, c_N is the Casimir invariant of the $SU(N)$ gauge group in the fundamental representation.

We have calculated the glueball masses [2–4] of 2 + 1 dimensional $U(1)$, $SU(2)$, and $SU(3)$ gauge theory, which shows that the Hamiltinian of LGT with exact ground state is effective in studying glueball masses.

There have been some papers [6–11] discussing glueball masses of 2 + 1 dimensional $SU(2)$ theory. The weak coupling perturbation theory shows that the scaling behaviors is $a \sim g^2$, and the

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latest results [10] from Monte Carlo simulation show that the scaling behavior $am = (2.15 \pm 0.2)g^2$ in the range of $4.5 \leq 4/g^2 \leq 5.5$. We have calculated the glueball mass [3] of 2 + 1 dimensional $SU(2)$ theory using H_1, and it shows that the scaling behavior $am = 2.28\, g^2$ can be extended to the deep weak coupling region of $1/g^2 \sim 7$.

However, as the scaling behavior of 2 + 1 dimensional $SU(N)$ theory is $a \sim g^2$ where $g^2 = e^2 a$, e is the charge), for 2 + 1 dimensional $SU(N)$ theory, we cannot show that H_1 possesses the same continuum limit as thatofWilson action (but the difference between them is at most a finite quantity) when $a \to 0$.

Recently, we have also proposed a new from of lattice Hamiltonian with exact ground state, and obtained a Hamiltonian with correct continuum limit in both 2 + 1 and 3 + 1 dimensions [5]. For $SU(2)$ theory, it is:

$$H_2 = \frac{g^2}{2a} e^{-R} E_l^a e^{2R} E_l^a e^{-R} \tag{2}$$

where $R = (x/2)\mathrm{Tr}U_p + (y/2)(\mathrm{Tr}U_p)^2$, $x = -8/5g^4$ and $y = 2/5g^4$.

Its exact ground state is:

$$|\Psi_0\rangle = e^R |0\rangle \tag{3}$$

where $|0\rangle$ is defined by $E_l |0\rangle = 0$.The energy of the ground state is zero.

The lowest excited state of $SU(2)$ gauge field is the static $J^{pc} = 0^{++}$ state. For this excited state, we choose the trial function:

$$\Phi_n(x) = \frac{1}{\sqrt{r}} \mathrm{Tr}\, U_{np}(x) \tag{4}$$

where $U_{np}(x)$ is an $n \times n$ square Wilson loop with a corner located at x, $n = 1, 2, ..., N$ and N is the total number of trial functions.

The glueball masses are determined by solving the eigenvalue equation:

$$\det \| W_{mn} - \lambda D_{mn} \| = 0 \tag{5}$$

where

$$W_{mn} = -\left\langle \sum_{lax} [E_l^a, \Phi_m(0)][E_l^a, \Phi_n(x)] \right\rangle_0 \tag{6}$$

$$D_{mn} = \left\langle \sum_x \Phi_m(0)\Phi_n(x) \right\rangle_0 - \langle \Phi_m(0) \rangle_0 \left\langle \sum_x \Phi_n(x) \right\rangle_0 \tag{7}$$

$$\lambda = 2am\beta = 2am/g^2 \tag{8}$$

For the fundamental representation of $SU(2)$ group, U_p can be parametrized by:

$$U_p = \cos\phi_p + i\sigma \cdot n \sin\phi_p \tag{9}$$

where $\boldsymbol{n} = (\sin\theta_p \cos\phi_p, \sin\theta_p \sin\phi_p, \cos\theta_p)$, $0 \le \theta_p, \psi_p \le \pi$, $0 \le \phi_p \le 2\pi$, and the measure is $d\Omega = \sin^2\psi_p\, d\psi_p \sin\theta_p\, d\theta_p\, d\phi_p/2\pi$. It is easy to obtain:

$$Z = \langle \Psi_0 | \Psi_0 \rangle = \prod_p z \tag{10}$$

$$z = \int e^{2x\cos\psi_p + 4y\cos^2\psi_p} d\Omega$$

$$= \sum_{n=0}^{\infty} \frac{y^n}{n!} \frac{d^{2n}}{dx^{2n}} \int e^{2x\cos\psi_p} d\Omega = \sum_{n=0}^{\infty} \sum_{k=n}^{\infty} z(k,n) \tag{11}$$

$$U \equiv \langle \cos\psi_p \rangle_0 = z^{-1} \sum_{n=0}^{\infty} \sum_{k=n+1}^{\infty} z(k,n)(k-n)/4y$$

$$A \equiv \langle \cos^2\psi_p \rangle_0 = z^{-1} \sum_{n=0}^{\infty} \sum_{k=n}^{\infty} z(k,n) n/4y \tag{12}$$

$$\langle n_i \rangle_0 = 0 \tag{13}$$

$$\langle n_i n_j \rangle_0 = \delta_{ij}/3 \tag{14}$$

where $z(k, n) = (2k)! 4^{2k-2n} y^{2k-n} / [n! k! (k+1)! (2k-2n)!]$
let

$$B_n = \langle \mathrm{Tr}(U_{1p} U_{2p} \cdots U_{np}) \mathrm{Tr}(U_{1p} U_{2p} \cdots U_{np}) \rangle_0 \tag{15}$$

and using the relations $\sigma^a_{ij}\sigma^a_{kk} = 2\delta_{il}\delta_{jk} - \delta_{ij}\delta_{kl}$ and $\mathrm{Tr}U_p = \mathrm{Tr}U_p^+$, we can obtain the recurrence formula:

$$B_n = B_{n-1}(4A-1)/3 + 4(1-A)/3 \tag{16}$$

and $B_1 = \langle \mathrm{Tr}U_p \mathrm{Tr}U_p \rangle_0 = 4A$.

From these, the variational matrix elements can be obtained as: $(n \ge m)$

$$W_{mn} = m(n-m-1)U^{n^2-m^2}(4 - B_{m^2}) + 2\sum_{i=1}^{m} i U^{n^2+m^2-2im}(4 - B_{im}) \tag{17}$$

$$D_{mn} = 4\sum_{ij=1}^{m} (U^{n^2+m^2-2ij} B_{ij} - 4U^{n^2+m^2}) + (n-m-1)^2 (U^{n^2-m^2} B_{m^2} - 4U^{n^2+m^2}) + 4(n-m-1)\sum_{i=1}^{m} (U^{n^2+m^2-2im} B_{im} - 4U^{n^2+m^2}) \tag{18}$$

Solving the eigenvalue equation for N = 1, 2, 3, 4, 5, 8, 15, and 30 respectively, the curves βam vs. $1/g^2$ are obtained and shown in Fig.1. Scaling behavior $am = 2.28\, g^2$ in the interval $1.3 < 1/g^2 < 7$ is observed. It is the same as that of H_1 [3], but the scaling region of βam appears later than that

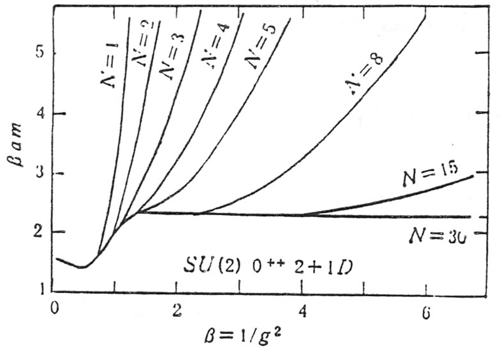

FIGURE 1.

of H_1 (for H_1, the scaling region of βam appears as $1/g^2 > 1$). The difference between the results of two Hamiltonian H_1 and H_2 exists only in the intermediate coupling region. Another calculation of ours shows that the string tension for H_1 and H_2 is different, perhaps the same mass gap for them appears by chance.

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