High Energy Physics and Nuclear Physics

Volume 13, No. 1

A New Method for the Determination of the Spin of $\xi(2230)$*

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In this paper the photon angular distributions for the moments of process $e^+e^- \to J/\psi \to \gamma B(J^\eta)$, $B(J^\eta) \to P_1P_2$ are derived. It provides a new way to determine the spin of $\xi(2230)$.

1. INTRODUCTION

A new state $\xi(2230)$ was reported in the radiative decay $J/\psi \to \gamma K\bar{K}$ by the MARK III group [1]:

$$e^+e^- \to J/\psi \to \gamma\xi \quad \xi \to K^+K^-,\ K_s^0K_s^0, \tag{1}$$

However, Another group, the DM2 group also analyzed the same process without observing significant structure at 2.23 GeV in the $K\bar{K}$ mass spectrum [2].

The MARK III group performed a spin analysis of the $\xi(2230)$ using the maximum likelihood technique [3]. The procedure is similar to that used for the analysis of $\theta/f_2\ \xi(1720)$. However, it was not certain whether the spin of $\xi(2230)$ is $J = 2$ or 4**.

We pointed out in Ref. [4] that in the general helicity formalism there exists an insensitive region in determining the spin ($J = 2$ or 4) of $\theta/f_2(1720)$ while using the angular distribution. Because the data of θ/f_2 just fall into this insensitive region we can not conclude that the spin of is

*Supported by the national natural Science Foundation of China.
Received on May 24, 1988.

**For a system of two pseudoscalar mesons the spin, parity and c-parity of $\xi(2230)$ should be J^{PC} = (even)$^{++}$.

θ/f_2 just 2. So far the indetermination of the $\xi(2230)$ spin is due to insufficient data and the insensitive region into which the existing data are fallen.

This paper attempts to give a new method to determine the spin of ξ(and θ/f_2) by using the generalized formalism of moment analysis [5].

2. ANGULAR DISTRIBUTION

We consider a reaction in which a boson resonance B with spin J and parity η is produced

$$e^+ + e^- \rightarrow J/\psi \rightarrow \gamma + B(J^\eta), \tag{2}$$

the resonance B then decays into two pseudoscalar mesons P_1 and P_2 (η must be +)

$$B(J^+) \rightarrow P_1 + P_2. \tag{3}$$

The angular distribution in a general helicity formalism is given by

$$W_J(\theta_\gamma,\theta,\phi) \propto \sum_{\Lambda\Lambda'} I(\lambda_J,\lambda_J')A_{\lambda_\gamma\Lambda}A_{\lambda_\gamma\Lambda'}D^{J*}_{-\Lambda,0}(\phi,\theta,0)D^{J}_{-\Lambda',0}(\phi,\theta,0), \tag{4}$$

where

$$A_{\lambda_\gamma\Lambda} \sim \langle \gamma_{\lambda_\gamma} B_\Lambda | T | \psi_{\lambda_J} \rangle \tag{5}$$

is the helicity amplitude and λ_γ, Λ and λ_J are the helicities of the photon, B, and J/ψ respectively;

$$I(\lambda_J,\lambda_J') \propto \frac{1}{4} \sum_{rr'} \langle \psi_{\lambda_J} | T | e_r^+ e_{r'}^- \rangle \langle \psi_{\lambda_J'} | T | e_r^+ e_{r'}^- \rangle^*, \tag{6}$$

and (θ, ϕ) describes the direction of the momentum of particle P_1 in the rest frame of B.

We choose a coordinate system in which the z axis is parallel to the photon direction and e^+e^- beams are in the x-z plane. Since $p \ll m_e$ we obtain easily in the rest frame of J/ψ.

$$\begin{aligned}
&I(1,1) = I(-1,-1) \approx p^2(1+\cos^2\theta_\gamma) \\
&I(1,0) = I(0,1) = -I(-1,0) = -I(0,-1) \approx \frac{1}{\sqrt{2}} p^2 \sin^2\theta_\gamma, \\
&I(1,-1) = I(-1,1) \approx p^2\sin^2\theta_\gamma, \\
&I(0,0) \approx 2p^2 \sin^2\theta_\gamma,
\end{aligned} \tag{7}$$

where $p = |p_+| = |p_-|$, p_+ and p_- are momenta of the positron and he electron respectively, θ_γ is the angle between the photon and positron beam.

3. GENERALIZED MOMENT ANALYSIS

Decay modes for example $B(J^\eta) \rightarrow B_1(1^-) + B_2(1^-)$, where B_1 and B_2 mesons in turn decay into 2 pseudoscalar mesons (p_1p_2 and p_3p_4) were studied in Ref. [5], where the corresponding moments and linear relations among different moments for definite spin-parity combinations of the parent bosons were given. According to these relations we can determine the spin and parity of B. We note that some relations are very effective. However, we know that the observed decay modes of ξ (and θ/f_2) are just two pseudoscalar mesons channels. Therefore we cannot use all relations of Ref. [5] to determine the spin of $\xi(2230)$ (and θ/f_2).

We generalize the moment analysis for the process

$$e^+e^- \to J/\psi \to \gamma + B(J^\eta) \quad \lfloor\!\longrightarrow P_1 + P_2. \tag{8}$$

Introduce the photon angular distribution for the moments defined as follows

$$H_J(\theta_\gamma, LM) = \int W_J(\theta_\gamma, \theta, \phi) D^L_{M0}(\phi, \theta, 0) \sin\theta d\theta d\phi. \tag{9}$$

This is an experimentally measurable quantity. From Eq.(4) we have

$$\begin{aligned} H_J(\theta_\gamma, LM) &= \frac{4\pi}{2J+1} \sum_{\Lambda\Lambda'} I(\lambda_J, \lambda_J') A_{\lambda_\gamma\Lambda} A_{\lambda_\gamma\Lambda'} (J - \Lambda' LM \mid J - \Lambda) \\ &\quad \cdot (J0L0 \mid J0) \\ &= \frac{4\pi}{2J+1} t^{M*}_{J,L}(\theta_\gamma)(J0L0 \mid J0), \end{aligned} \tag{10}$$

where we use the notation $j_1m_1j_2m_2 \mid j_3m_3$ for the usual Clebsch-Gordan coefficients and the multipole parameter is given by

$$t^{M*}_{J,L}(\theta_\gamma) = \sum_{\Lambda\Lambda'} I(\lambda_J, \lambda_J') A_{\lambda_\gamma\Lambda} A_{\lambda_\gamma\Lambda'} (J - \Lambda' LM \mid J - \Lambda). \tag{11}$$

We take L to be even so that $t^{M*}_{J,L}(\theta_\gamma)$ is real. We can express $H_J(\theta_\gamma, LM)$ in terms of Eq.(7) and the helicity amplitudes ratios x, y. In the case of $L = 2$, we have

$$\begin{aligned} H_2(\theta_\gamma, 22) &= H_2(\theta_\gamma, 2-2) \propto -\frac{16\pi}{35} p^2 y \sin^2\theta_\gamma, \\ H_2(\theta_\gamma, 21) &= -H_2(\theta_\gamma, 2-1) \propto -\frac{4\sqrt{2}\pi}{35} p^2(x - \sqrt{6}xy) \sin 2\theta_\gamma, \\ H_2(\theta_\gamma, 20) &\propto \frac{16\pi}{35} p^2[x^2 \sin^2\theta_\gamma + (1 - y^2)(1 + \cos^2\theta_\gamma)] \sim 1 + A_1 \cos^2\theta_\gamma, \end{aligned} \tag{12}$$

$$A_1 = \frac{1 - y^2 - x^2}{1 - y^2 + x^2}; \tag{13}$$

$$\begin{aligned} H_4(\theta_\gamma, 22) &= H_4(\theta_\gamma, 2-2) \propto -\frac{16\sqrt{15}\pi}{231} p^2 y \sin^2\theta_\gamma, \\ H_4(\theta_\gamma, 21) &= -H_4(\theta_\gamma, 2-1) \propto -\frac{8\sqrt{15}\pi}{693} p^2 \left(x - \frac{9}{\sqrt{10}} xy\right) \sin 2\theta_\gamma, \end{aligned}$$

$$H_4(\theta_\gamma, 20) \propto \frac{272\pi}{693} p^2 \left[x^2 \sin^2\theta_\gamma + \frac{10}{17}(1 + 0.4y^2)(1 + \cos^2\theta_\gamma)\right]$$
$$\sim 1 + A_2 \cos^2\theta_\gamma, \tag{14}$$

$$A_2 = \frac{\frac{10}{17}(1 + 0.4y^2) - x^2}{\frac{10}{17}(1 + 0.4y^2) + x^2}. \tag{15}$$

We note that $H_2(\theta_\gamma, 20)$ and $H_4(\theta_\gamma, 20)$ which are comparable with the photon angular distribution in the usual helicity formalism, are different, where the photon angular distributions are the same in spite $J = 2$ or 4:

$$W_J(\theta_\gamma) \sim 1 + A\cos^2\theta_\gamma$$
$$A = \frac{1 + y^2 - 2x^2}{1 + y^2 + 2x^2}. \tag{16}$$

For $L = 4$ we have

$$H_2(\theta_\gamma, 40) \propto -\frac{64\pi}{105} p^2 \left[x^2 \sin^2\theta_\gamma - 0.75\left(1 + \frac{1}{6}y^2\right)(1 + \cos^2\theta_\gamma)\right]$$
$$\sim -(1 + A_3 \cos^2\theta_\gamma), \tag{17}$$

$$A_3 = \frac{-0.75\left(1 + \frac{1}{6}y^2\right) - x^2}{-0.75\left(1 + \frac{1}{6}y^2\right) + x^2};$$

$$H_4(\theta_\gamma, 40) \propto \frac{144\pi}{1001} p^2 \left[x^2 \sin^2\theta_\gamma + \left(1 - \frac{11}{18}y^2\right)(1 + \cos^2\theta_\gamma)\right] \tag{18}$$

$$\sim 1 + A_4 \cos^2\theta_\gamma, \tag{19}$$

$$A_4 = \frac{\left(1 - \frac{11}{18}y^2\right) - x^2}{\left(1 - \frac{11}{18}y^2\right) + x^2}. \tag{20}$$

The difference between $H_2(\theta_\gamma, 40)$ and $H_4(\theta_\gamma, 40)$ is more conspicuous. Therefore we can use these photon angular distributions for the moments to determine the spin of ξ(and θ/f_2).

4. CONCLUSION

We know that the helicity amplitude ration x and y for ξ and θ/f_2 production in the radiative decay of J/ψ are Ref. [6]

$$\begin{array}{lll} \xi(J=2) & x=-0.67\pm\begin{array}{c}0.14\\0.16\end{array} & y=0.13\pm\begin{array}{c}0.21\\0.19\end{array} \\ \xi(J=4) & x=1.29\pm\begin{array}{c}0.62\\0.30\end{array} & y=0.4\pm\begin{array}{c}0.76\\0.39\end{array} \\ \theta/f_2 & x=-1.07\pm0.16 & y=-1.09\pm0.15. \end{array} \tag{21}$$

Using these values for x and y (omitting the errors) from Eqs.(13),(15),(18) and (20) we have

	A_1	A_2	A_3	A_4
$\xi(J=2)$	0.373		3.96	
$\xi(J=4)$		-0.453		-0.297
θ/f_2	-1.39	-0.138	-8.29	-0.614.

Applying Eqs.(12),(14), (17) and (19) the photon angular distributions for the moments are calculated and shown in Fig.1, 2, 3, and 4. For $\theta/f_2(1720)$, as shown in Fig.2, the behaviors of $H_2(\theta_\gamma, 40)$ and $H_4(\theta_\gamma, 40)$ are very different, this means the photon angular distributions for these moments are very sensitive for different spin values (J = 2 and 4) of θ/f_2. For $\xi(2230)$ the behaviors of these angular distributions are very different too (see Fig.3, 4). We expect that using this generalized moment analysis to reexamine there data on processes $J/\psi\rightarrow\gamma K\bar{K}$, $J/\psi\rightarrow\gamma\pi\pi$ and $J/\psi\rightarrow\gamma\eta\eta$ experimentalist might obtain some valuable results. We hope also a definite conclusion for the spin of $\xi(2230)$ can be made soon. Moreover, after determining the spin of $\xi(2230)$ (and θ/f_2) we can fit the photon angular distributions for the moments to get more precise values of the helicity amplitudes ratios x and y.

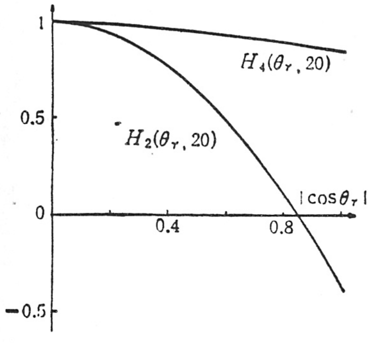

FIGURE 1 The photon angular distributions for the moments of
$e^+ + e^- \rightarrow J/\psi \rightarrow \gamma\theta/f_2$, $\hookrightarrow P_1P_2$
(JLM) = (220) , (420) .

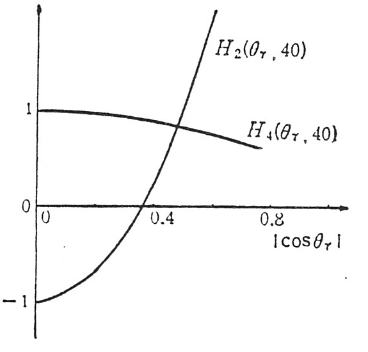

FIGURE 2 The photon angular distributions for the moments of
$e^+ + e^- \rightarrow J/\psi \rightarrow \gamma\theta/f_2$ $\hookrightarrow P_1P_2$
(JLM) = (240), (440).

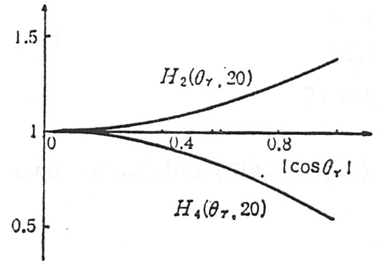

FIGURE 3 The photon angular distributions for the moments of
$e^+ + e^- \to J/\psi \to \gamma\xi$,
$\quad\quad\quad\quad\quad\quad\quad\quad\ \ \lfloor\!\to P_1P_2$
$H_J(\theta_\gamma, LM)$, (JLM) = (220), (420)

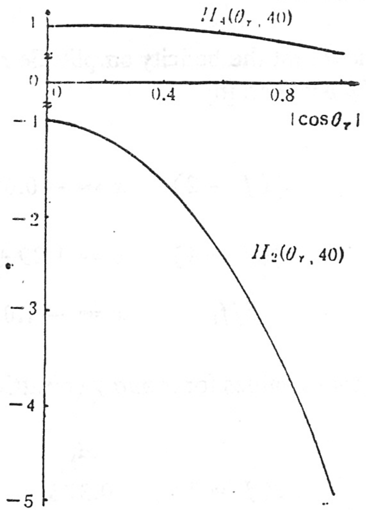

FIGURE 4 The photon angular distributions for the moments of
$e^+ + e^- \to J/\psi \to \gamma\xi$,
$\quad\quad\quad\quad\quad\quad\quad\quad\ \ \lfloor\!\to P_1P_2$
$H_J(\theta_\gamma, LM)$, (JLM) = (240), (440)

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